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Z_2 DEFINITION

Group with 2 elements $\{e \quad g\}$

$$ge = eg = g \qquad g^2 = e$$

MULT. TABLE

	e	g
e	e	g
g	g	e

$$\Rightarrow R(e) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbb{1}$$

$$\Rightarrow R(g) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_x$$

GROUP ALGEBRA $:- \mathbb{C}^2$

$$|e\rangle = |0\rangle \qquad |g\rangle = |1\rangle$$

$$R(e)\,|0\rangle = |0\rangle \qquad R(e)\,|1\rangle = |1\rangle$$

$$R(g)\,|0\rangle = |1\rangle \qquad R(g)\,|1\rangle = 0$$

IRREDUCIBLE REPRESENTATION

$$W R(g) W^{+} = \begin{pmatrix} \boxed{1} & 0 \\ 0 & \boxed{-1} \end{pmatrix} = \sigma_z = \begin{matrix} |+\rangle\langle+| \\ - |-\rangle\langle-| \end{matrix}$$

$$W: \quad \frac{|0\rangle + |1\rangle}{\sqrt{2}} \to |+\rangle$$

$$\frac{|0\rangle - |1\rangle}{\sqrt{2}} \to |-\rangle$$

" ELECTRIC BASIS"

SYMMETRIC ENTANGLERS

Def : $U : \sum_g |g\rangle\langle g| \otimes R(g)$

Ex 1) $R(g) = \ominus$

$$U_\ominus = |0\rangle\langle 0| \otimes 1 + |1\rangle\langle 1| \otimes -1 = \sigma_z$$

Ex 2) $R(g) = \mathbb{C}(G)$

$$U_R = |0\rangle\langle 0| \otimes \mathbb{1} + |1\rangle\langle 1| \otimes \sigma_x = \text{CNOT} \longrightarrow$$

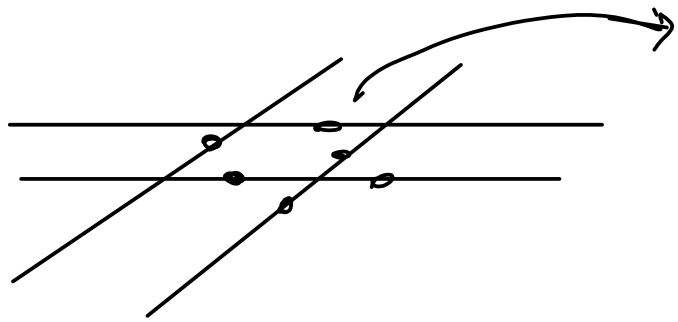
IN THE ELECTRIC BASIS

$$W U_- W^\dagger = |+\rangle\langle -| \otimes 1 + |-\rangle\langle +| \otimes 1 = \sigma_x$$

$$W U_R W^\dagger = \mathbb{1} \otimes P^+ + \sigma_x \otimes P^-$$

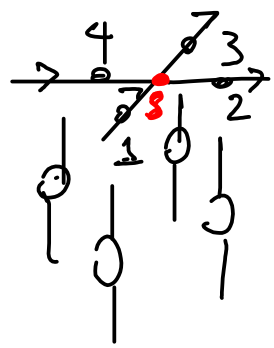
$\longleftarrow$ CNOT

THE $\mathbb{Z}_2$ LOCAL SYMMETRY



$$\mathbb{C}^2 = \{|0\rangle, |1\rangle\}$$

LOCAL ROTATION:



$$|g_1\, g_2\, g_3\, g_4\rangle$$

$$\downarrow \; h$$

$$|g_1 h^{-1} \;\; h g_2 \;\; h g_3 \;\; g_4 h^{-1}\rangle$$

$$R_1^R(h^{-1}) \otimes R_2^L(h) \otimes R_3^L(h) \otimes R_4^R(h^{-1}) = A^s_h$$

Ex:

$$A^s(e) = \mathbb{1}_{s_1} \otimes \mathbb{1}_{s_2} \otimes \mathbb{1}_{s_3} \otimes \mathbb{1}_{s_4}$$

$$A^s(g) = A^s = \sigma^x_{s_1} \otimes \sigma^x_{s_2} \otimes \sigma^x_{s_3} \otimes \sigma^x_{s_4}$$

and in the "ELECTRIC BASIS"

$$A(g) = \sigma^z_{s_1} \otimes \sigma^z_{s_2} \otimes \sigma^z_{s_3} \otimes \sigma^z_{s_4}$$

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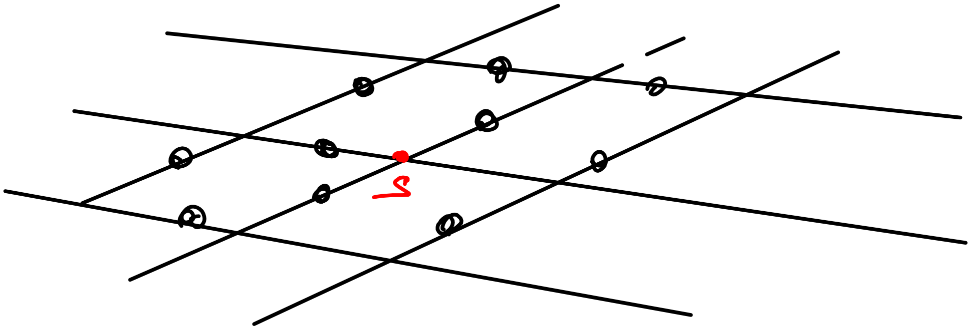
GAUGE INVARIANT HILBERT SPACE



$$|\psi\rangle \in \left(\mathbb{C}^d\right)^{\otimes N}$$

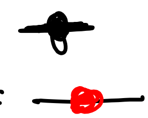
$$A_s|\psi\rangle = |\psi\rangle$$

$\forall s$

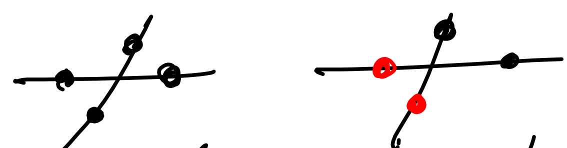
What does it mean?

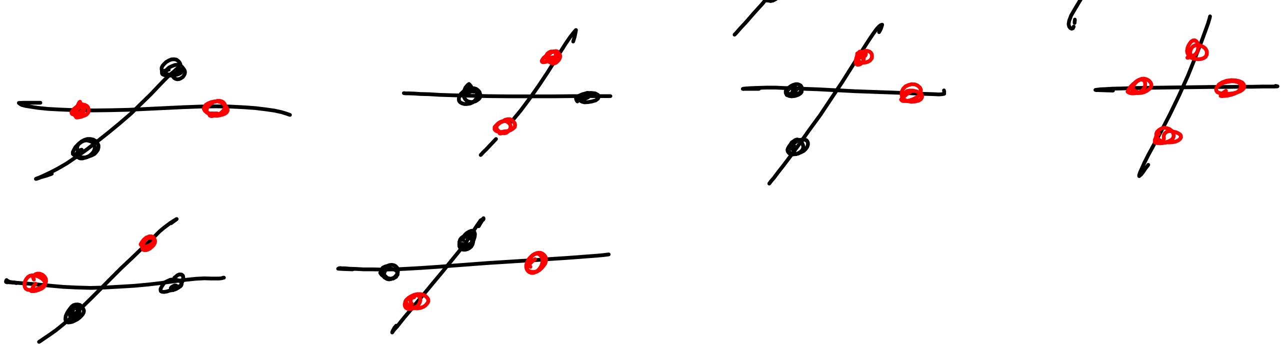
In the electric base

$|+\rangle =$ $|-\rangle =$



$$A_s|\psi\rangle = |\psi\rangle$$





Among the 2^4 possible configurations

restricts to 2^3

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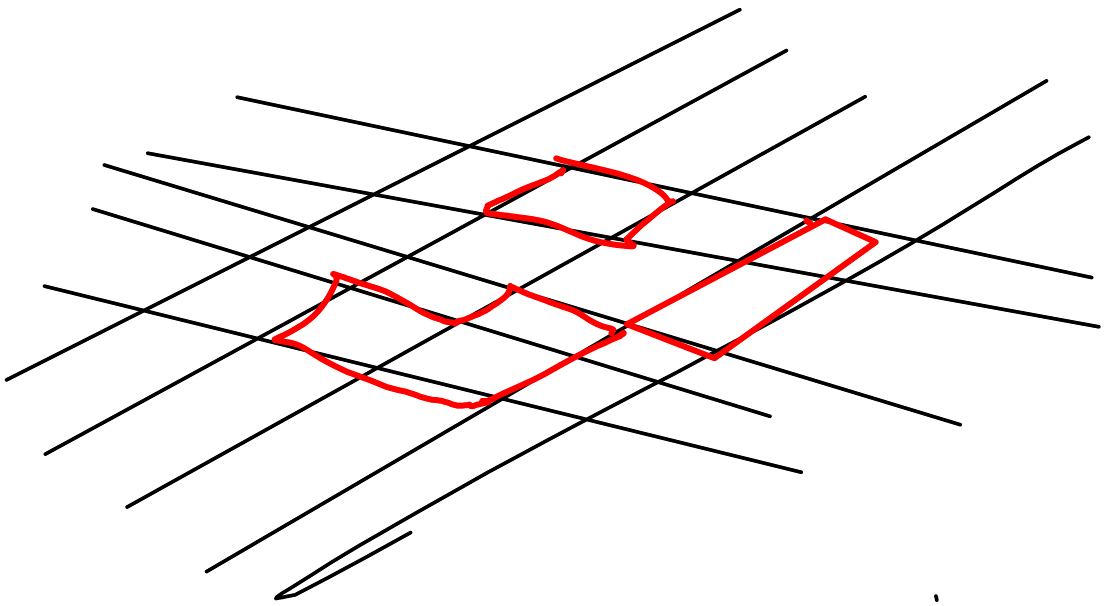
PHYSICAL INTERPRETATION



" $\sigma_x^i = e^{i E^i}$ "

$$\downarrow$$

$$\sigma_x^{s_1}\otimes\sigma_x^{s_2}\otimes\sigma_x^{s_3}\otimes\sigma_x^{s_4} = e^{i\left(E^1+E^2+E^3+E^4\right)}$$

$$= e^{i \nabla_{\mathrm{mod}\, 2} E}$$

$$A_s|\psi\rangle = |\psi\rangle \quad \Rightarrow \quad A_s = \mathbb{1}$$

$$\Rightarrow \boxed{\nabla\cdot E = 0}$$

"GAUSS LAW" $\nabla E = \rho$

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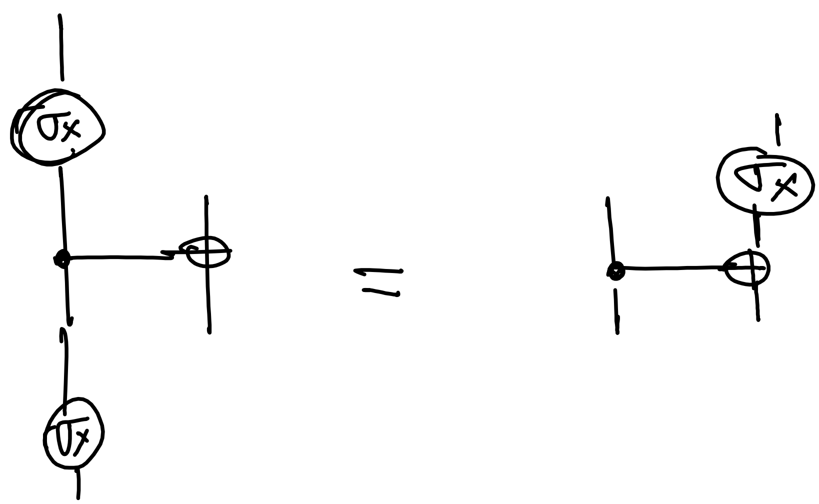
COMMUTATION RELATIONS

$$R(g)_1 U_{[1,2]} R^{\dagger}(g)_1 = \sum_h |gh\rangle\langle gh|_1 \otimes R_2(h)$$

$$= \sum_{h'} |h'\rangle\langle h'| \otimes R(g^{-1}h')$$

$$= R(g^{-1})_2 \, U_{[1,2]}$$

Ex)



HAMILTONIAN

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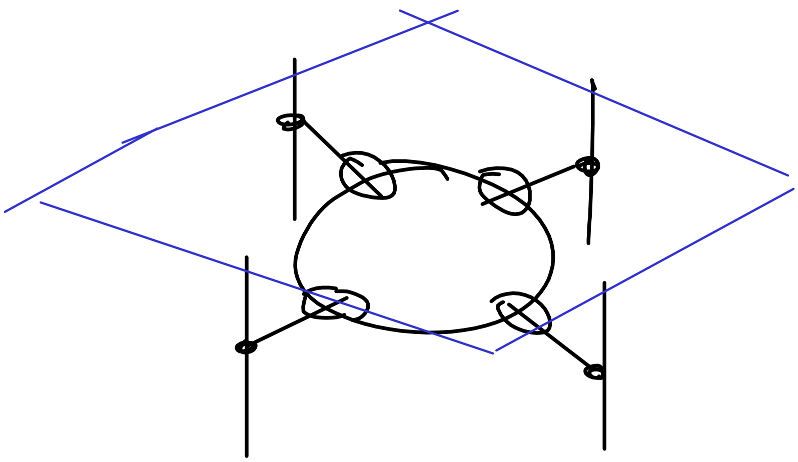
$$[H, A_s] = 0 \qquad \forall s$$

1) σ_x : $A_s = \sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3 \otimes \sigma_x^4$

2) $t_2^2\, U_{p_1} U_{p_2} U_{p_3} U_{p_4} = B_p$



$|0000) \rightarrow 2\,|0000)$

$|0110) \rightarrow 2\,|0110)$ ....

$|0100) \rightarrow 0$

$$H = \frac{1}{g^2} \sum_p B_p + \sum_s \sigma_x^s$$

PHASE DIAGRAM

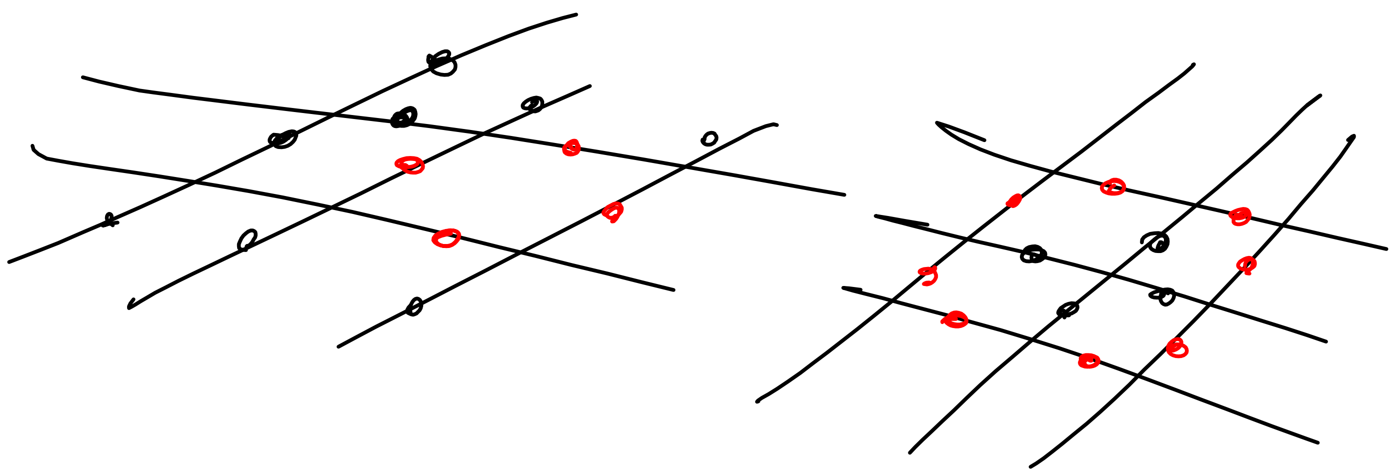
$$H = -\frac{1}{g^2}\sum_p B_p - \sum_s \sigma_x$$

$g \to \infty \Rightarrow |\Omega\rangle = \prod_i |+\rangle^i$

$g \to 0$

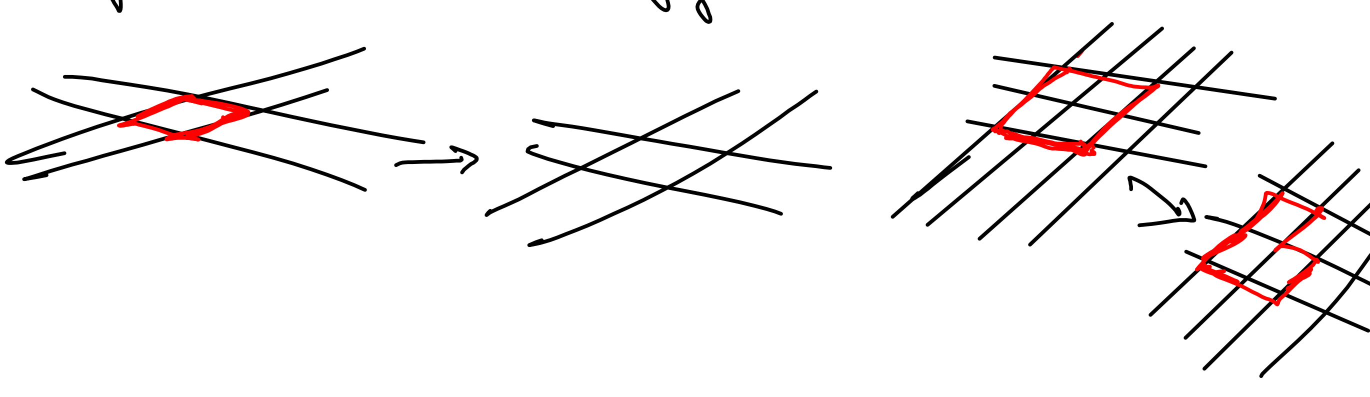
Bp select the gauge invariant states

In the electric basis $\mathbb{1} + \sigma_x^{\otimes 4}$



CLOSED LOOP OF ELECTRIC FLUXES

Bp mixes all configurations



WHAT ABOUT INTERMEDIATE REGIMES $g \sim 1$

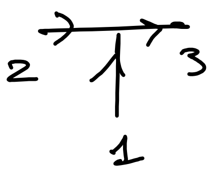
$$H = \frac{1}{g^2} \sum_p B_p + \sum_s \sigma_x^s$$

VARIATIONAL ANSATZ in G. inv. H. Space

1) ground STATE at $g \to 0$

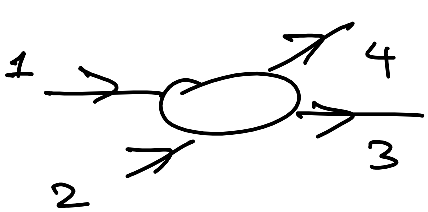
Use 2 tensors

C :

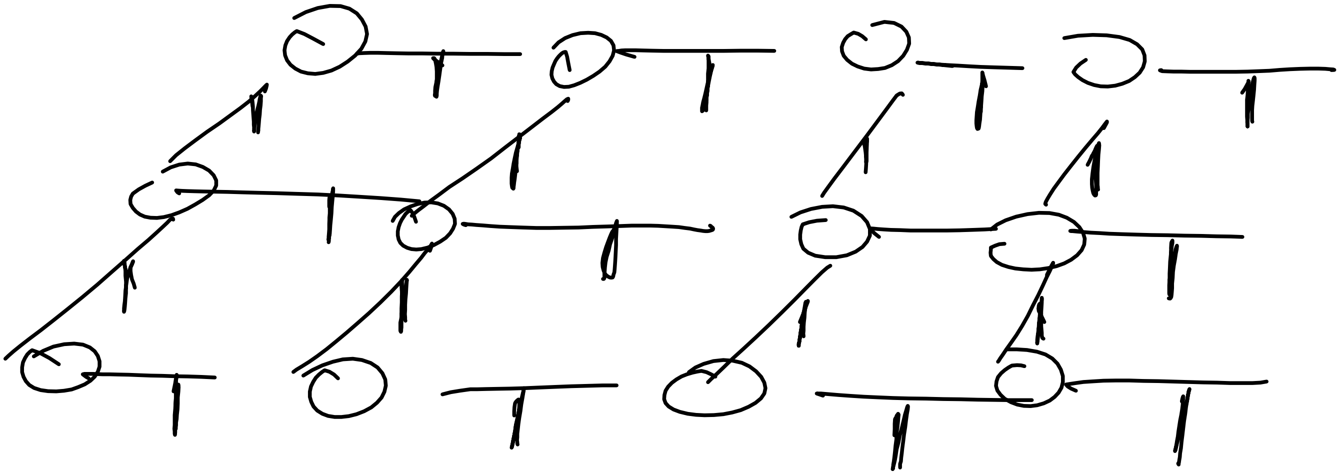


$$= |+\rangle\langle+|^{1\,2} \otimes \langle+|^{3} + |-\rangle\langle-|^{1\,2} \otimes \langle-|^{3}$$

G :



$$= |++\rangle\langle++|^{12\,34} + |++\rangle\langle--| + |+-\rangle\langle+-| + |-+\rangle\langle-+| + |+-\rangle\langle-+| + |-+\rangle\langle+-| + |--\rangle\langle++| + |--\rangle\langle--|$$

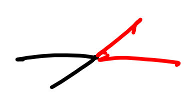


IN GENERAL DECORATED TENSOR NETWORK

$$C: \quad \mathbb{C}^2 \longrightarrow \left(\mathbb{C}^{x_0} \otimes \mathbb{C}^{x_0}\right) \oplus \mathbb{C}^{x_1} \otimes \mathbb{C}^{x_1}$$

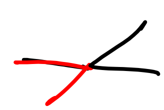
$$C^{(0\alpha),(0\beta),0} \, |(0\alpha)\rangle\langle(0\beta)| \, \langle 0| \; +$$
$$C^{(1\alpha),(1\beta),1} \, |(1\alpha)\rangle\langle(1\beta)| \, \langle 1|$$

$$\mathcal{G}: \quad \mathbb{C}^{x_0} \otimes \mathbb{C}^{x_0} \to \mathbb{C}^{x_1} \otimes \mathbb{C}^{x_1}$$



$$\oplus$$

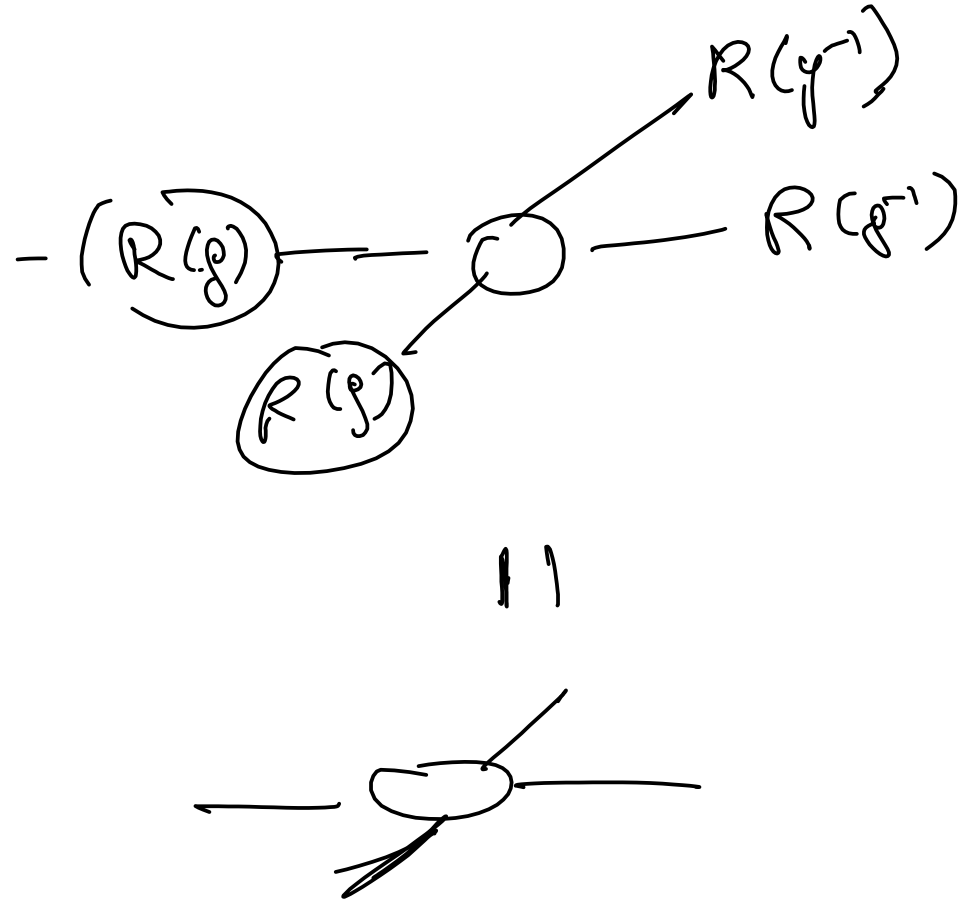
$$\mathbb{C}^{x_1} \otimes \mathbb{C}^{x_1} \to \mathbb{C}^{x_0} \otimes \mathbb{C}^{x_0}$$



$$\oplus$$

$$\vdots$$

$\mathcal{G}$:



$\mathcal{G}$:

$\mathbb{Z}_2$ EVEN TENSOR