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SYMMETRIES and SUPERPOSITIONS

HAMILTONIANS with GLOBAL DISCRETE SYMMETRIES,
can have DEGENERATE GROUND STATEs (few)

Symmetric perturbation
only splits exponentially little in system size degenerate ground state

Non symmetric perturbation splits linearly degenerate-ground state

SYMMETRY BREAKING, superposition is FRAGILE

HAMILTONIAN with LOCAL DISCRETE SYMMETRIES can

CAN HAVE HIGHLY degenerate ground state

Symmetric perturbation
splits independently of the system size

Non-symmetric perturbation
splits finite value degenerate ground state

NO SYMMETRY BREAKING, superposition ROBUST

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ON SPONTANEOUS SYMMETRY BREAKING

$$H = \sum_i \sigma_z^i \sigma_z^{i+1} \quad ; \quad G = \prod_i \sigma_x^i$$

$$\sigma_z |\uparrow\rangle = |\uparrow\rangle \qquad \sigma_z |\downarrow\rangle = -|\downarrow\rangle$$

$$[H, G] = 0$$

$$H' = H - \varepsilon_G G$$

Has two eigenvectors

1) $|\Omega_1\rangle = |\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\downarrow\rangle \; ; \; E_1 = N - \varepsilon_G$

2) $|\Omega_2\rangle \quad |\downarrow\downarrow\downarrow\downarrow\rangle - |\uparrow\uparrow\uparrow\uparrow\rangle \; ; \; E_2 = N + \varepsilon_G$

$$\Delta = 2\varepsilon_G .$$

The same eigenvectors are selected by using

$$T = \varepsilon_T \sum_i \sigma_x^i$$

$$[T, G] = 0$$

$$\left(\langle\uparrow|^{\otimes N}\right) T^N \left(|\downarrow\rangle^{\otimes N}\right) = 1$$

$$\Downarrow$$

Producing the same effect H^T

$$\begin{array}{c|cc} & |\uparrow\rangle^{\otimes N} & |\downarrow\rangle^{\otimes N} \\ \hline (|\uparrow\rangle)^N & 0 & \varepsilon_T^N \\ (|\downarrow\rangle)^N & \varepsilon_T^N & 0 \end{array}$$

$$H^T = \sum_i \sigma_z^i \sigma_z^{i+1} + \varepsilon_T \sum_i \sigma_x^i$$

HOW STABLE is the SUPERPOSTION WITH respect to a small perturbation?

$$H^{T,h} = H^T + h \sum_i \sigma_z^i$$

Notice that

$$[\sigma_z , G] \neq 0$$
$$[\sigma_z , T] \neq 0$$

If we work at finite ε_T the gap is $(\varepsilon_T)^L$

We define the magnetization as

$$M = \frac{\partial}{\partial h} \langle \psi | H^{T,h} | \psi \rangle \Big|_{h=0}$$

what state do we use?

If $h < \varepsilon^L$ $\Rightarrow$ Unique ground state $\Rightarrow$ and

$$O = \left(\frac{\langle 0|^N \mp \langle 1|^N}{\sqrt{2}} \right) \cdot M \left(\frac{|0\rangle^N \mp |1\rangle^N}{\sqrt{2}} \right)$$

But we are interested in the thermodynamic limit,

Even if for small L $\epsilon^{L} > W$ the gap is closing exponentially and we need for large enou
L to use again the degenerate Perturbation theory

Now the degeneracy get lifted as

$$|S_1\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\rangle \to L + WL$$
$$|S_2\rangle \quad |\downarrow\downarrow\downarrow\downarrow\downarrow\rangle \qquad L - WL$$

We compute M on the appropriate ground state

$$(\langle\uparrow|)^N \epsilon_T \sum \sigma^i_x |\uparrow\rangle^N \sim \epsilon_T^2$$

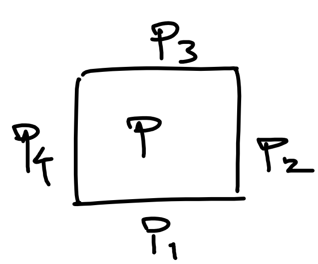
that does not depend on h

$$\frac{\partial}{\partial h}\langle\psi|M|\psi\rangle\Big|_{h=0} = \cancel{\epsilon N} + \cancel{\epsilon} N + \epsilon_T^2 - N$$

We have a FINITE MAGNETIZATION $\Rightarrow$ SPONTANEOUS SYMMETRY BREAKING

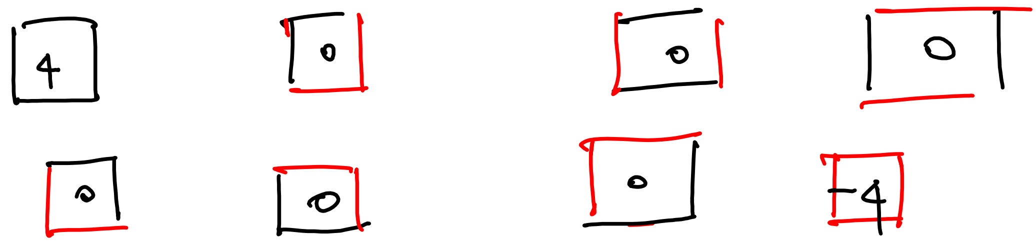
GAUGE THEORY, LOCAL SYMMETRY

$$H = \sum_P \sigma_z^{P_1}\sigma_z^{P_2}\sigma_z^{P_3}\sigma_z^{P_4}$$



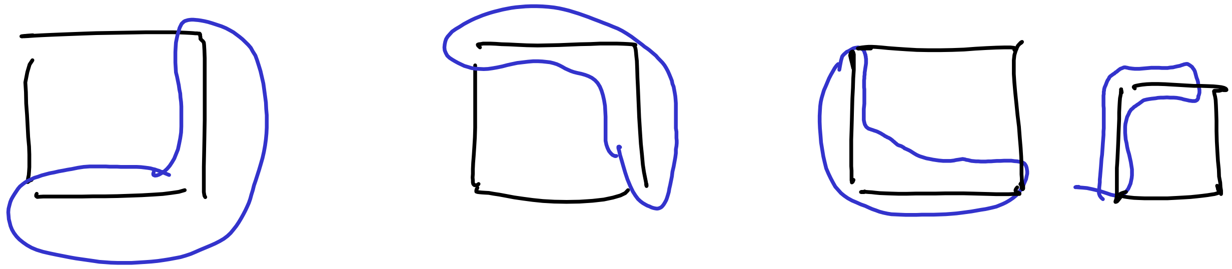
Now we see that H has a huge degeneracy $\{|\Omega_i\rangle\}$

$|\uparrow\uparrow\uparrow\uparrow\rangle \quad |\downarrow\downarrow\uparrow\uparrow\rangle \quad \ldots$



There are several operators (LOCAL) that commute with H

$\sigma_x^{P_1}\sigma_x^{P_2} \qquad \sigma_x^{P_2}\sigma_x^{P_3} \qquad \sigma_x^{P_1}\sigma_x^{P_4}$



We can add these operators to the Hamiltonian

$$H + \varepsilon_G \sum_i G^i$$

$$[H, G^i] = 0 \qquad \forall i$$

The ground state becomes the uniform superpostion of the degenerate ground states

$$|\Omega_G\rangle = \frac{1}{\sqrt{\#G}} \sum_i |\Omega_i\rangle$$

Rather than putting the symmetry operators we could have added a transverse fiedl T

$$H = H + \varepsilon_T \sum_i \sigma_x^i$$

The ground state is the same than above

$$[H, G] = 0$$

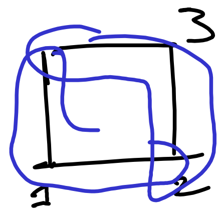
$$[G, T] = 0$$

$$[H, T] \neq 0$$

IMPORTANT OBSERVATION

$$G_1 |\psi_G\rangle = |\psi_G\rangle$$

$$G_3 |\psi_G\rangle = |\psi_G\rangle = G_3 G_1 |\psi_G\rangle$$



LOCAL SYMMETRY $\longrightarrow$ GLOBAL SYMMETRY

Now differently from the globally symmetric case the splitting

$$|\Omega_T\rangle = E_0 + \varepsilon_T^2$$

does not depend on the size

In order to address the stability of the superposition

$$H_M = H + \varepsilon_T T + h M$$

$$M = \sum_i \sigma_z^i \qquad [M, G] \neq 0$$

and

$$\langle \Omega_G | M | \Omega_G \rangle = 0$$

Since the ground state is symmetric

$$E_h = E_G + O(h^2)$$

NO SYMMETRY BREAKING