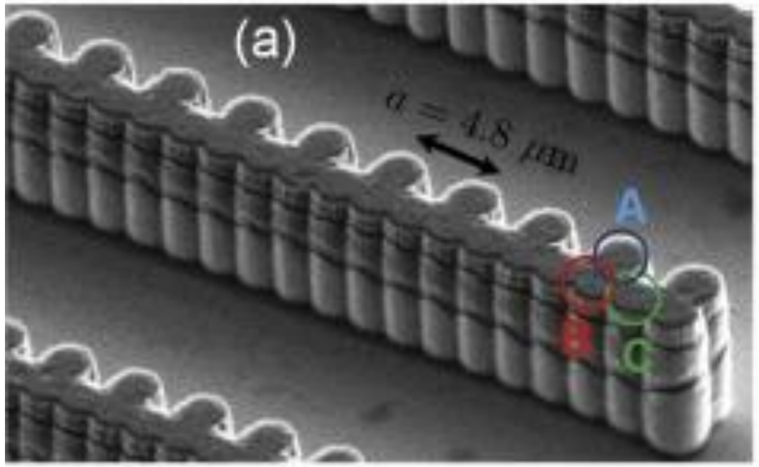
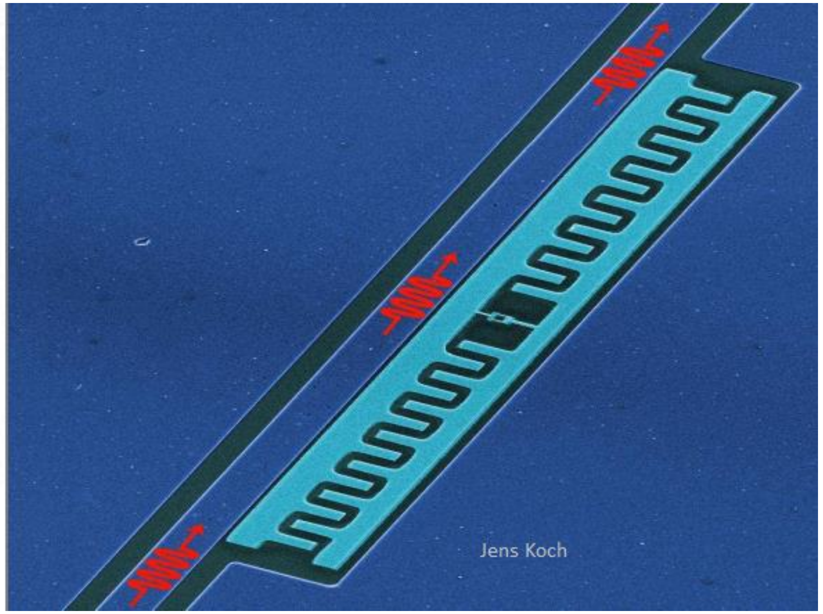
Gaussian trajectory approach to dissipative phase transitions: the case of quadratically-driven photonic lattices

Wouter Verstraelen, Michiel Wouters, Riccardo Rota, Vincenzo savona



A. Amo & J.Bloch, CRR 17(8), 2016

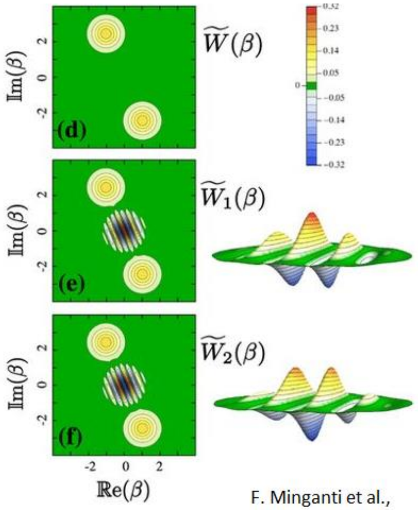


Bose-Hubbard lattice with two-photon driving and single-photon losses

- $\mathbb{Z}_2$-symmetry $\{\hat{a}_i\} \leftrightarrow \{-\hat{a}_i\}, \quad \{\hat{a}^\dagger{}_i\} \leftrightarrow \{-\hat{a}^\dagger{}_i\}$

$$\hat{H} = \sum_i -\Delta\, \hat{a}^\dagger{}_i\, \hat{a}_i + \frac{U}{2}\, \hat{a}^{\dagger}{}_i^2\, \hat{a}_i^2 + \frac{G}{2}\, \hat{a}^{\dagger}{}_i^2 + \frac{G^*}{2}\, \hat{a}_i^2 - \sum_{\langle ij \rangle} \frac{J}{z} (\hat{a}^\dagger{}_i\, \hat{a}_j + \hat{a}^\dagger{}_j\, \hat{a}_i), \qquad (1)$$

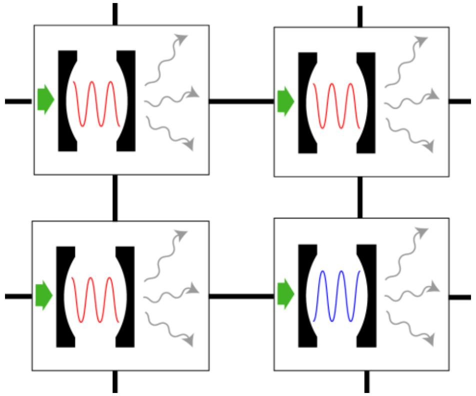
$$\hat{\mathcal{L}} = -i\left[\hat{H}, \cdot\right] + \gamma \sum_j \hat{a}_j \cdot \hat{a}^\dagger{}_j - \frac{\gamma}{2}\{\hat{a}^\dagger{}_j\, \hat{a}_j, \cdot\}. \qquad (2)$$



F. Minganti et al., SciRep 6:26987(2016)

Single site *i*: state can be written in terms of 'spin states'

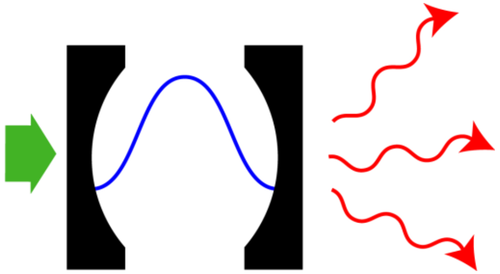
- $|\alpha_0\rangle_i, |-\alpha_0\rangle_i \sim |\leftarrow\rangle_i, |\rightarrow\rangle_i$ $(\alpha_i = \langle \hat{a}_i \rangle \neq 0)$
- $\left|\mathcal{C}^{(\pm)}_{\alpha_0,i}\right\rangle \propto |\alpha_0\rangle \pm |-\alpha_0\rangle \sim |\downarrow\rangle_i, |\uparrow\rangle_i.$ $\left(\Pi = \left\langle e^{i\pi\, \hat{a}^\dagger\, \hat{a}} \right\rangle \neq 0\right)$

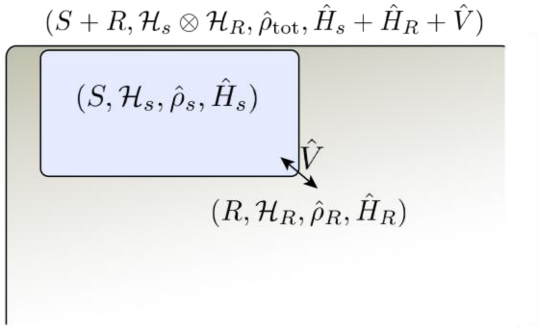


- Extended system ~ Spin lattice

- The vacuum is a $|\mathcal{C}^{(+)}\rangle$state

- Spontaneous breaking of $\mathbb{Z}_2$-symmetry at large *G*
  [VS, PRA **96**, *033826 (2017)]*



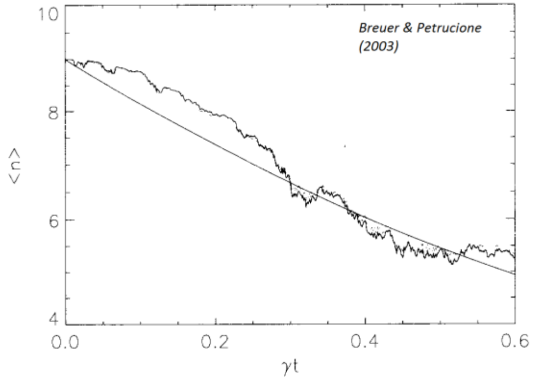


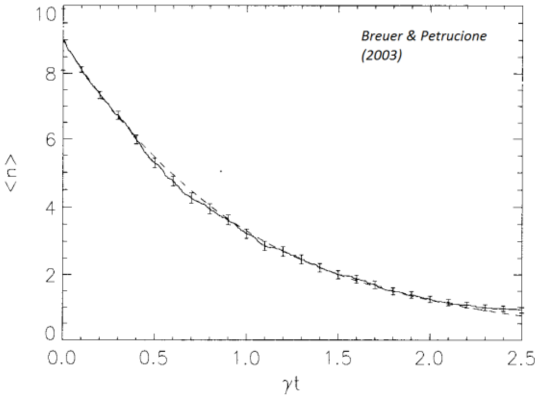
Losses lead to a (Markovian) open quantum system.

Described by

- Master equation $\partial_t \hat{\rho} = \hat{\mathcal{L}}\, \hat{\rho}$
- Stochastic quantum trajectories
  - E.g. Heterodyne unraveling

$$\mathrm{d}|\widetilde{\psi}\rangle = \left(-i\,\hat{\mathrm{H}}\,\mathrm{d}t - \tfrac{\gamma}{2}\,\hat{\mathrm{a}}^\dagger\,\hat{\mathrm{a}}\,\mathrm{d}t + \gamma\,\langle\hat{\mathrm{a}}^\dagger\rangle\,\hat{\mathrm{a}}\,\mathrm{d}t + \sqrt{\gamma}\,\hat{\mathrm{a}}\,\mathrm{d}Z^*\right)|\widetilde{\psi}\rangle$$





Trajectory for expectation values:

$$\begin{aligned}\mathrm{d}\left\langle\hat{\mathrm{O}}\right\rangle =& i\left\langle\left[\hat{\mathrm{H}},\hat{\mathrm{O}}\right]\right\rangle\mathrm{d}t \\ & -\frac{\gamma}{2}\left\langle\hat{\mathrm{n}}\,\hat{\mathrm{O}}\right\rangle\mathrm{d}t - \frac{\gamma}{2}\left\langle\hat{\mathrm{O}}\,\hat{\mathrm{n}}\right\rangle\mathrm{d}t + \gamma\left\langle\hat{\mathrm{a}}^\dagger\,\hat{\mathrm{O}}\,\hat{\mathrm{a}}\right\rangle\mathrm{d}t \\ & + \sqrt{\gamma}\left(\left\langle\hat{\mathrm{a}}^\dagger\,\hat{\delta}_O\right\rangle\mathrm{d}Z + \left\langle\hat{\delta}_O\,\hat{\mathrm{a}}\right\rangle\mathrm{d}Z^*\right),\end{aligned}$$

where $\hat{\delta}_O = \hat{\mathrm{O}} - \left\langle\hat{\mathrm{O}}\right\rangle$

- Close Hierarchy with Wick's theorem
  = **Gaussian ansatz** for $|\psi\rangle$
- Gives coupled SDEs for

$$\{\alpha_i = \langle\hat{\mathrm{a}}_i\rangle\},\left\{u_{ij} = \left\langle\hat{\delta}_i\,\hat{\delta}_j\right\rangle\right\},\left\{v_{ij} = \left\langle\hat{\delta}_i^\dagger\,\hat{\delta}_j\right\rangle\right\}$$

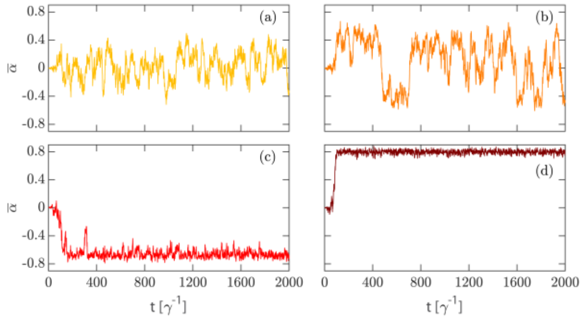
- -> Quadratic scaling with #modes

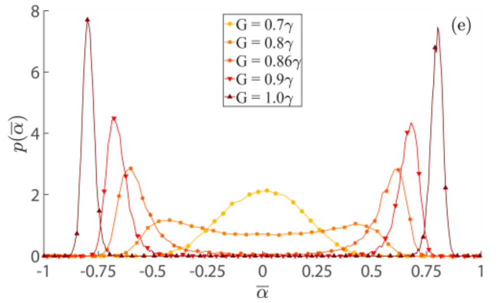
Further reading on the method:

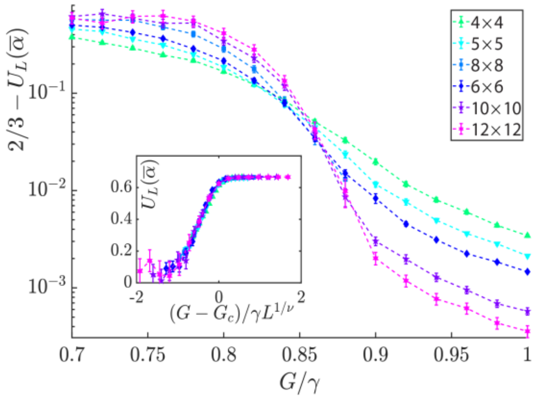
- *WV and MW, App. Sci 8 (9) 1427 (2018)*
- *WV, Gaussian quantum trajectories for the variational simulation of open quantum systems, with photonic applications,* PhD-thesis *(2020)*

Study of the dissipative PT in quadratically driven Kerr lattices:

- Define order parameter
  $$\overline{\alpha} := N^{-1} \sum_i \mathrm{Im}\{\alpha_i\}$$
- Tracked directly with individual trajectories
- U=J=-Δ =γ
- Observe symmery breaking for increasing *G* values
  - Individual trajectories (top)
  - distribution (bottom)





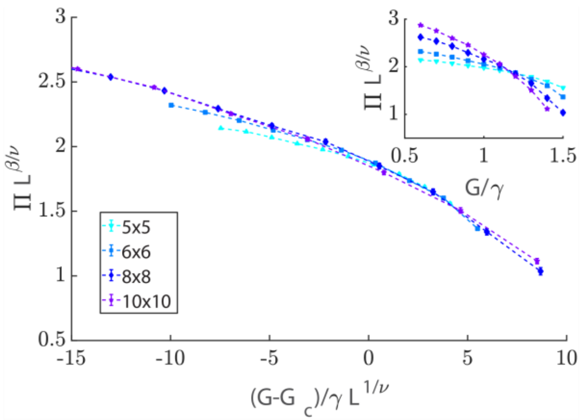


Quantitative analysis of distribution: rescaling of Binder cumulant

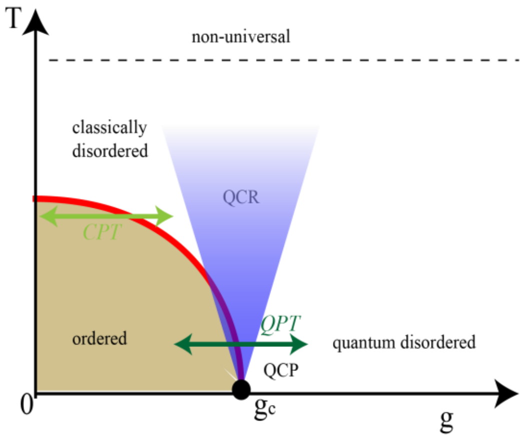
- Extract critical exponent ν=1
- Universality class of *classical* 2D Ising model

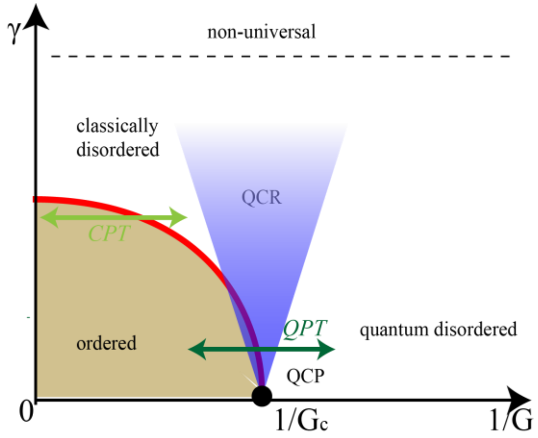
Regime of lower losses (U=40γ, J=-Δ =20γ)

- ν≈0,63
- Universality class of *Quantum* 2D Ising model
- Reproduction of result from *RR et al., PRL 122, 110405 (2019)*



Crossover between quantum and classical universality depending on γ = analogous to quantum phase transitions at finite temperature

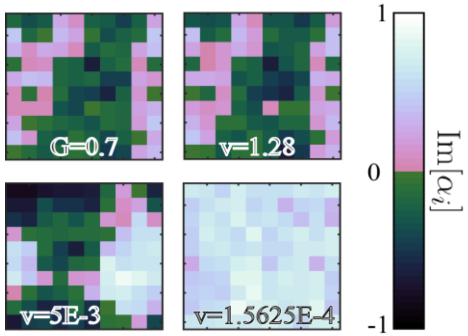


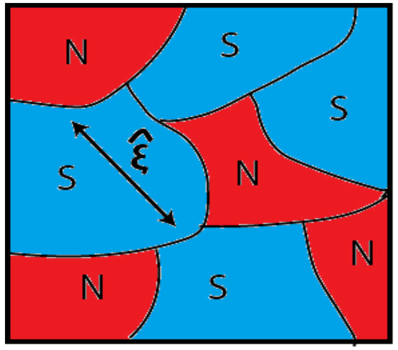


Further reading: *WV, RR, VS and MW, PRResearch 2, 022037(R) (2020)*

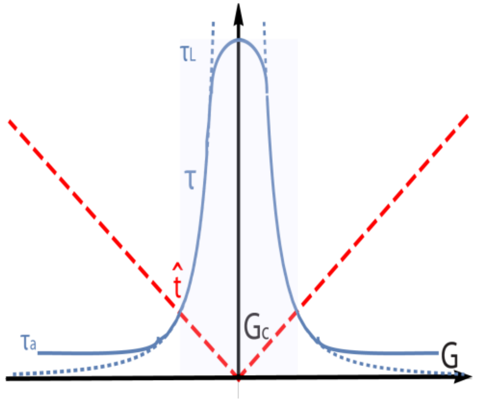
Now, quench G from disordered phase G_0 to G_c as $G(t)=G_0+vt$

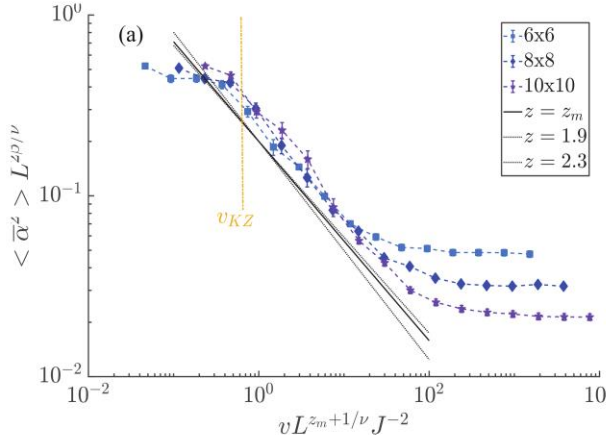
- *Formation of domains with size dependent on v*





- Known as the Kibble-Zurek Mechanism
- Finite-size scaling *[C-W Liu et al. PRB 89, 054307 (2014)]* allows extraction of dynamical critical exponent z
- Find *z ≈2.18*: Metropolis dynamics

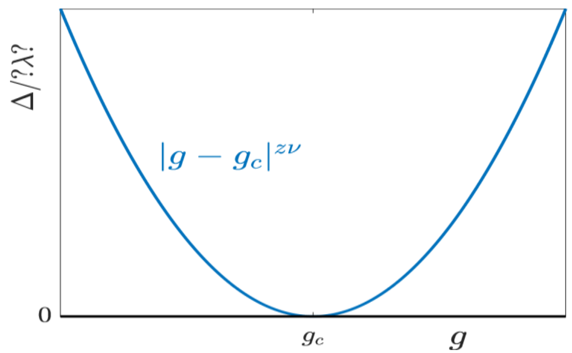




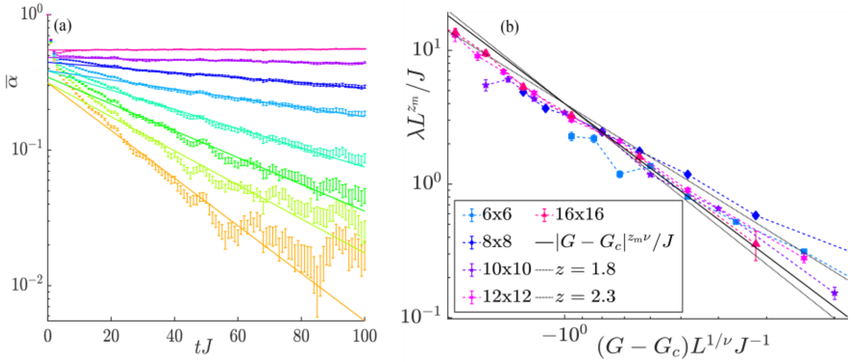
In QPT, the gap scales as

$$\Delta_H \sim \xi^{-z} \;\&\; \Delta_H \sim |g-g_c|^{z\nu} \quad (1)$$

- What about the Liouvillian gap λ in a DPT?



- Extract λ from slow decay to the NESS
- Result: straightforward generalization of (1)
- Same *z* as from the Kibble-Zurek effect



Further reading: *WV and MW, PRA 101, 043826 (2020)*