Identification of topological phases using RBMs

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Motivation

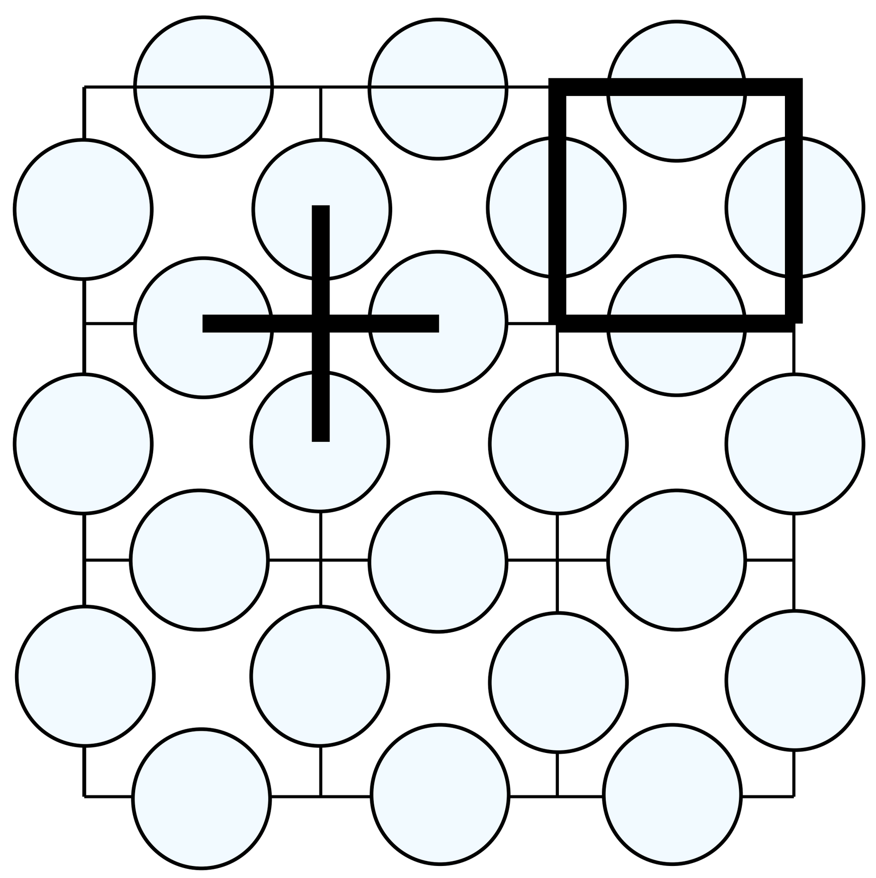
- Restricted Boltzmann machines (RBMs) have proven to efficiently represent wide class of states with rich topological orders
- Can topological phase transitions be encaptured? We straightforwardly modify the RBM structure to allow for more flexible representation of correlations → Modification allows to reproduce the toric code phase diagram and improves RBM accuracy also for other models

Model

Toric code

$$A_s = \prod_{i\in s} \sigma_i^x \qquad B_p = \prod_{i\in p} \sigma_i^z$$

$$H_{\mathrm{TC}} = -\sum_s A_s - \sum_p B_p$$



- Toric ground state $|\mathrm{TC}\rangle$: exactly solvable

Understanding the phase diagram: Toric code with fields

$$H = H_{\mathrm{TC}} + h_x \sum_i \sigma_i^x + h_y \sum_i \sigma_i^y + h_z \sum_i \sigma_i^z$$

- in general not exactly solvable, QMC sign problem
  → explore variational wave-functions

A Yu Kitaev, Ann. Phys. 2003

Modifying RBMs

RBM structure

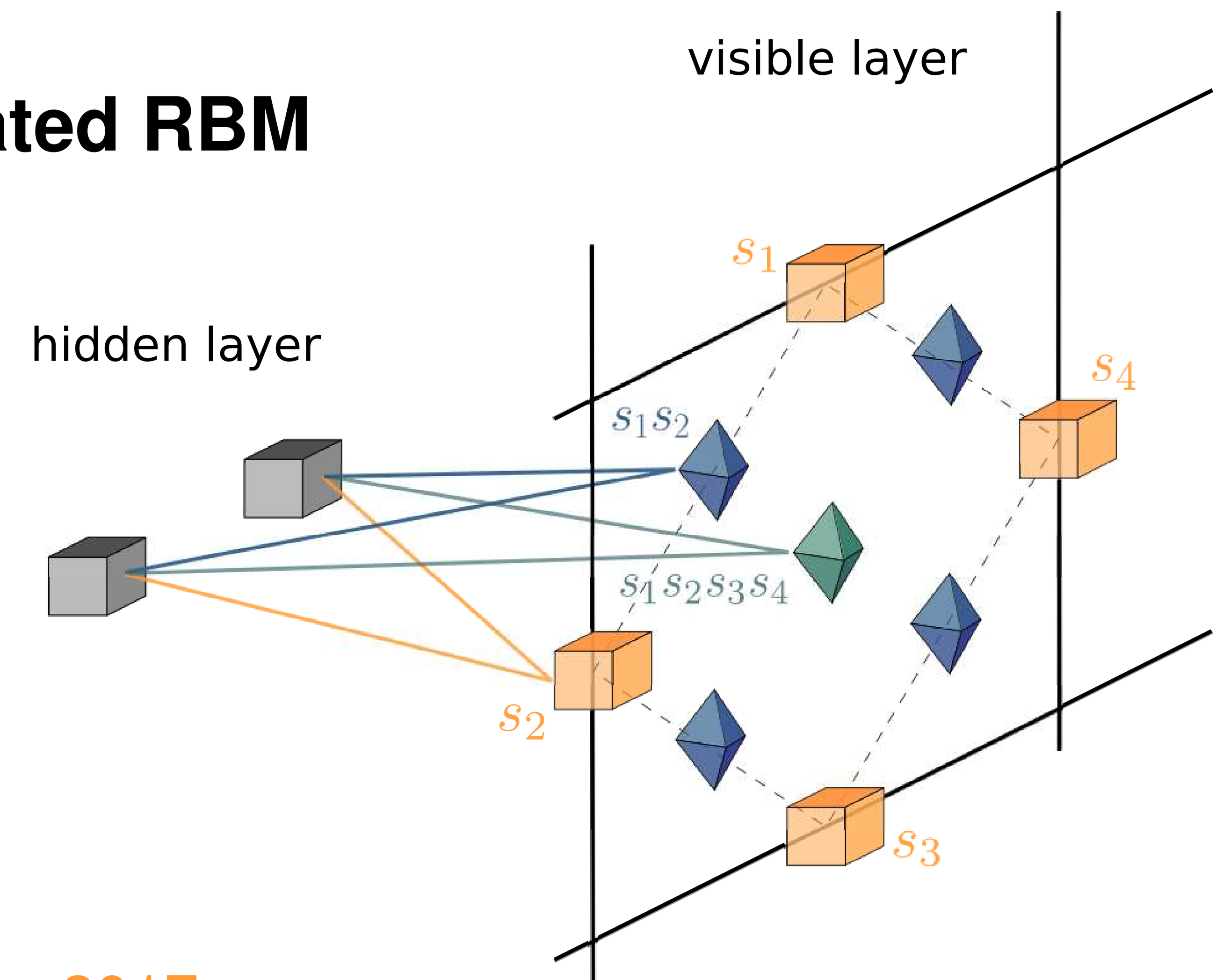
$$\Psi(S) = \sum_h e^{\sum_k a_k s_k + \sum_j b_j h_j + \sum_{k,j} W_{k,j} s_k h_j} \quad \rightarrow |TC\rangle \text{ is exact RBM}$$

- Toric code + fields: RBM not efficient

Modify structure: correlated RBM

Solution:

introduce visible neurons, values determined by correlations $\langle s_i..s_l\rangle$



G. Carleo and M. Troyer, Science 2017

DL Deng, X. Li and S. Das Sarma, Phys. Rev. B 2017

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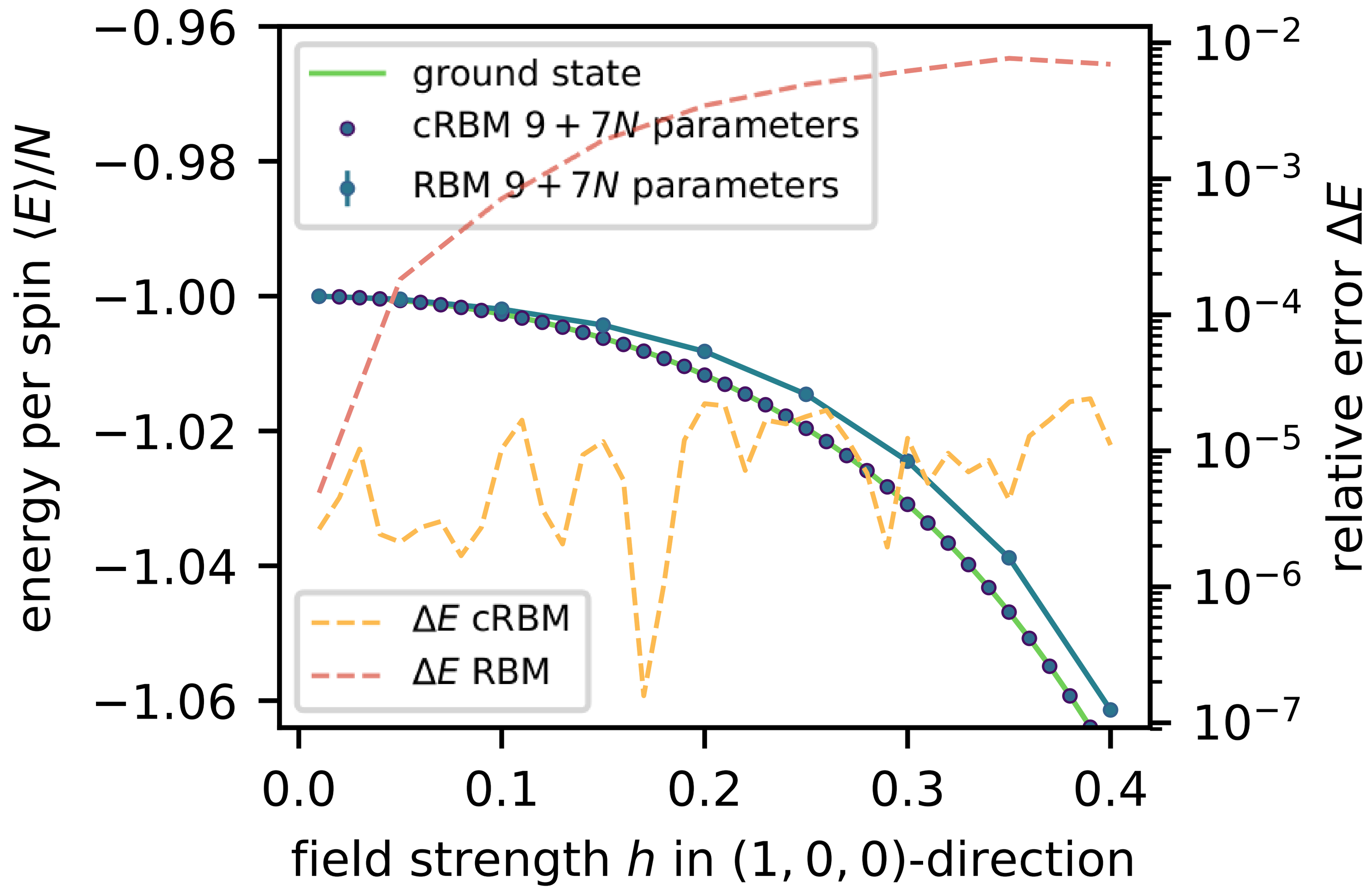
Results I: energies

Hamiltonian: $H = H_{\mathrm{TC}} + h_x \sum_i \sigma_i^x + h_y \sum_i \sigma_i^y + h_z \sum_i \sigma_i^z$

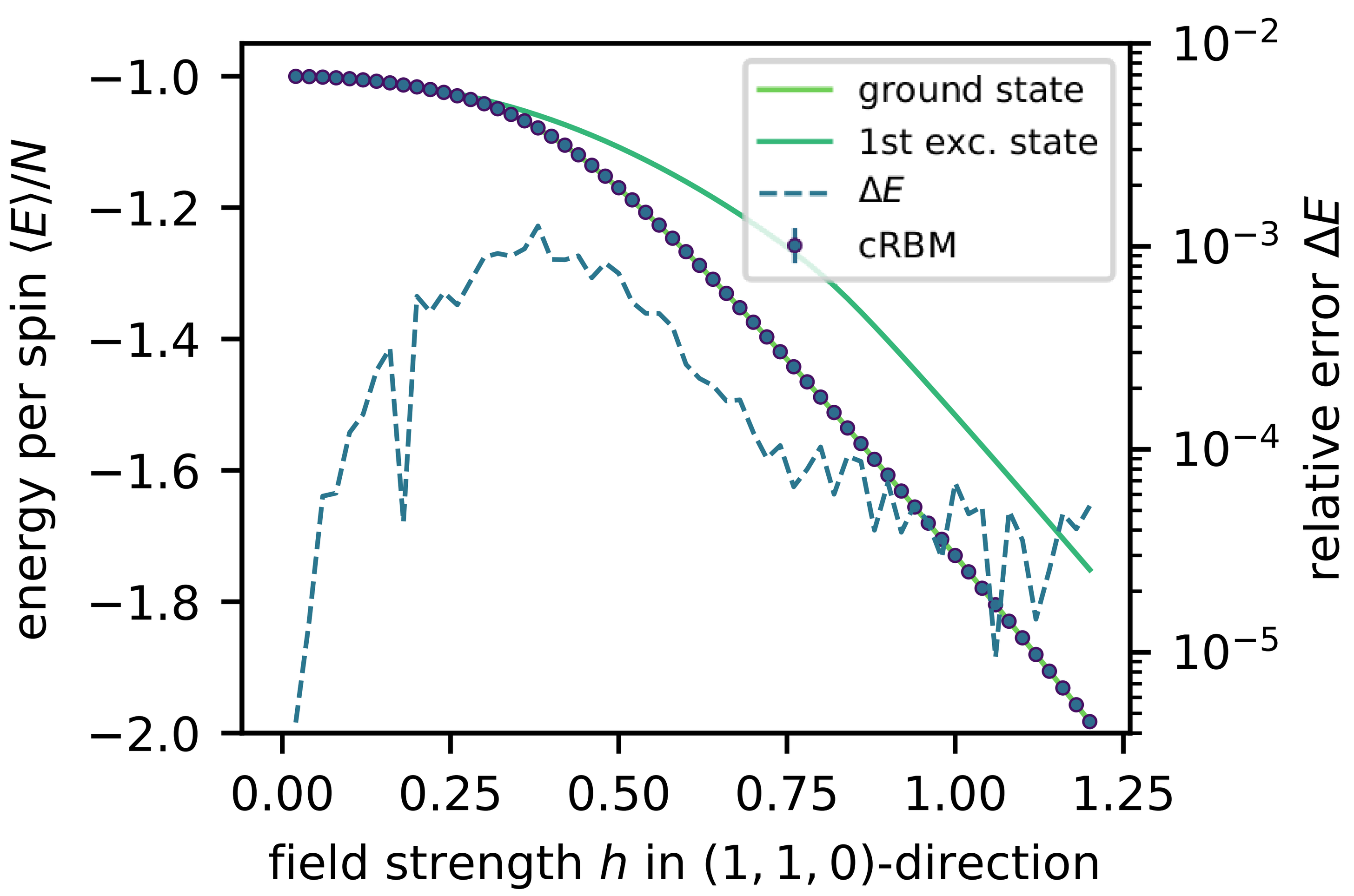
Comparison: RBM and correlated RBM

- compare with ED: $L = 3$ (18 spins)
- symmetrize ansatz: use translational symmetries



$\rightarrow$ variational ansatz including correlators is more efficient

Precision in different field sectors

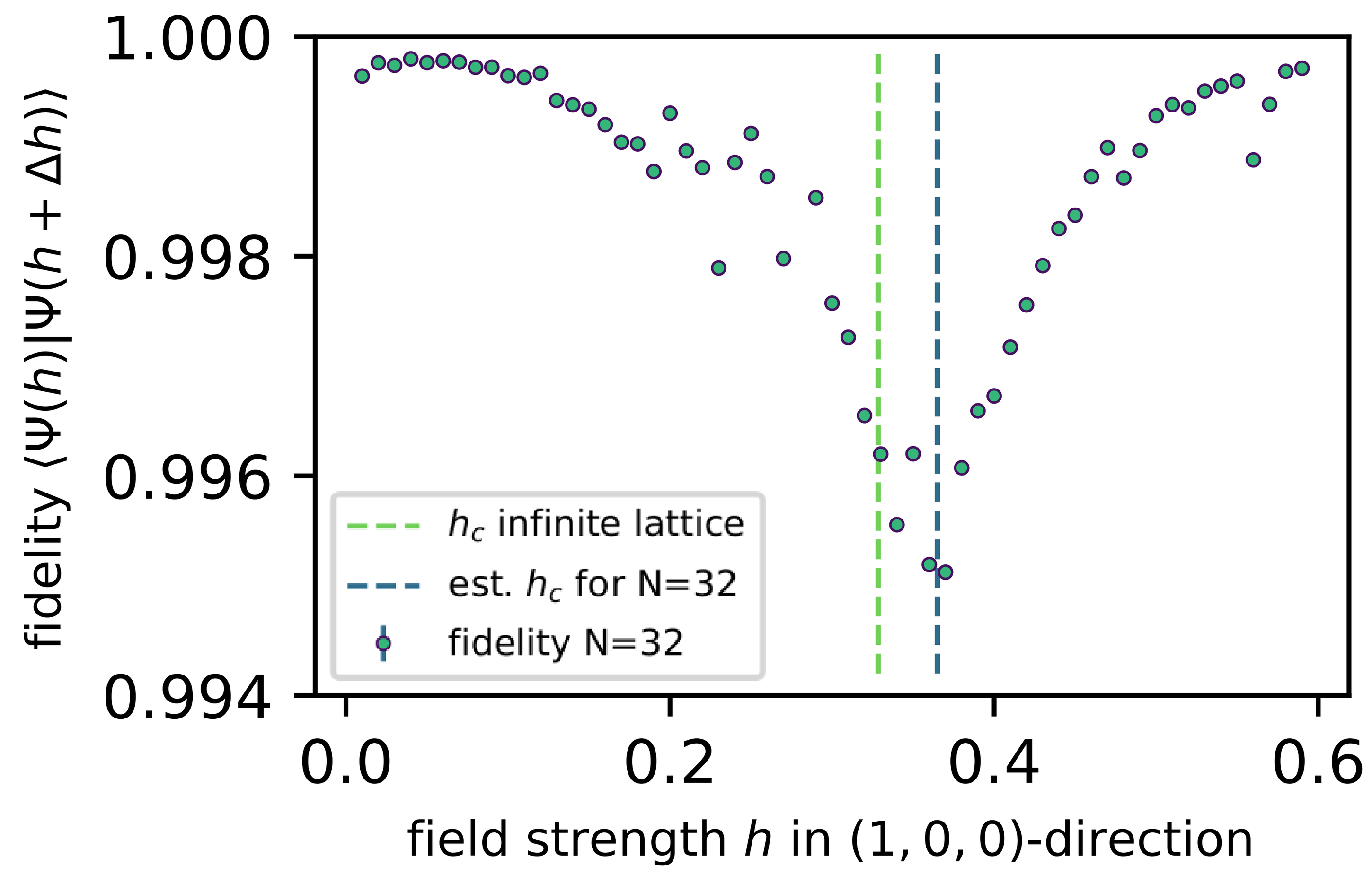


- precision still high in presence of y-fields, but lowest at phase transition

Results II: top. phase transition

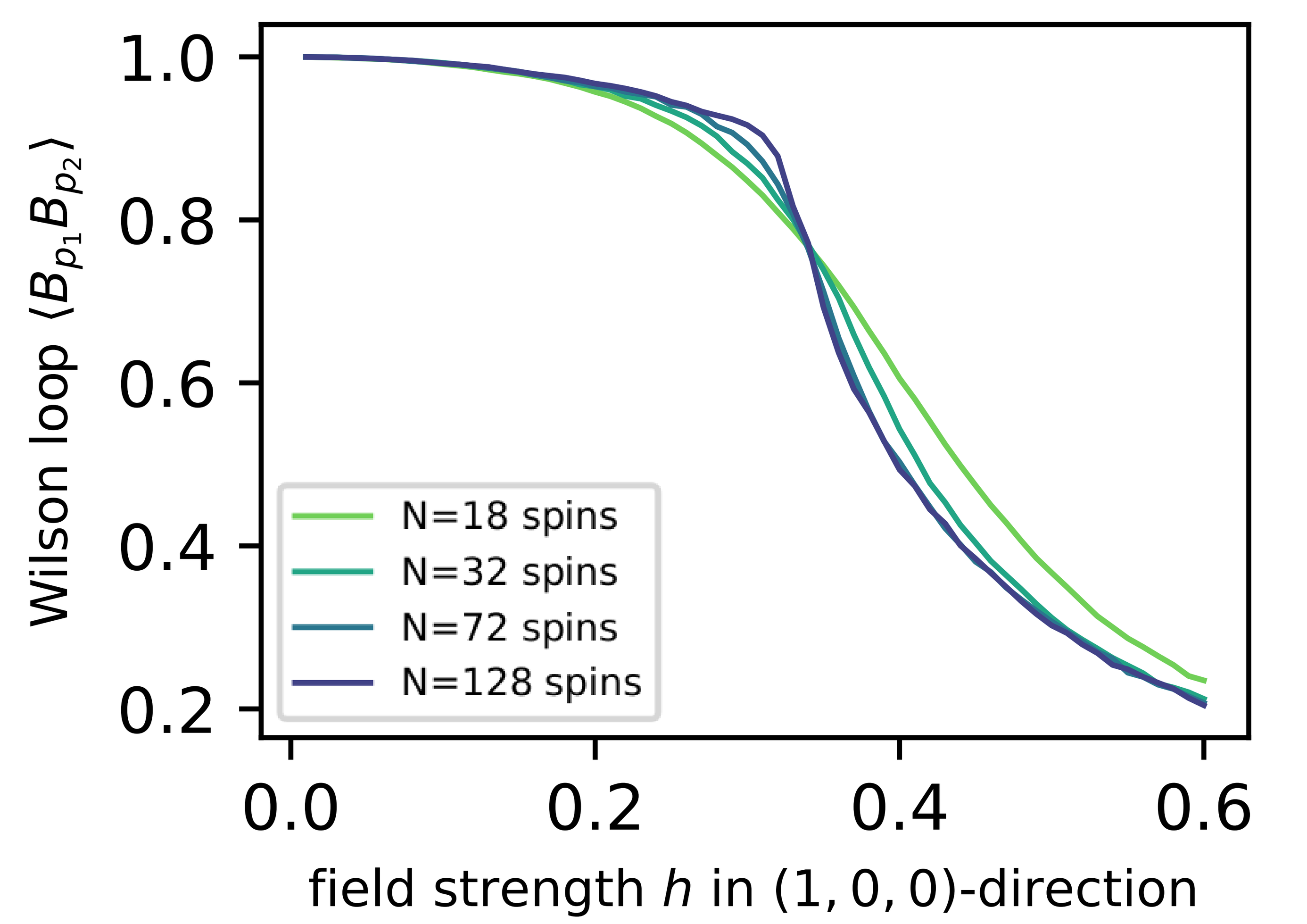
Fidelity: detecting phase transitions

- second order QPT detectable via minimum in fidelity
  $F(\Delta h) = \langle \Psi(h) | \Psi(h + \Delta h) \rangle$



Wilson loops: detecting topological order

- examine Wilson loop $\langle B_{p_1} B_{p_2} \rangle$ scaling



$\rightarrow$ correlated RBM can encapture topological phase transition

Explore representational power

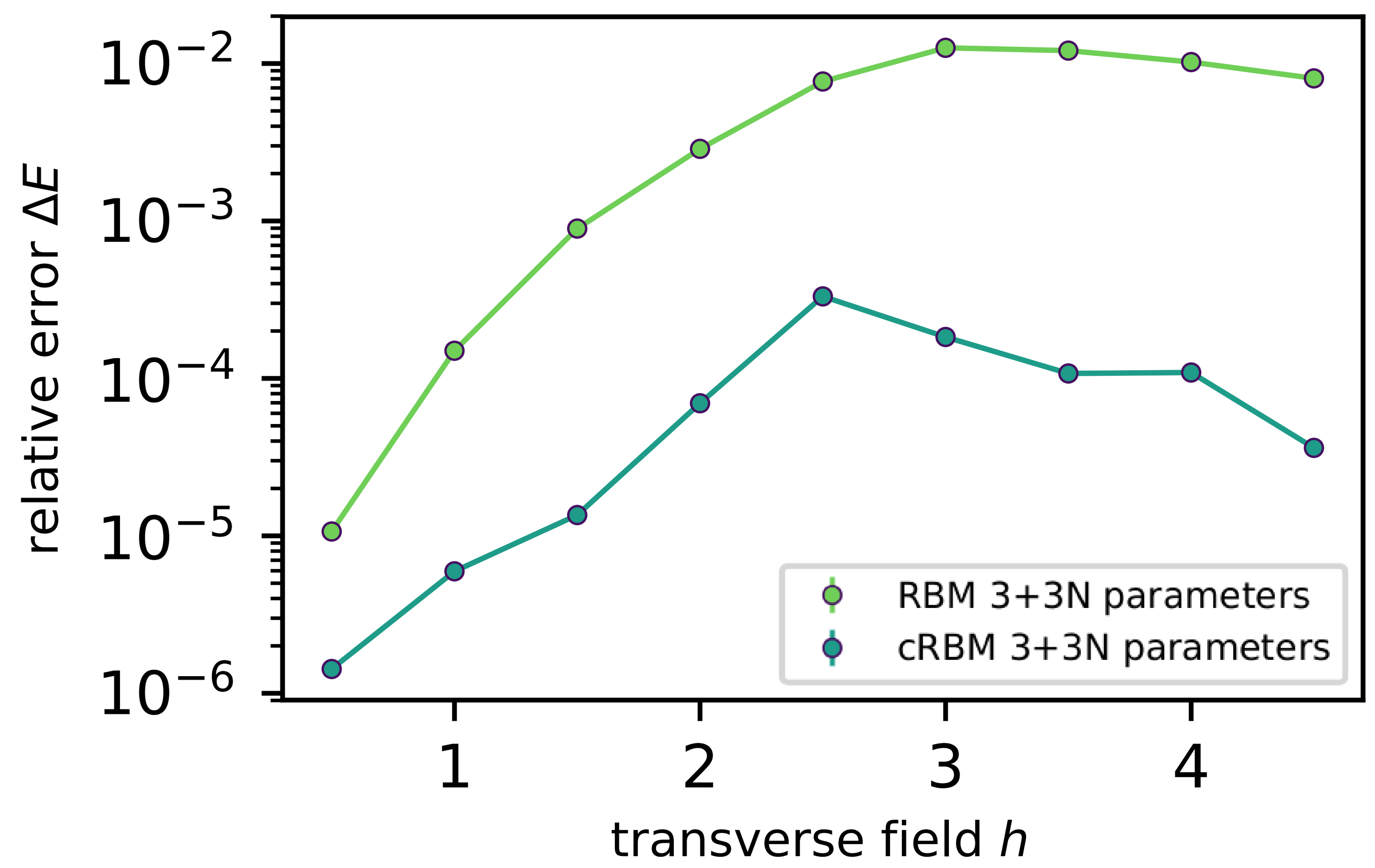
Can we improve the RBM accuracy by including correlators also for different models?

Toy model: transverse field Ising 2D

$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + h \sigma_i^x$$

Comparison: RBM and correlated RBM

- use small amount of hidden neurons to compare efficient representation, no symmetries imposed
- correlated RBM with only bond correlators $s_i s_j$



$\Longrightarrow$ precision order of magnitude better, when correlators are included!

- explore class of models where correlators improve efficiency
- analyse scaling

Learning phase transitions

Identifying phase transitions is one of the key questions in theoretical and experimental condensed matter physics alike. Can we find a neural-network based tool to detect quantum phase transitions that is generic, unbiased and accessible to typical numerical and experimental techniques?

Phase transition from measurements

Neural networks: find transitions *without prior knowledge*

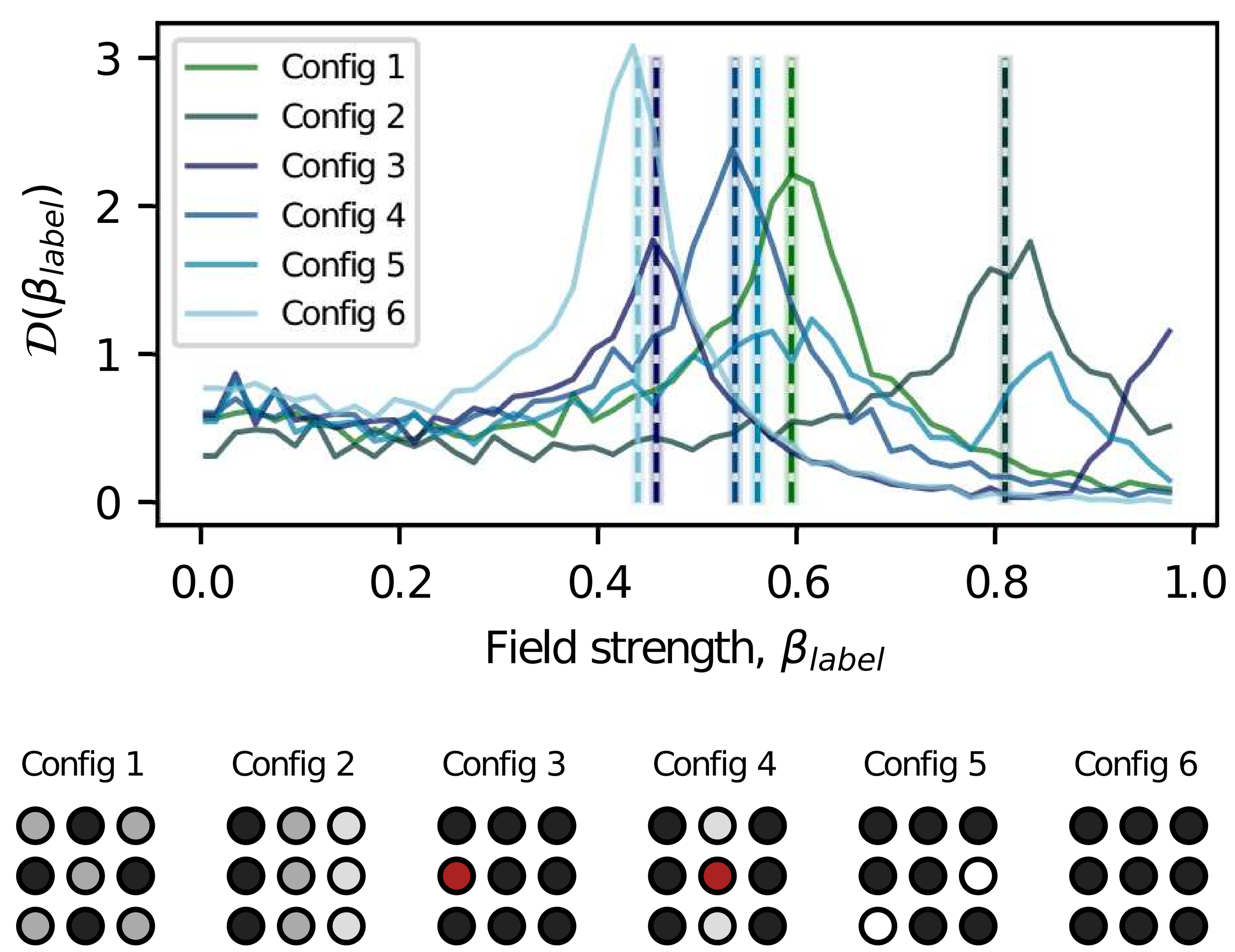
Train network to reproduce continuous parameter β (field on TC)

- Idea: difference in network performance in the two phases

$\Longrightarrow$ deriv. of the network prediction diverges at phase transition!

Network input

- Recognize patterns: feed in measured projection on spin-configurations S



- Compare with phase transition found with fidelity $\langle \Psi(\beta) | \Psi(\beta + \delta\beta) \rangle$

$\Longrightarrow$ topological phase transition identified by neural network!

E. Greplova, A. Valenti *et al.*, NJP (2020)