Quantum aspects of magnetic fragmentation in pyrochlore spin ice

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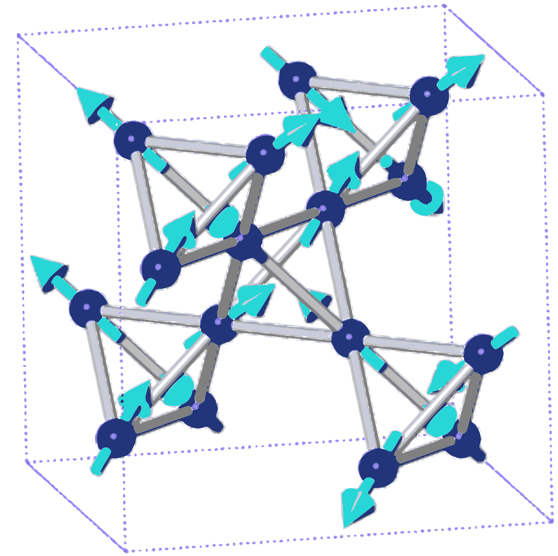
Quantum spin ice



- XXZ Hamiltonian on the pyrochlore lattice:

$$\mathrm{H} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_{\pm} \left(S_i^+ S_j^- + S_i^- S_j^+ \right)$$

- Classical spin ice $J_{\pm} = 0$: frustration with local constraint

$$\mathcal{Q}_{\mathbf{r}} = \eta_{\mathbf{r}} \sum_{\mu} S^z_{\mathbf{r}\mu} = 0$$

- Mapping to a compact U(1) lattice gauge theory [2], [5] :

$\Phi^\dagger_{\mathbf{r}},\ (\Phi_{\mathbf{r}})$ raising (lowering) operators for $\mathcal{Q}_{\mathbf{r}}$

$$S^+_{\mathbf{r}\mu} = \Phi^\dagger_{\mathbf{r}} e^{iA_{\mathbf{r}\mu}} \Phi_{\mathbf{r}+\hat{\mu}}$$

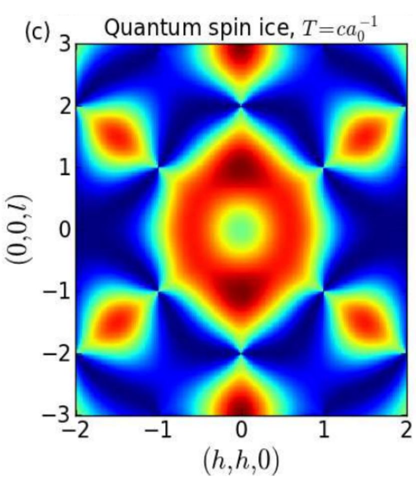
$i,\ j \in$ Pyrochlore lattice
$\mathbf{r} \in$ Diamond lattice with sublattices A, B

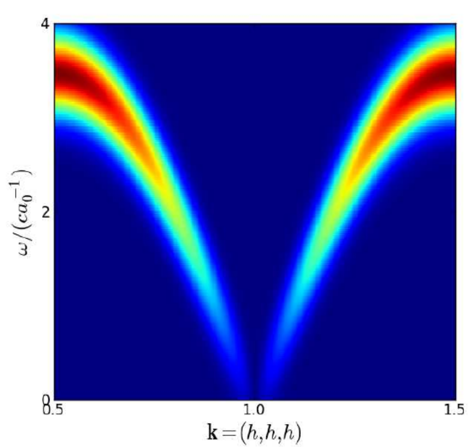
$$\Rightarrow \mathrm{H} = \frac{J_{zz}}{2} \sum_{\mathbf{r}} \mathcal{Q}^2_{\mathbf{r}} - J_{\pm} \sum_{\substack{\mathbf{r}\in A \\ \mu\neq\nu}} \Phi^\dagger_{\mathbf{r}+\hat{\mu}} \Phi_{\mathbf{r}+\hat{\nu}} e^{-i(A_{\mathbf{r}\mu} - A_{\mathbf{r}\nu})} - J_{\pm} \sum_{\substack{\mathbf{r}\in B \\ \mu\neq\nu}} \Phi^\dagger_{\mathbf{r}-\hat{\mu}} \Phi_{\mathbf{r}-\hat{\nu}} e^{i(A_{\mathbf{r}\mu} - A_{\mathbf{r}\nu})}$$

On a bigger Hilbert space

$$\mathcal{H}_{tot} = \mathcal{H}_{\mathcal{Q}} \bigotimes \mathcal{H}_A$$

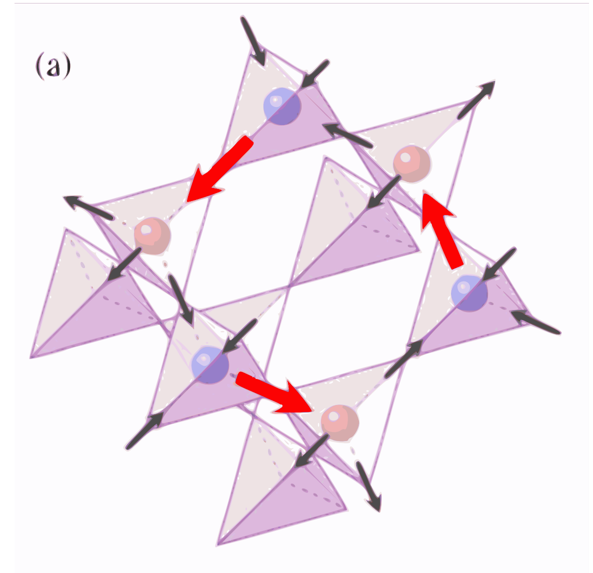
with Gauss' Law constraint





- Perturbative limit $J_{\pm} \ll J_{zz}$: spin loop terms → U(1) spin liquid [11]
- Emergent EM with photon-like excitation
- Signatures in magnetic structure factor [4]

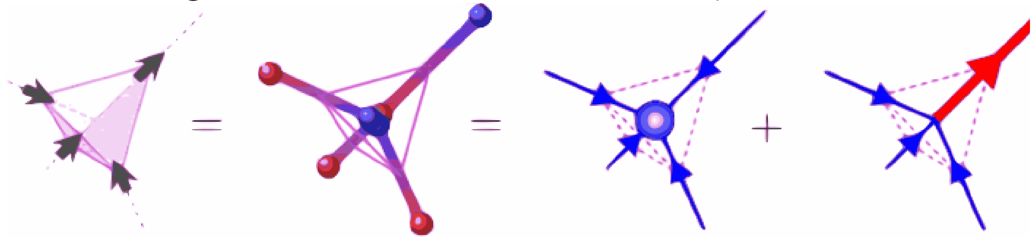
Fragmentation in spin ice [3]



- A staggered chemical potential can stabilize a monopole crystal:

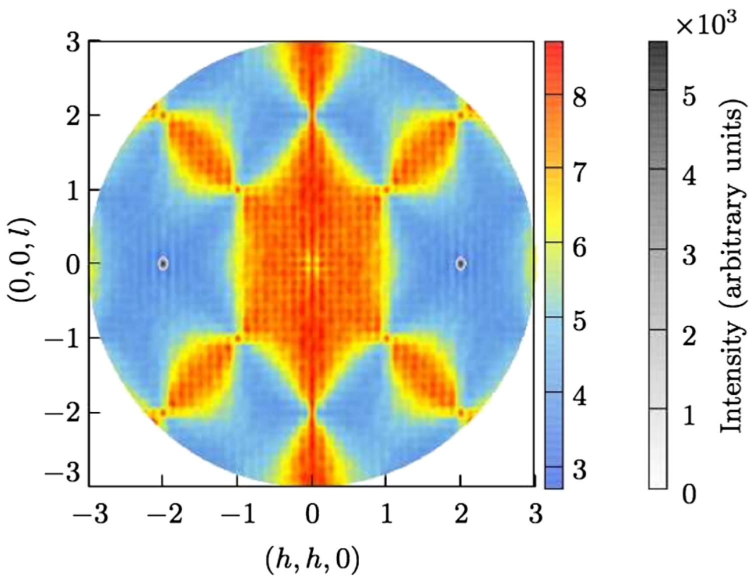
$$\mathrm{H} = \frac{J_{zz}}{2} \sum_{\mathbf{r}} \mathcal{Q}_{\mathbf{r}}^2 - J_{\pm} [\cdots] - \Delta \sum_{\mathbf{r}} \eta_{\mathbf{r}} \mathcal{Q}_{\mathbf{r}}$$

- Fragmentation of magnetization in a divergence full (longitudinal) and a divergence free (transverse) component



$$[M_{\mathbf{r}\mu}] \frac{a}{m} = (-1,-1,-1,1) = \left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right) + \left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{3}{2}\right)$$

- Monopole order + emergent field with dipolar correlations (classically)
- One bond of strength ±3/2 per diamond site → mapping to a dimer model
- Extensive entropy ↔ closed loop of dimer moves



- Consequence on the classical structure factor: Braggs peak + diffuse pinch-points

Effective low-energy theories

- If we add quantum fluctuations, does the emergent dipolar field order ?

➔ Effective theory for the degenerate subspace of ground states

- Projection on the GS subspace satisfies the "Effective Schrödinger equation":

$$\left[E_0 + \mathcal{P}\mathrm{H}_1 \sum_{n=0}^{\infty} \left(\frac{1}{E - \mathrm{H}_0} (1-\mathcal{P})\mathrm{H}_1 \right)^n \mathcal{P} \right] |\Psi_0\rangle = \mathrm{H}_{eff} |\Psi_0\rangle = E |\Psi_0\rangle$$

- Separation in diagonal and non-diagonal terms [7], [8] :

Diagonal terms

- Energy of a particular configuration of spins / dimers in the classically denegerate manifold
- At lowest order : potential energy of having a flippable 6-link loop $\mu \sim J_{\pm}^6 / J_{zz}^5$

Non-diagonal terms

- Energy cost of tunnelling to another configuration
- At lowest order: kinetic energy associated with flipping a 6-link loop $g \sim J_{\pm}^3 / J_{zz}^2$

- For quantum spin ice:

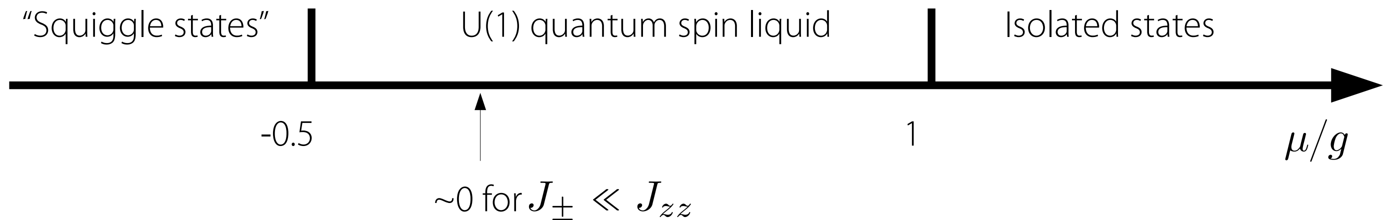
$$\mathrm{H}_{eff,\,s} = \mu_s \sum_{\hexagon} |\circlearrowleft\rangle\langle\circlearrowleft| + |\circlearrowright\rangle\langle\circlearrowright| - g_s \sum_{\hexagon} |\circlearrowleft\rangle\langle\circlearrowright| + |\circlearrowright\rangle\langle\circlearrowleft|$$

- For fragmented spin ice and emergent dimers:

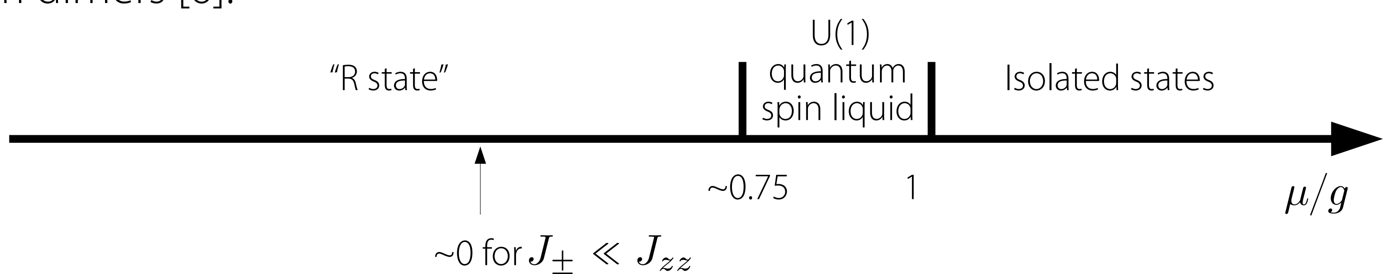
$$\mathrm{H}_{eff,\,d} = \mu_d \sum_{\hexagon} |\triangledown\rangle\langle\triangledown| + |\triangle\rangle\langle\triangle| - g_d \sum_{\hexagon} |\triangledown\rangle\langle\triangle| + |\triangle\rangle\langle\triangledown|$$

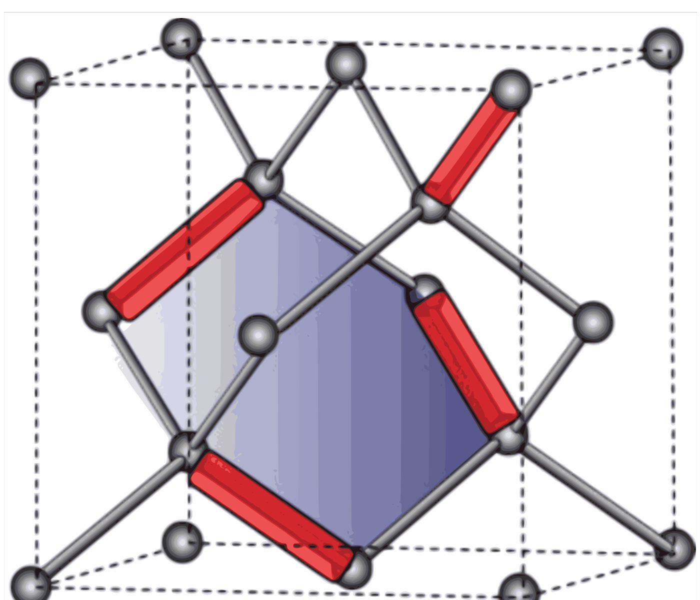
Possible quantum phases

- Depends on the value of $\mu/g \sim (J_{\pm}/J_{zz})^3$
- For quantum spin ice [4]:



- For quantum dimers [6]:





- In fragmented spin ice, **the emergent dipolar field is likely ordered**, with dimers maximising the number of flippable plaquettes
- Experimental signatures: structure factor of R state as one sector of the dimers + phase transition as T → 0

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