Introduction to Tensor Network States and Methods

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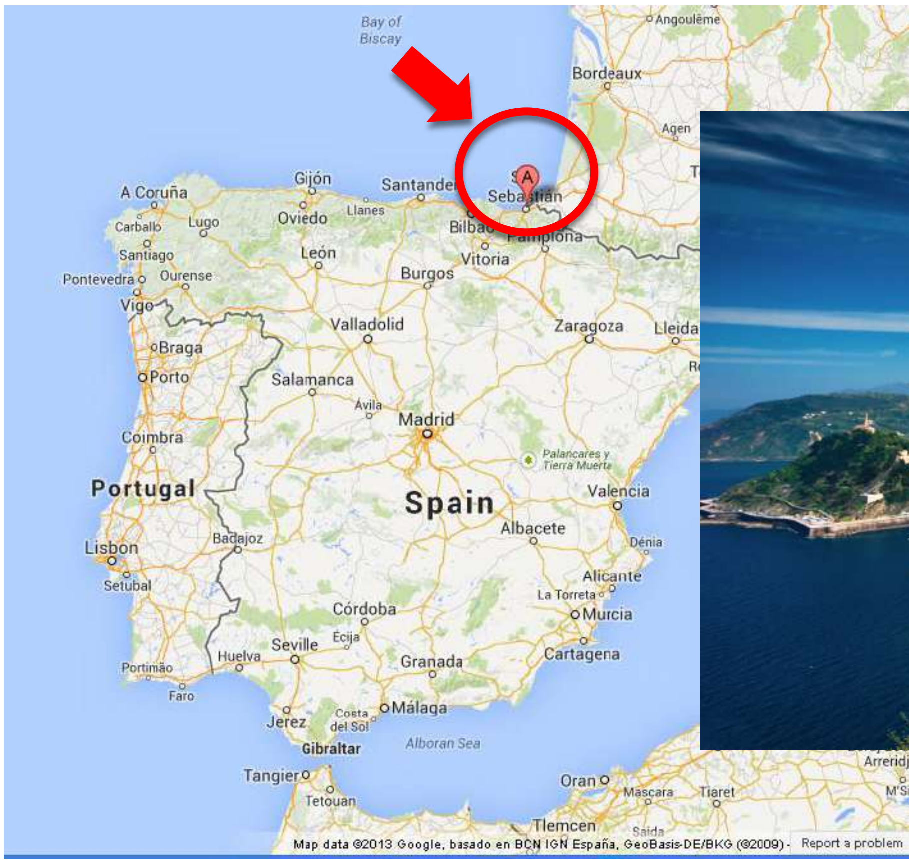
DIPC & Multiverse

Quantum @ Donostia – San Sebastián

MULTIVERSE COMPUTING



Some reviews

J. Eisert, Modeling and Simulation 3, 520 (2013), arXiv:1308.3318

N. Schuch, QIP, Lecture Notes of the 44th IFF Spring School 2013, arXiv:1306.5551

R. Orus, Annals of Physics 349 (2014) 117-158

R. Orus, Nature Reviews Physics 1, 538-550 (2019)

J. I. Cirac, F. Verstraete, J. Phys. A: Math. Theor. 42, 504004 (2009)

F. Verstratete, J. I. Cirac, V. Murg, Adv. Phys. 57,143 (2008)

J. Jordan, PhD thesis, www.romanorus.com/JordanThesis.pdf

G. Evenbly, PhD thesis, arXiv:1109.5424

U. Schollwöck, RMP 77, 259 (2005)

U. Schollwöck, Annals of Physics 326, 96 (2011)

Outline

1) Basics

2) 1d MPS

3) 2d PEPS

4) Numerical algorithms

5) MERA

6) Extras

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2) 1d MPS

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The main problem

- **Aim:** *efficient representation of a quantum many-body state*

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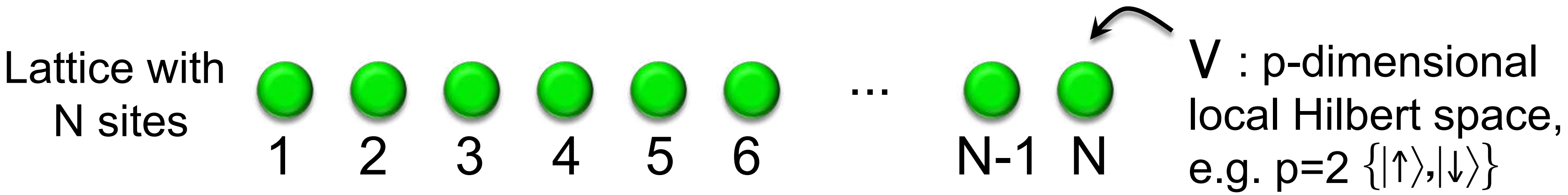

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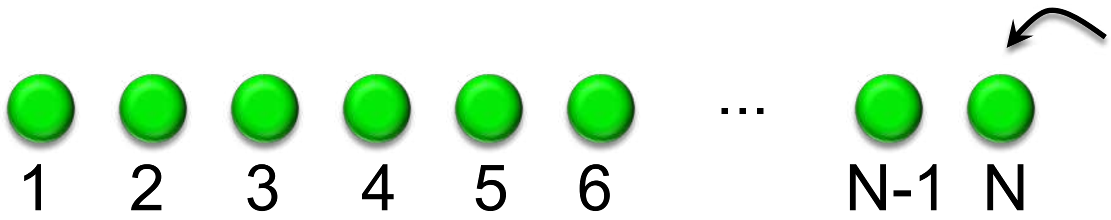
- **Aim:** *efficient representation of a quantum many-body state*

Lattice with N sites



V : p-dimensional local Hilbert space, e.g. p=2 $\{|\uparrow\rangle, |\downarrow\rangle\}$

Hilbert space

$$V \otimes V \otimes V \otimes V \otimes V \otimes V \otimes \cdots \otimes V \otimes V$$

dimension $O(p^N)$ grows exponentially with N

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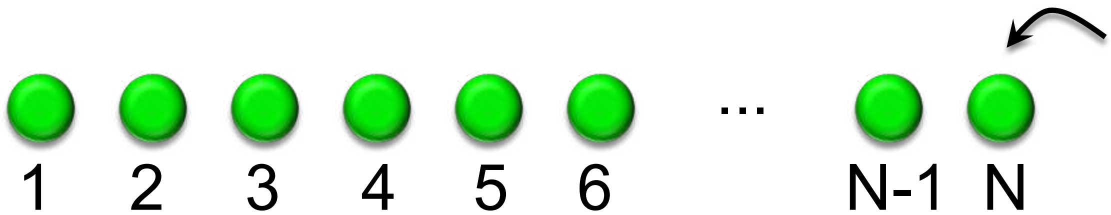

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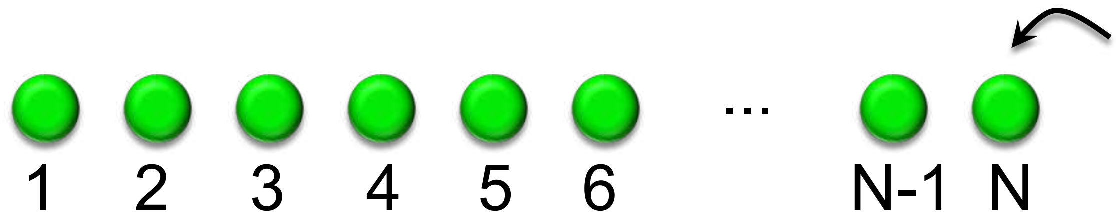

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$$|\Psi\rangle = \sum_{i's} \Psi_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

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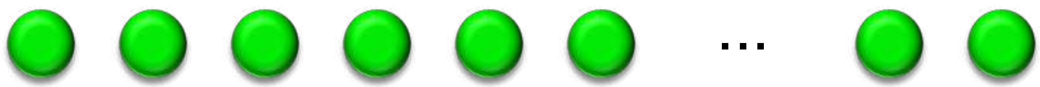

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Complexity **O(p^N) coefficients**

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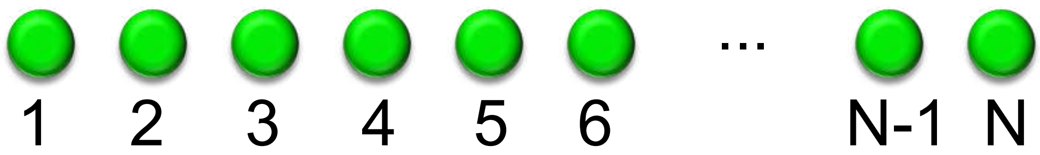

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Complexity **$O(p^N)$ coefficients** → **inneficient representation!**

Entanglement obeys area-law

Entanglement → key resource in quantum information

teleportation, quantum algorithms,
quantum error correction, quantum cryptography…

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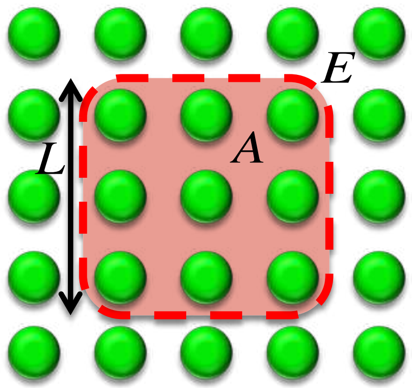
2d system



$$\rho_A = \mathrm{tr}_E\left(|\Psi\rangle\langle\Psi|\right)$$

Reduced density matrix of subsystem A

$$S(A) = -\mathrm{tr}\left(\rho_A \log \rho_A\right)$$

Entanglement entropy (von Neumann entropy)

For many ground states → $S(A) \sim L$

$(L > \xi)$

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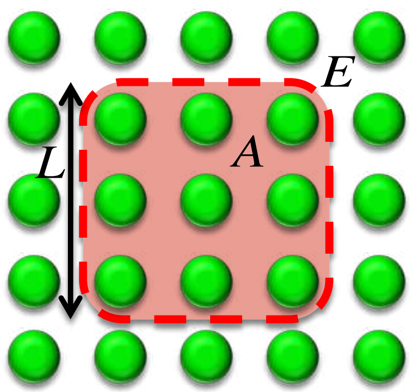

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In d dimensions

Generic state $S(A) \sim L^d$ (volume)

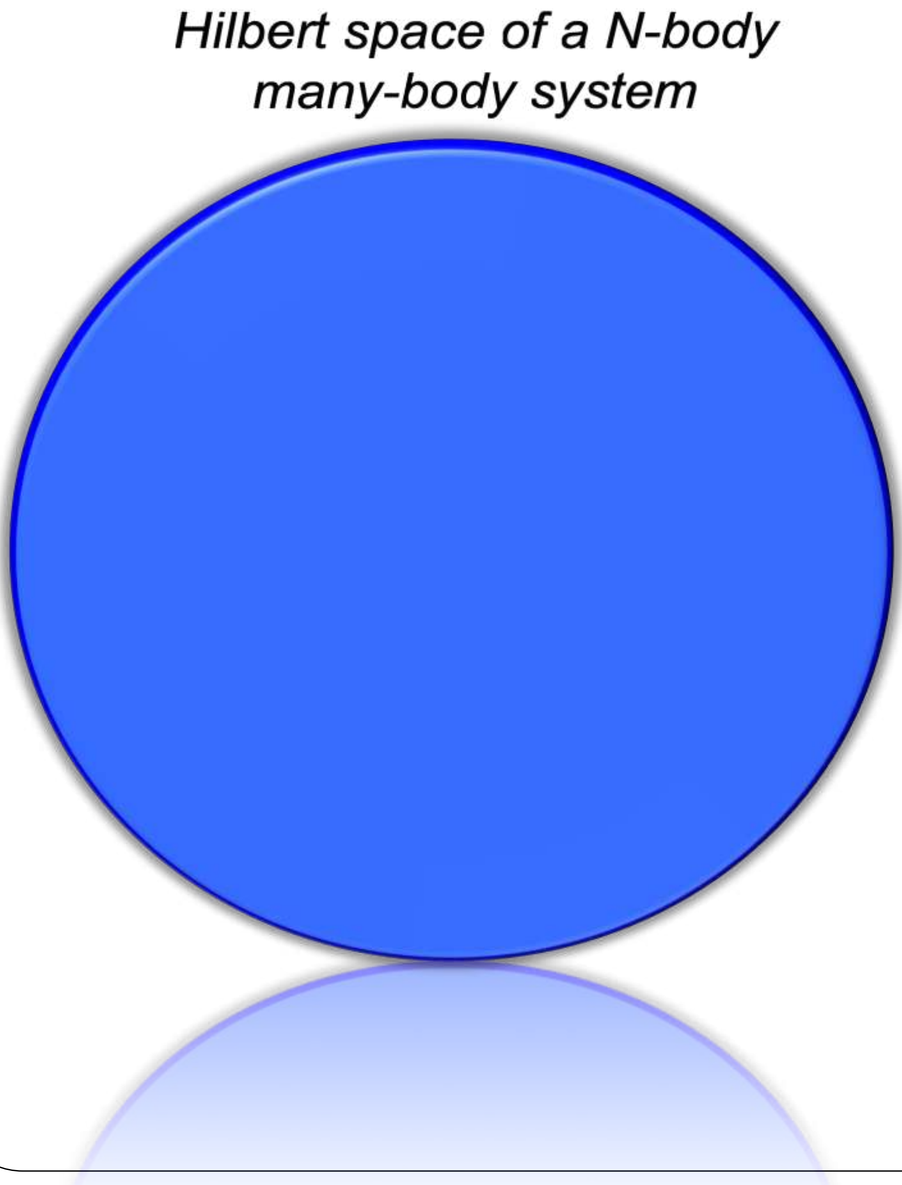
Ground states of (most) local Hamiltonians $S(A) \sim L^{d-1}$ (area)

Srednicki, Plenio, Eisert, Dreißig, Cramer, Wolf…

Locality of interactions ⇔ area-law

Many-body Hilbert space
is far too large

Hilbert space is a convenient illusion



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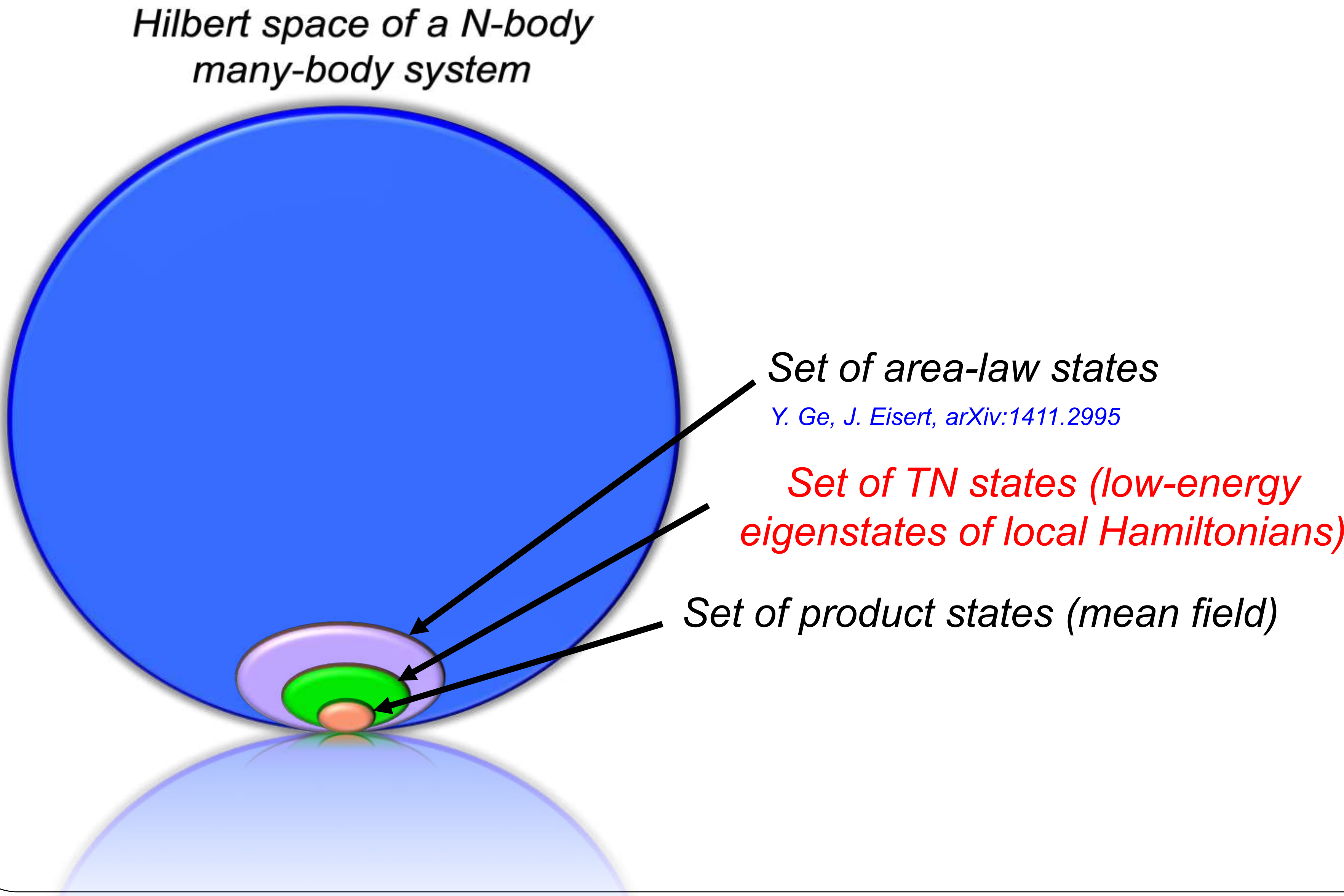
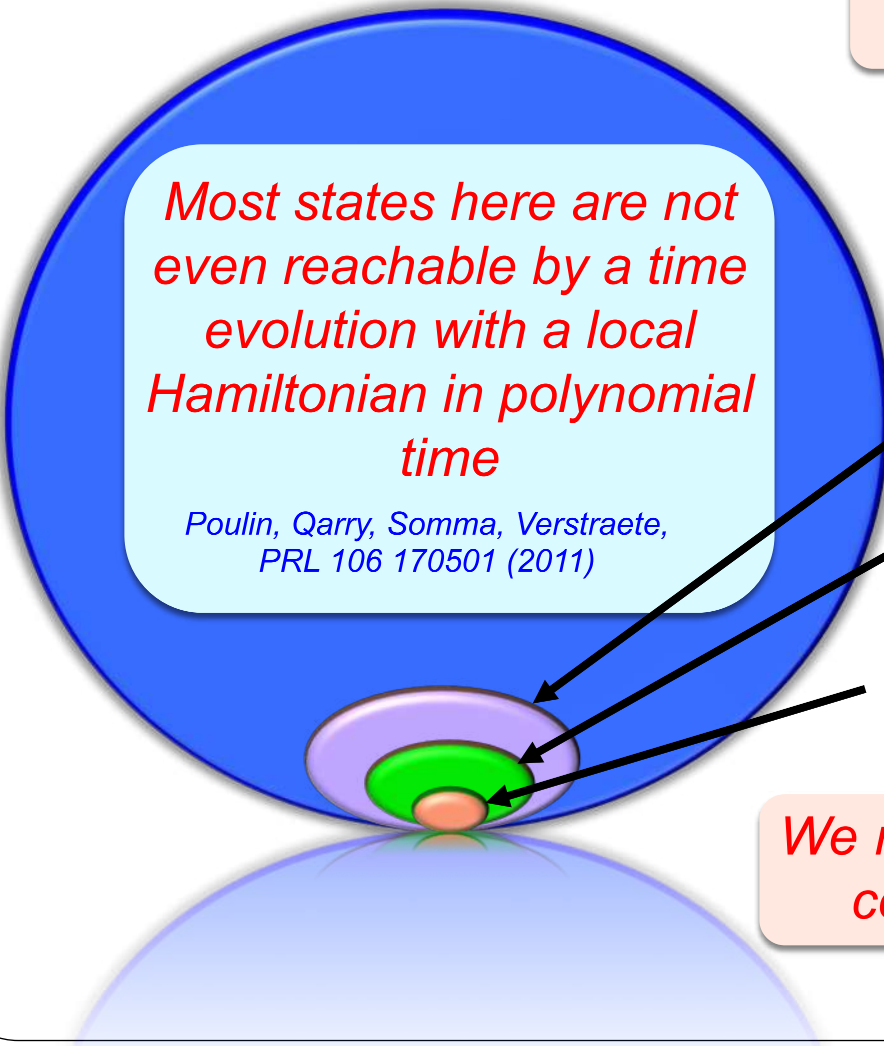


Hilbert space is a convenient illusion

Hilbert space of a N-body many-body system



"Exploration" time ~ $O(10^{10^{23}})$ sec.

Compare to…

Age of the universe ~ $O(10^{17})$ sec.

Set of area-law states

Y. Ge, J. Eisert, arXiv:1411.2995

Set of TN states (low-energy eigenstates of local Hamiltonians)

Set of product states (mean field)

We need a language to target the relevant corner of quantum states directly

Tensor Networks

A new language

$$\left|\Psi\right\rangle = \sum_{i's} \Psi_{i_1 i_2 \ldots i_N} \left|i_1\right\rangle \otimes \left|i_2\right\rangle \otimes \cdots \otimes \left|i_N\right\rangle$$

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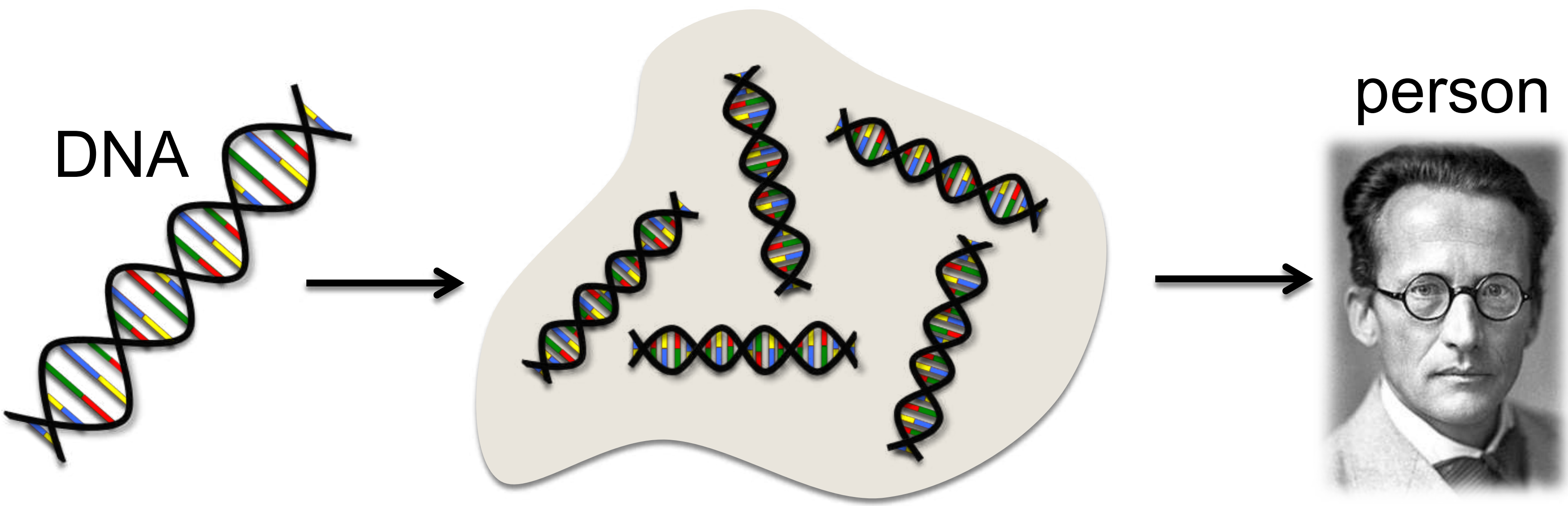

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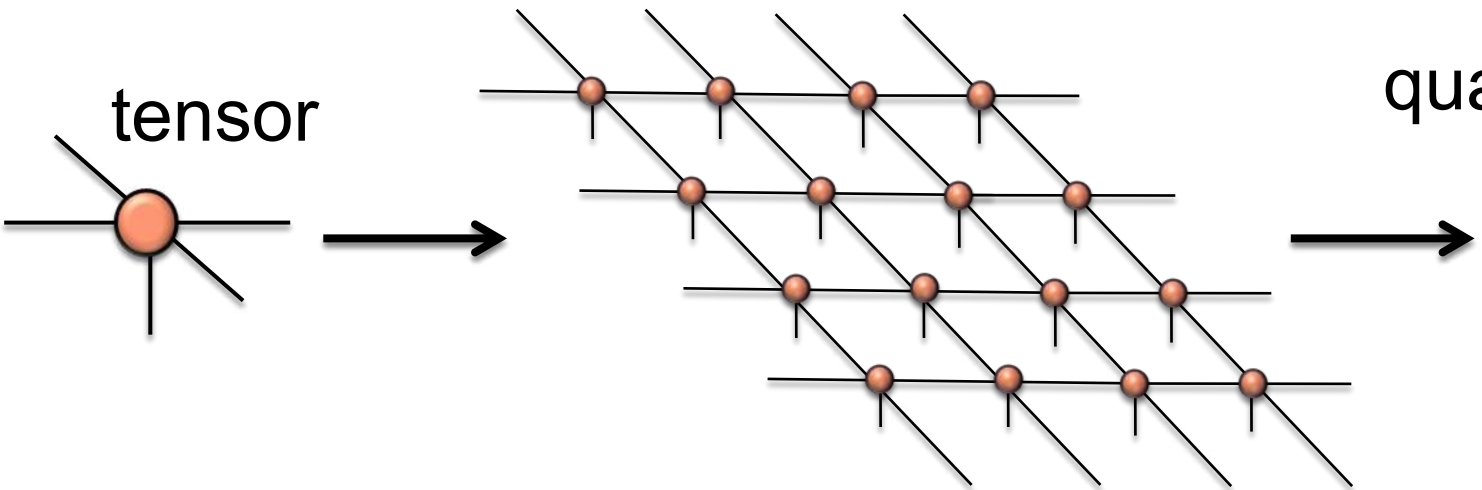

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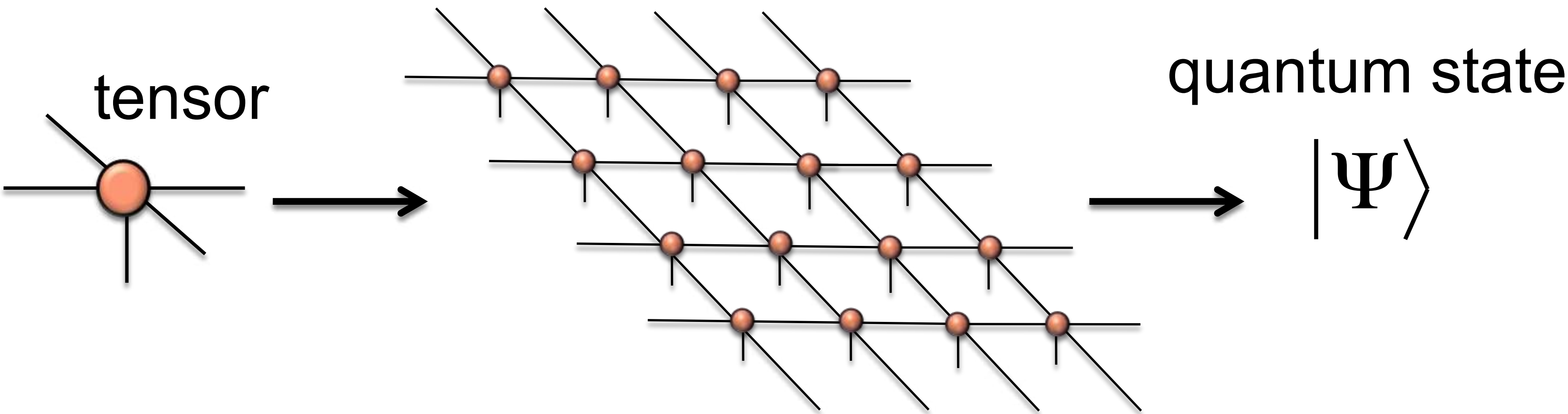


Tensors are local building blocks for the quantum state (like a DNA, or LEGO)

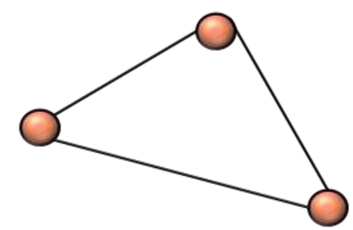
Tensor network diagrams

vector $\vec{v}$

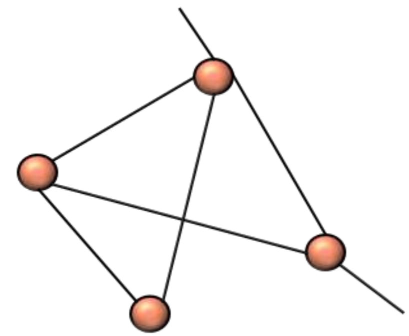
matrix A

matrix product AB

trace of matrix product $\mathrm{tr}(ABC)$



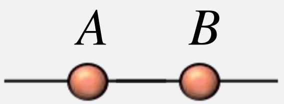
tensor contraction $f(A,B,C,D)$



Tensor Networks

*e.g. RO, Annals of Physics **349** (2014) 117–158*

$A \cdot B \rightarrow$



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p-level systems

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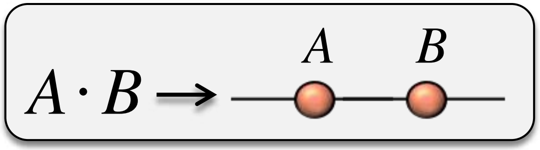

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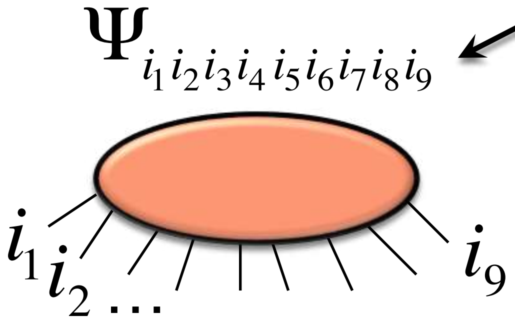

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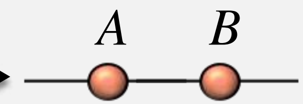

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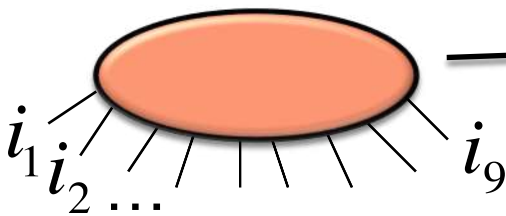
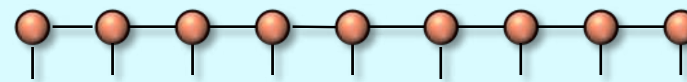
p-level systems

$\Psi_{i_1 i_2 i_3 i_4 i_5 i_6 i_7 i_8 i_9}$



1d

Matrix Product States (MPS)



DMRG, PWFRG, TEBD…

Tensor Networks

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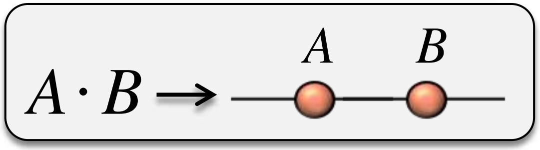

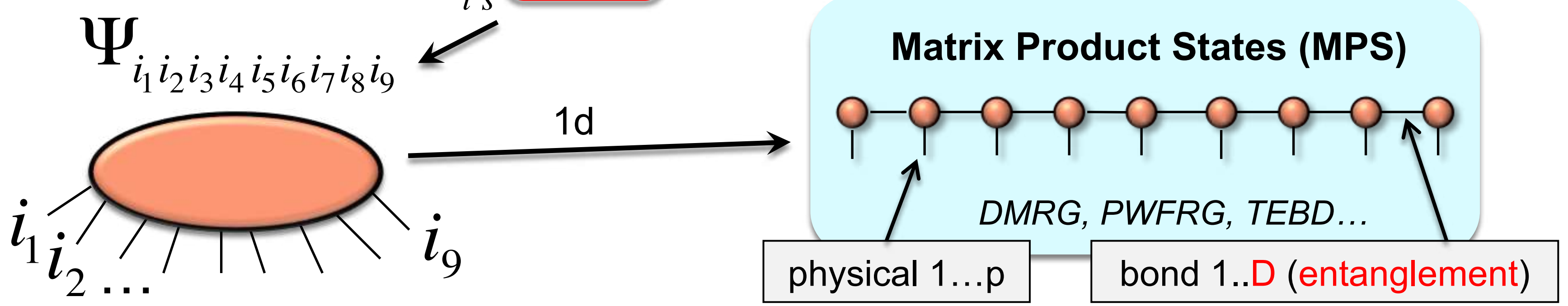

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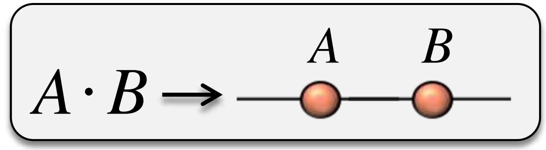

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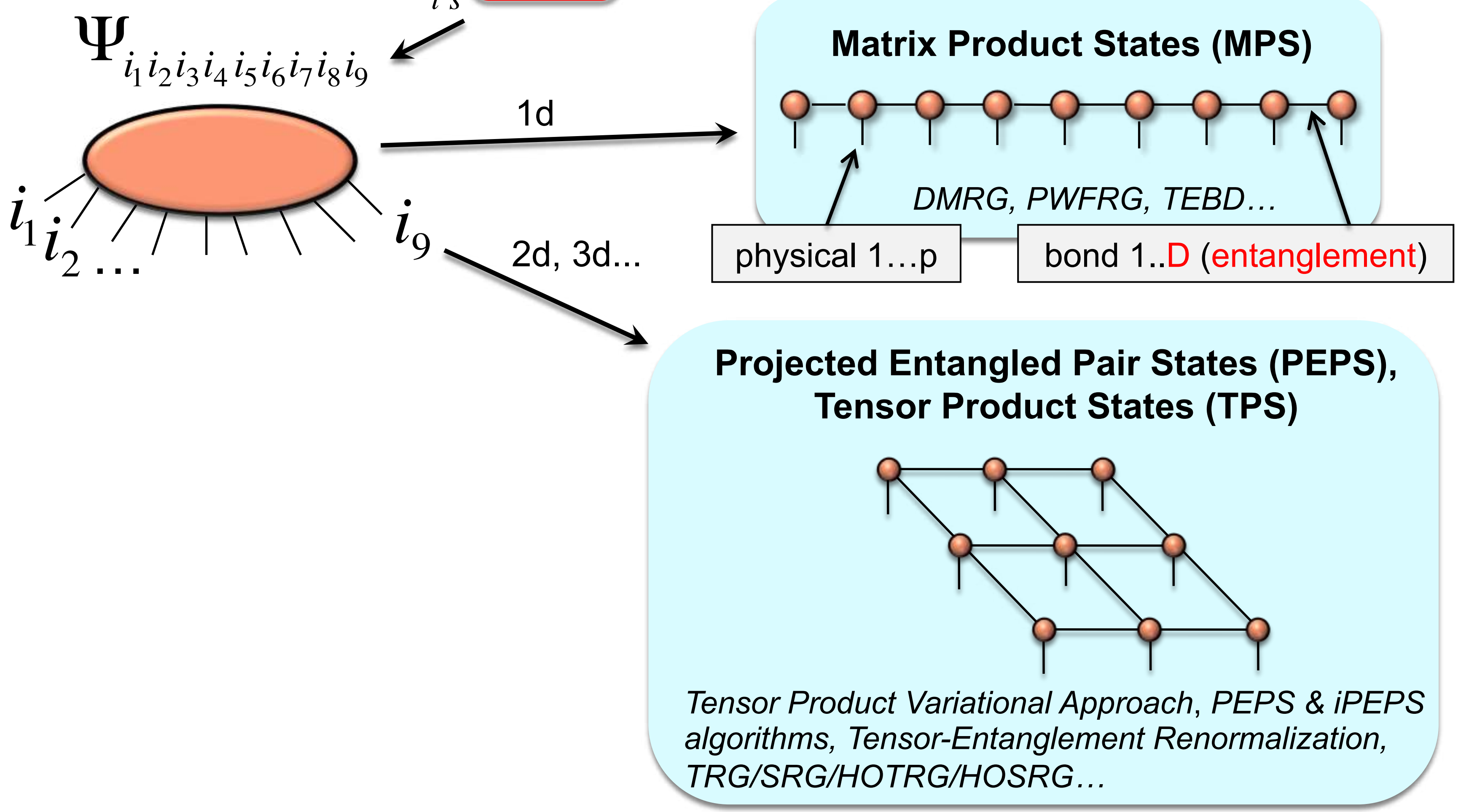

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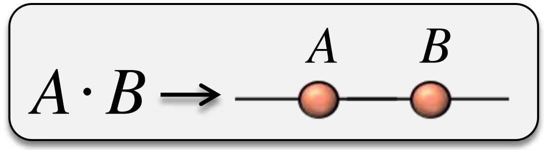

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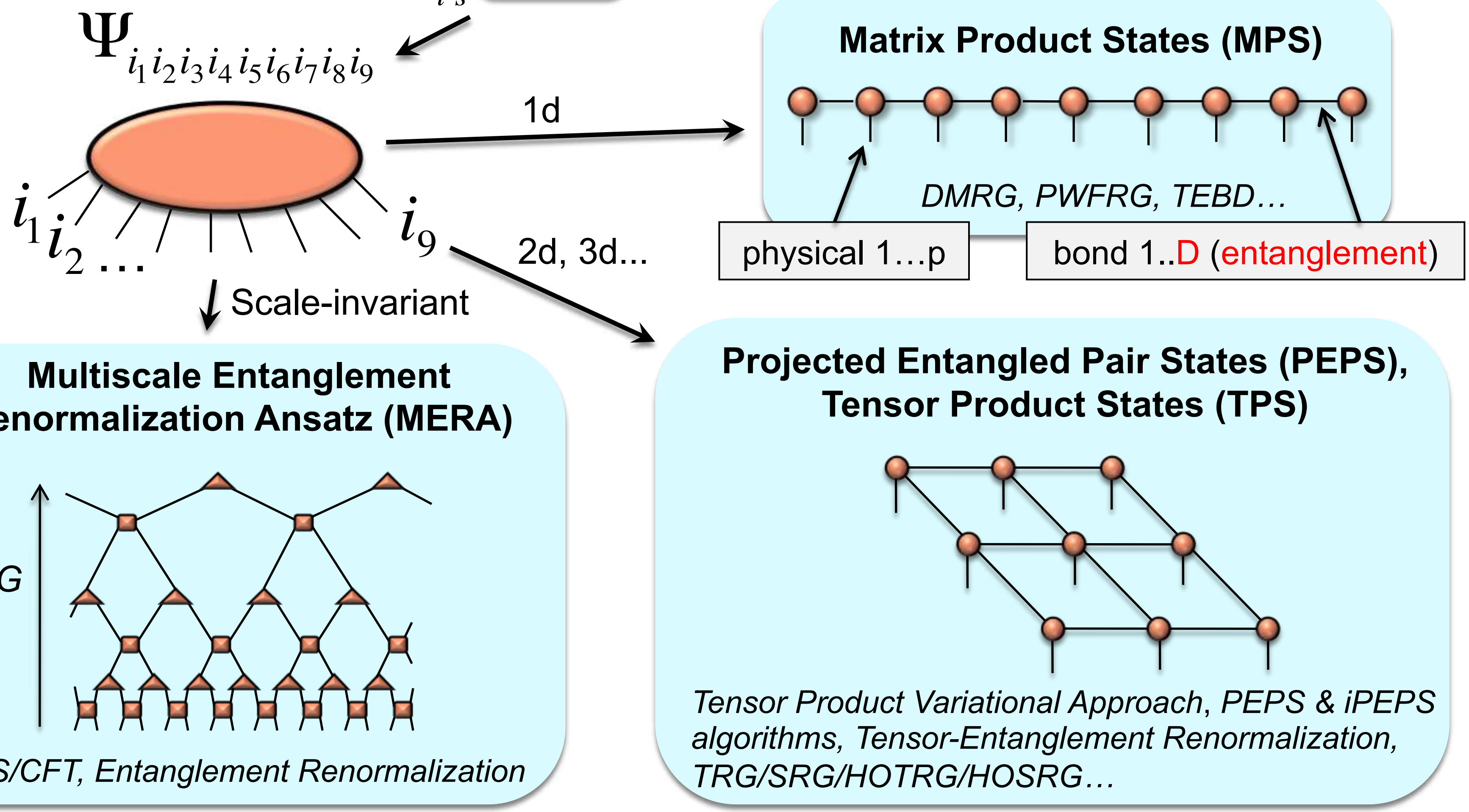


Efficient O(poly(N)), satisfy area-law, low-energy eigenstates of local Hamiltonians

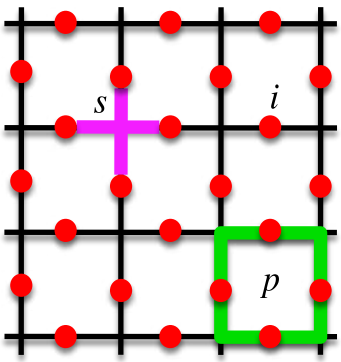
Two exact examples

$$H = -J\sum_{s} A_s - J\sum_{p} B_p \qquad A_s = \prod_{i\in s}\sigma_i^x \qquad B_p = \prod_{i\in p}\sigma_i^z$$

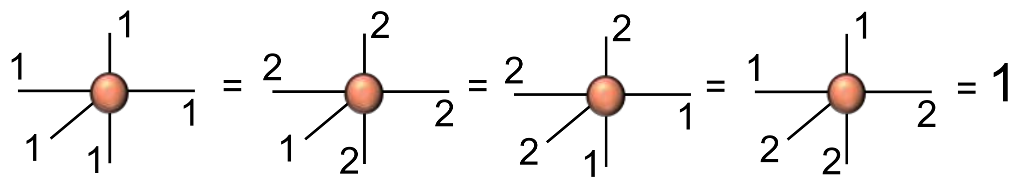
Toric Code

A. Y. Kitaev, Ann. Phys. 303, 2 (2003)

Z_2 Lattice Guage Theory



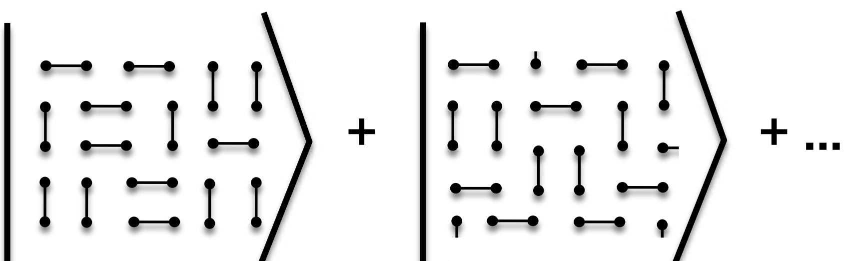
e.g., ground state is a D=2 PEPS



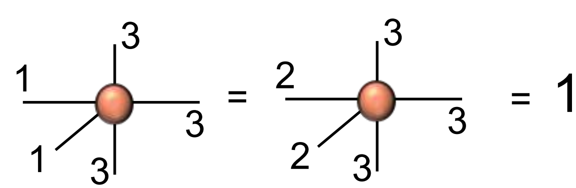
and another tensor rotated 90°

Short-range RVB

A. Anderson, 1987

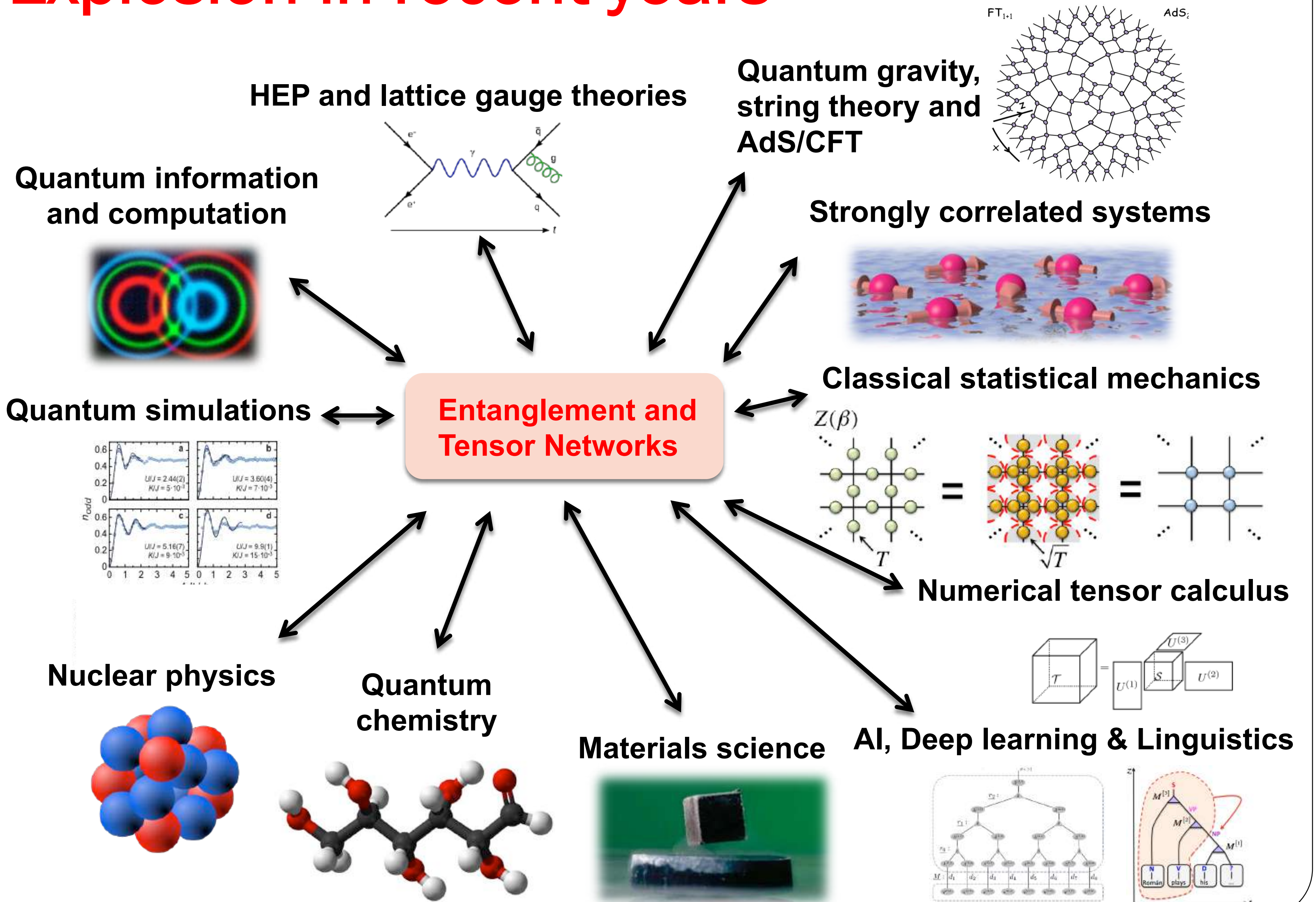


It's a D=3 PEPS



and rotations

Explosion in recent years



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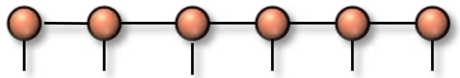

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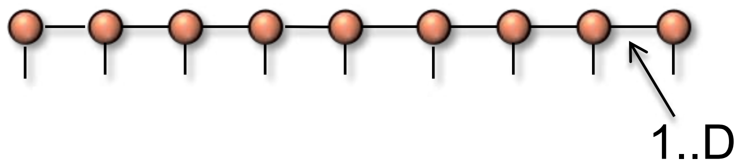
MPS obey 1d area-law

*e.g., U. Schollwöck, Ann. of Phys. **326**, 96 (2011)*

$|\Psi\rangle$



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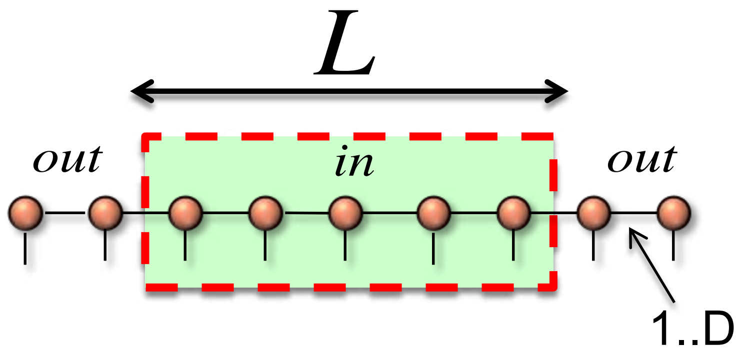

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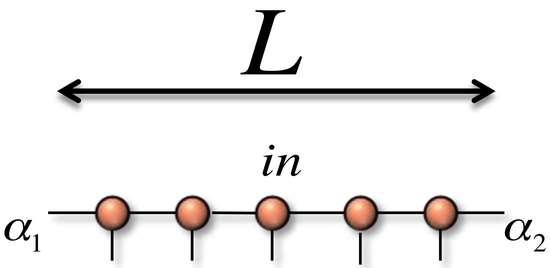

$$|in(\bar{\alpha})\rangle$$

$$\bar{\alpha} = 1, 2, \ldots, D^2$$

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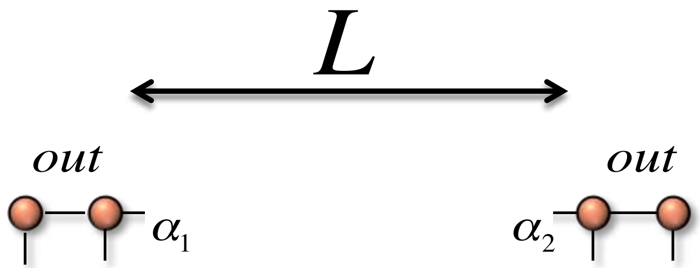

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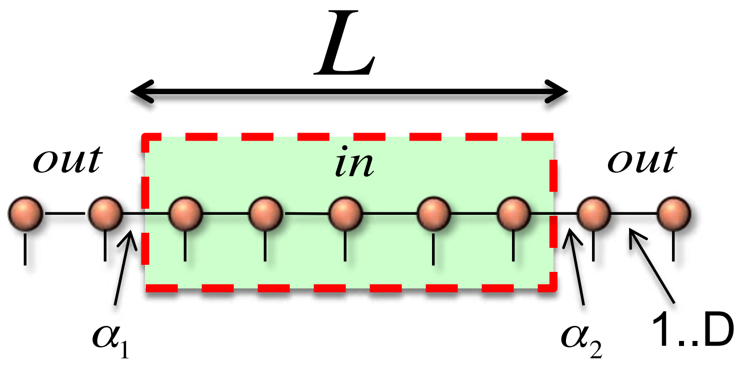

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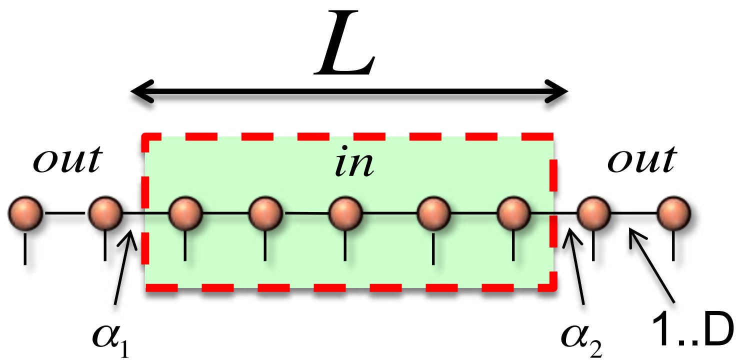

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$$\rho_{in} = \mathrm{tr}_{out}\left(|\Psi\rangle\langle\Psi|\right) = \sum_{\bar{\alpha},\bar{\alpha}'} X_{\bar{\alpha},\bar{\alpha}'} |in(\bar{\alpha})\rangle\langle in(\bar{\alpha}')|$$

$$X_{\bar{\alpha},\bar{\alpha}'} = \langle out(\bar{\alpha}')|out(\bar{\alpha})\rangle$$

$$rank(\rho_{in}) \le D^2$$

$$S(L) = -\mathrm{tr}\left(\rho_{in}\log\rho_{in}\right) \le \log(D)2$$

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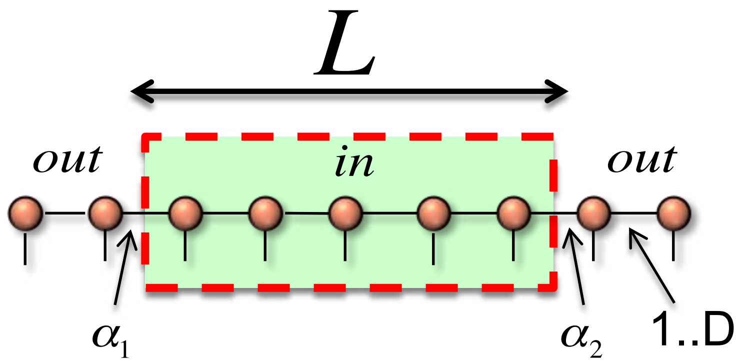

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$$S(L) = -\mathrm{tr}\left(\rho_{in} \log \rho_{in}\right) \leq \log(D)\,2$$

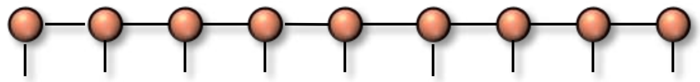
prefactor

size of the 0d boundary

MPS contraction is efficient

e.g., U. Schollwöck, Ann. of Phys. ***326****, 96 (2011)*

$|\Psi\rangle$



MPS contraction is efficient

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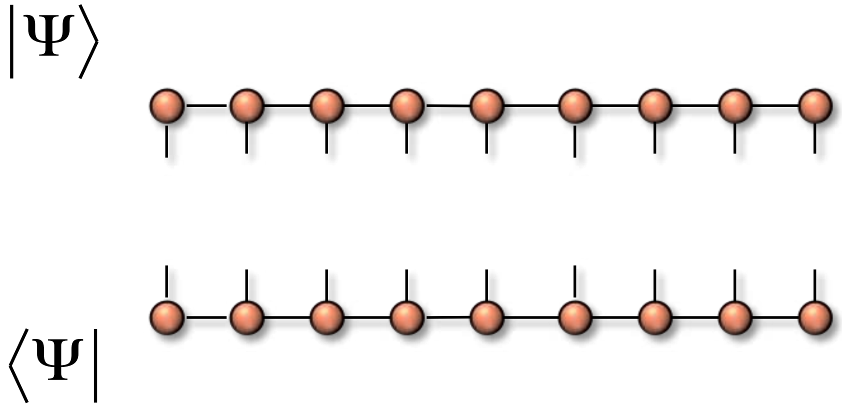

MPS contraction is efficient

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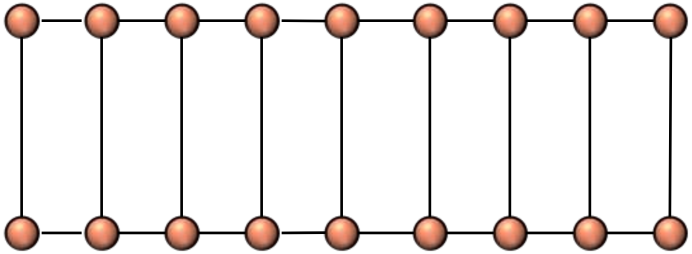
$$\langle \Psi | \Psi \rangle$$



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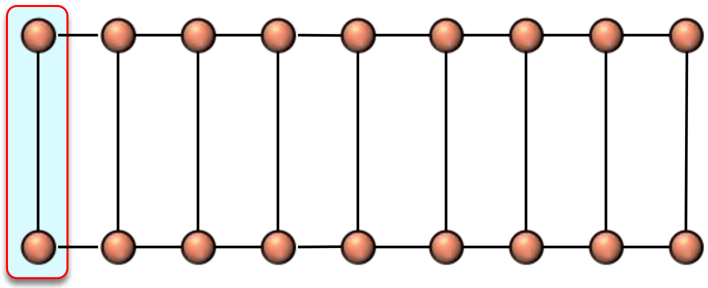

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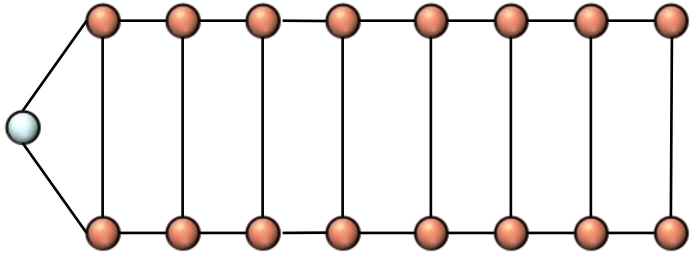
$$O(pD^2)$$

$$\langle \Psi | \Psi \rangle$$

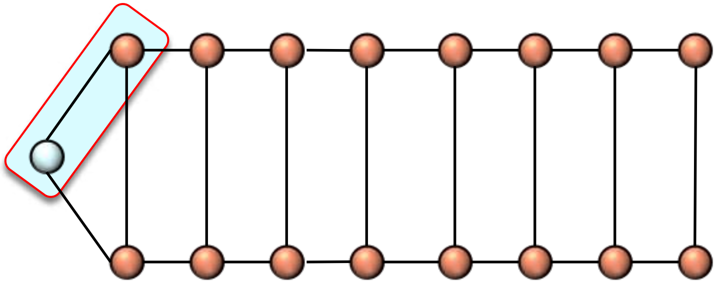


MPS contraction is efficient

e.g., U. Schollwöck, Ann. of Phys. ***326****, 96 (2011)*

$$O(pD^2)$$

$$\langle\Psi|\Psi\rangle$$

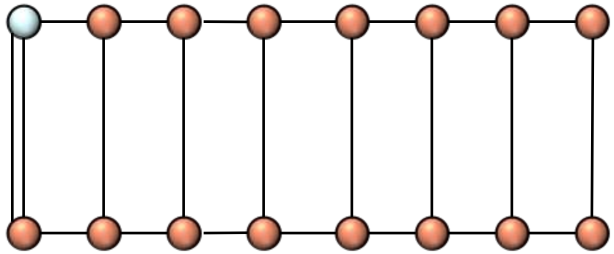


MPS contraction is efficient

*e.g., U. Schollwöck, Ann. of Phys. **326**, 96 (2011)*

$$O(pD^2) + O(pD^3)$$

$$\langle \Psi | \Psi \rangle$$



MPS contraction is efficient

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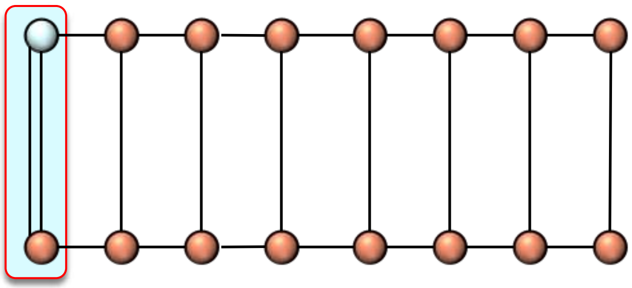
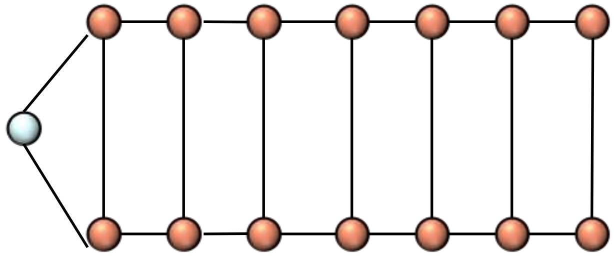


MPS contraction is efficient

e.g., U. Schollwöck, Ann. of Phys. ***326****, 96 (2011)*

$$O(pD^2) + O(pD^3) + O(pD^3)$$

$$\langle \Psi | \Psi \rangle$$

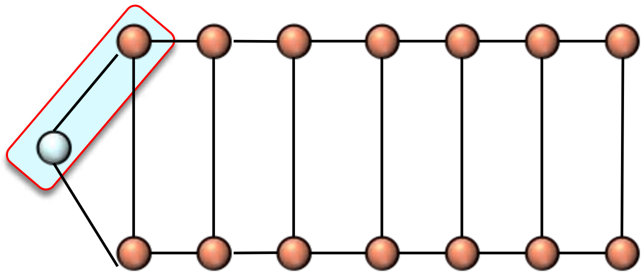


MPS contraction is efficient

*e.g., U. Schollwöck, Ann. of Phys. **326**, 96 (2011)*

$$O(pD^2) + O(pD^3) + O(pD^3) + \ldots$$

$\langle \Psi | \Psi \rangle$



... and so on...

Exact contraction in $O(NpD^3)$ time, and the same for expectation values of local observables

MPS corr. length is finite

*e.g., U. Schollwöck, Ann. of Phys. **326**, 96 (2011)*

$$C(r) = \left\langle O_i O'_{i+r} \right\rangle - \left\langle O_i \right\rangle \left\langle O'_{i+r} \right\rangle$$

MPS corr. length is finite

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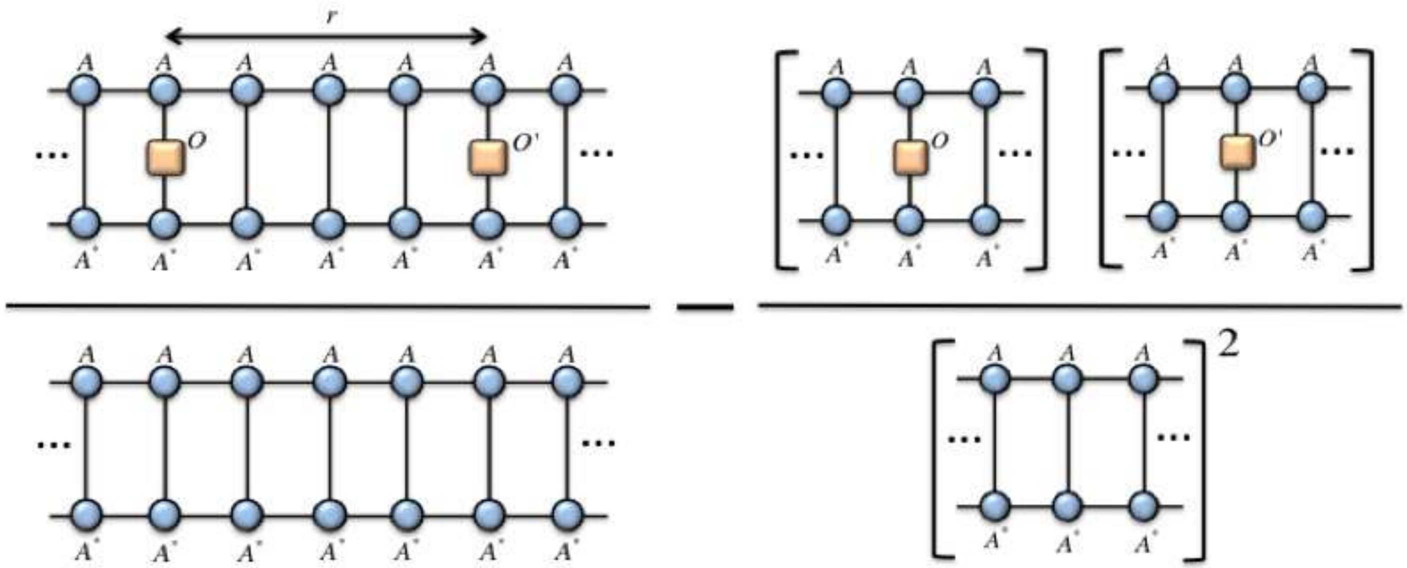
$$C(r) = \left\langle O_i O'_{i+r} \right\rangle - \left\langle O_i \right\rangle \left\langle O'_{i+r} \right\rangle$$



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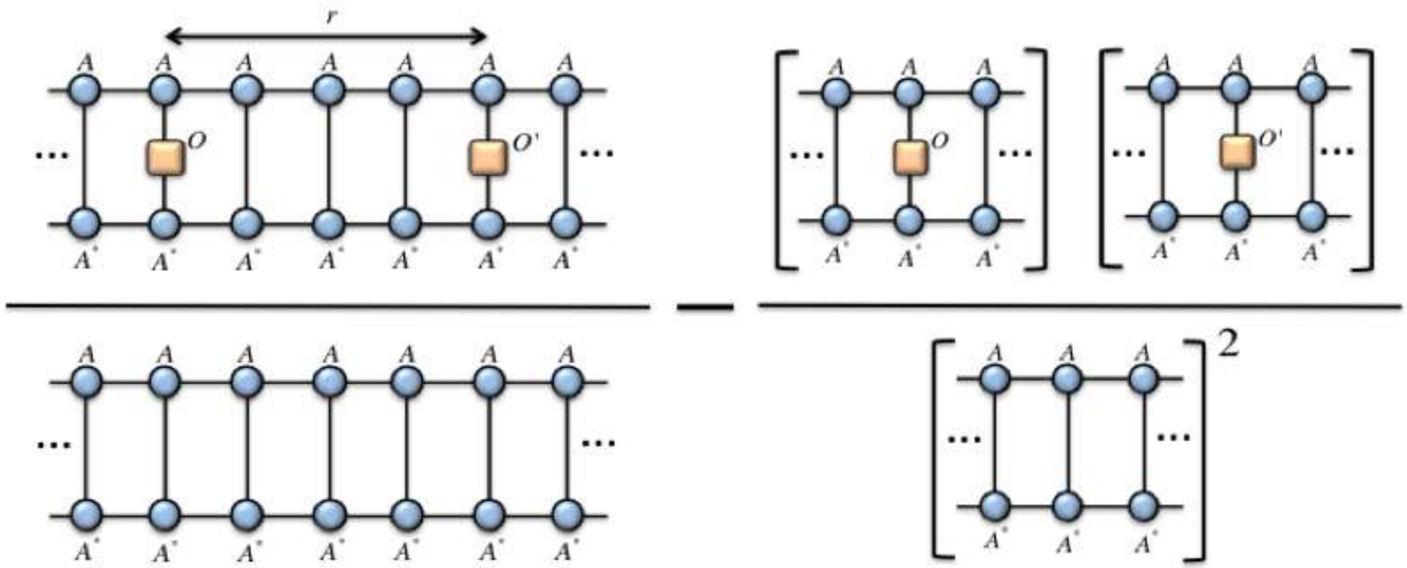
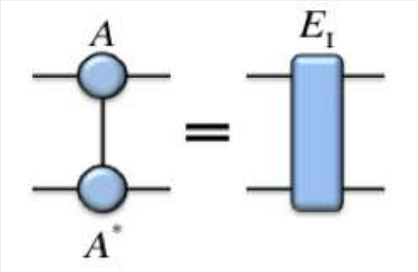


$$C(r) \approx f(r) a e^{-r/\xi}$$

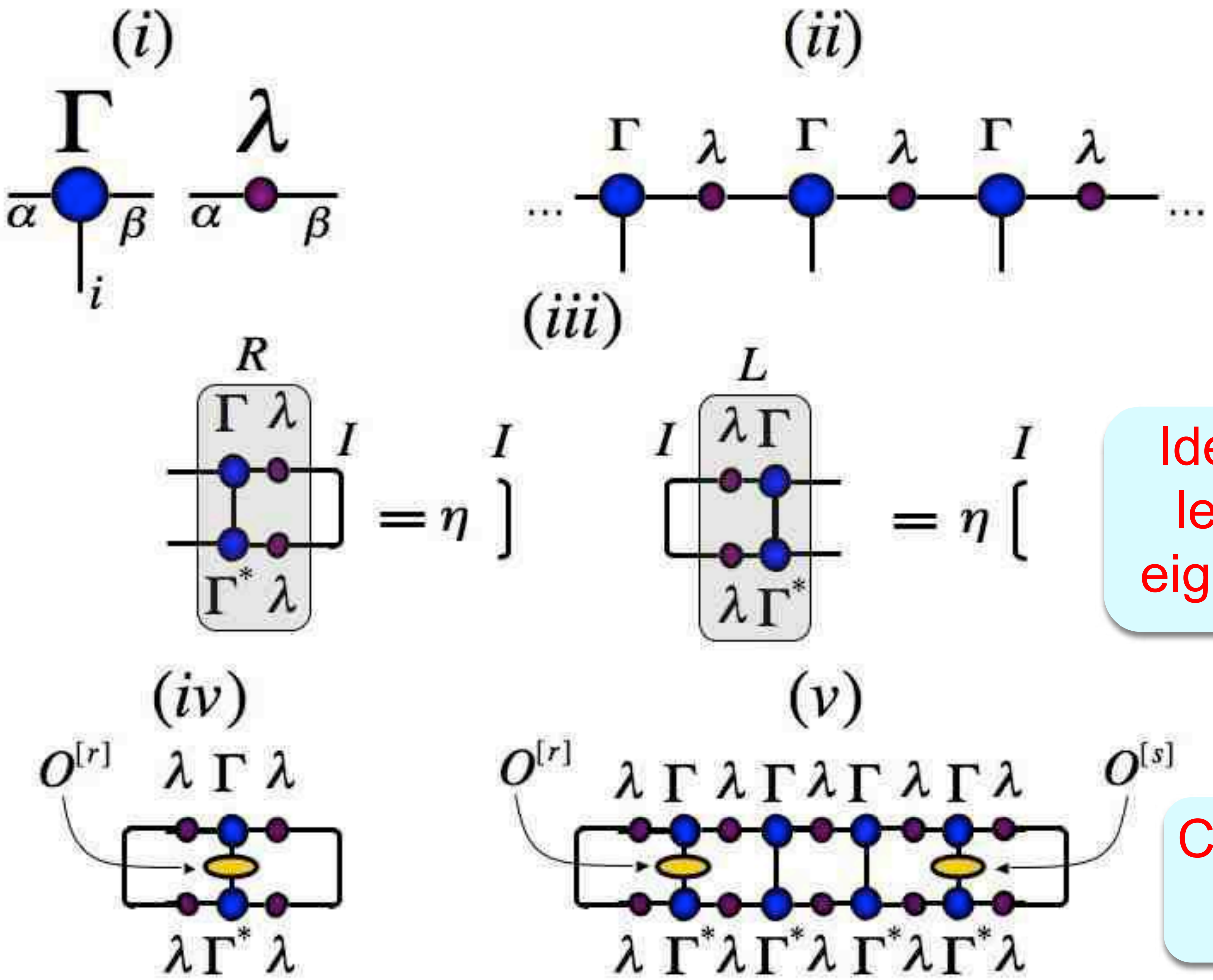
$$\xi \equiv -1/\log\left|\lambda_2/\lambda_1\right|$$

MPS transfer matrix



MPS canonical form

RO, G. Vidal, Phys. Rev. B ***78****, 155117 (2008)*

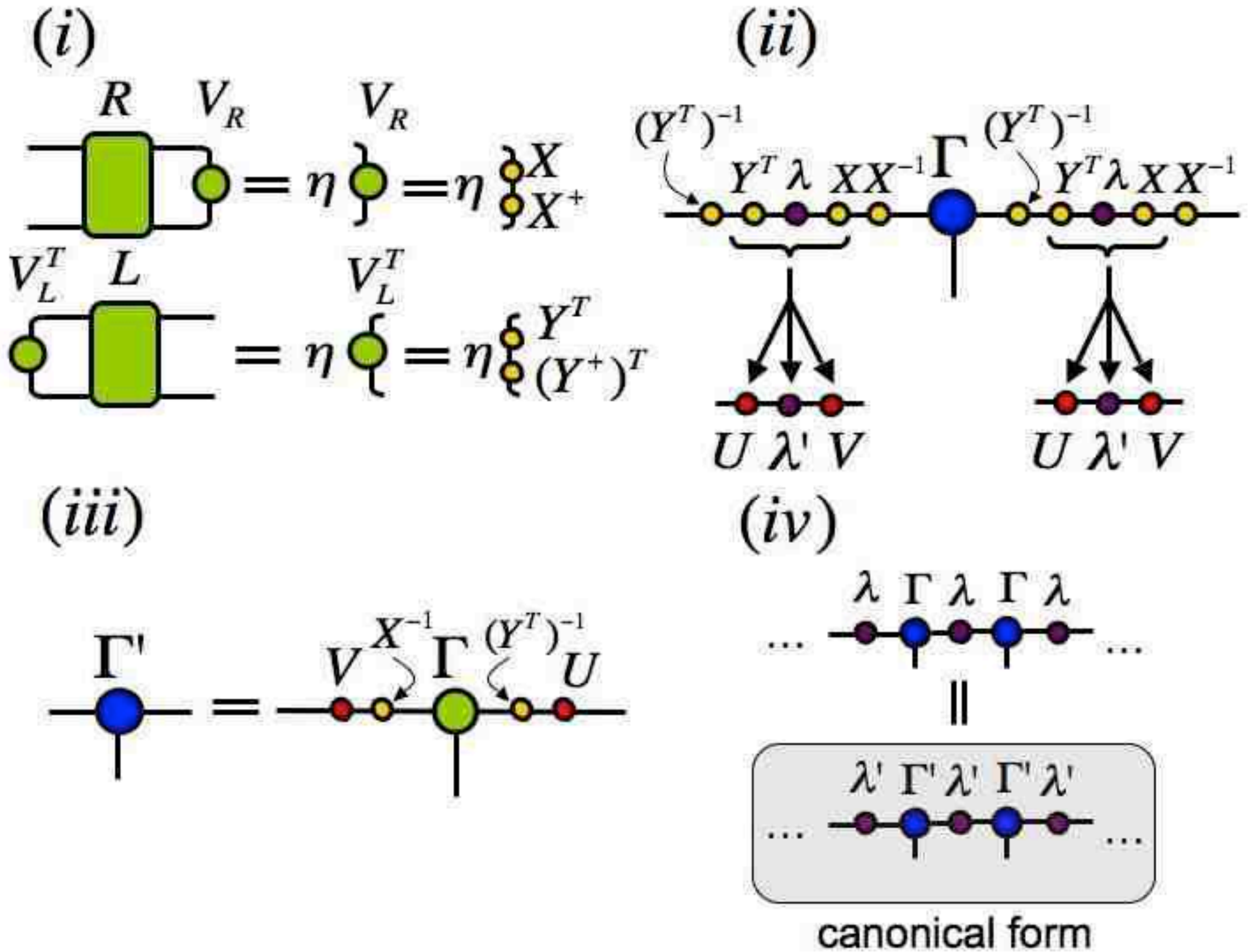


Identity is left/right eigenvector

Calculations simplify

Finding MPS canonical form

*RO, G. Vidal, Phys. Rev. B **78**, 155117 (2008)*

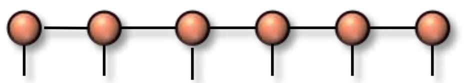


Bond indices correspond to orthonormal Schmidt basis

Outline

1) Basics

2) 1d MPS



3) 2d PEPS

4) Numerical algorithms

5) MERA

6) Extras

Outline

1) Basics

2) 1d MPS

3) 2d PEPS

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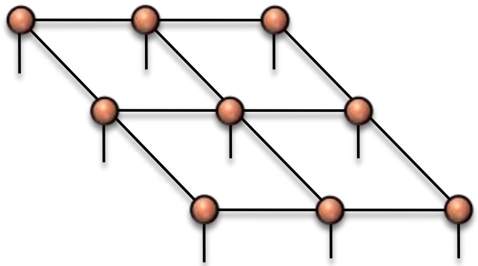
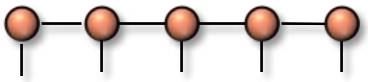
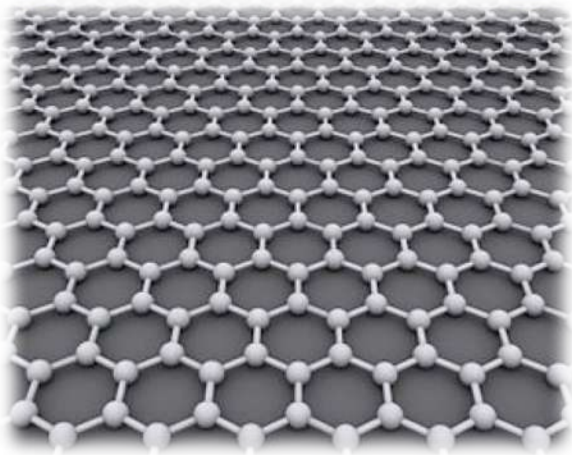
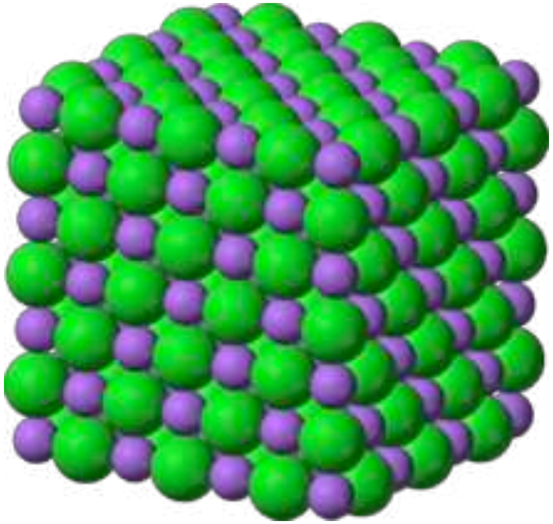


From MPS to PEPS

Matrix Product States (MPS)  *1d systems*

But we want to go beyond 1d systems!!!

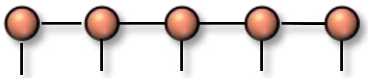




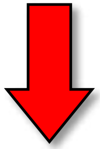
Very painful for DMRG...

From MPS to PEPS

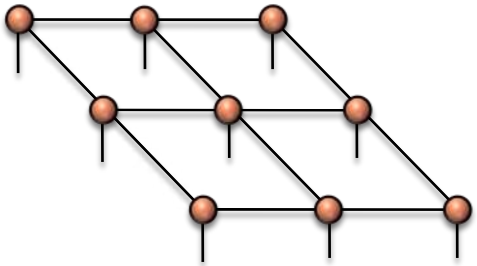
Matrix Product States (MPS)



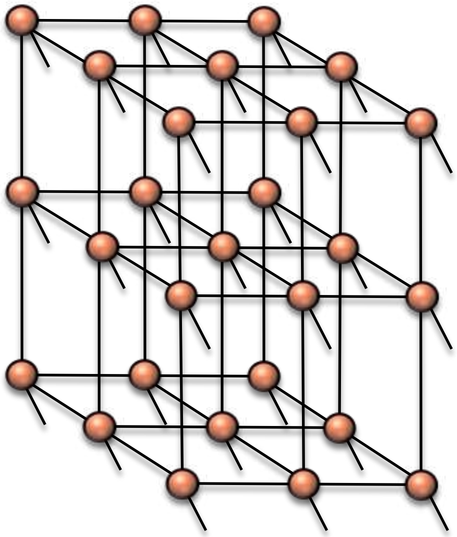
1d systems



Projected Entangled Pair States (PEPS),
Tensor Product States (TPS)



2d systems

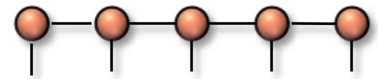


3d systems

and so on…

From MPS to PEPS

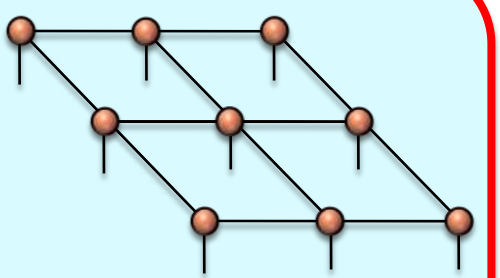
Matrix Product States (MPS)



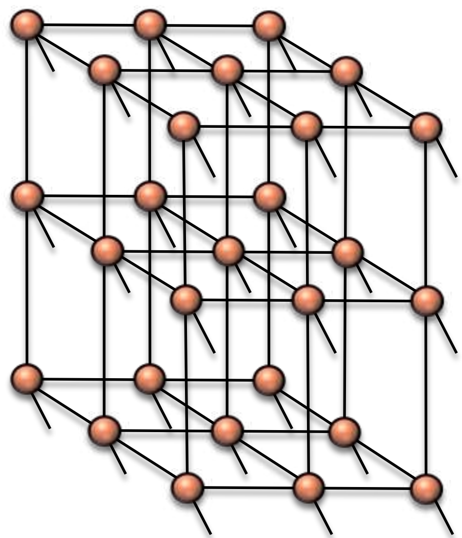
1d systems

Projected Entangled Pair States (PEPS),
Tensor Product States (TPS)

This lecture



2d systems



3d systems

and so on…

PEPS are not your friends…

MPS

PEPS are not your friends...

MPS

PEPS *(initially)*

PEPS are not your friends...

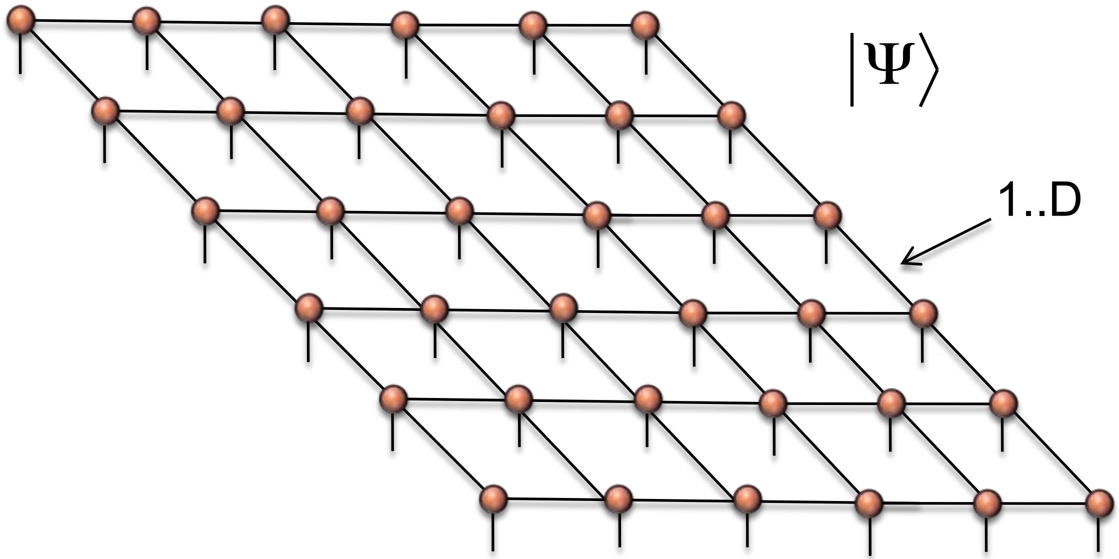
MPS

PEPS *(initially)*

PEPS *(in the end)*

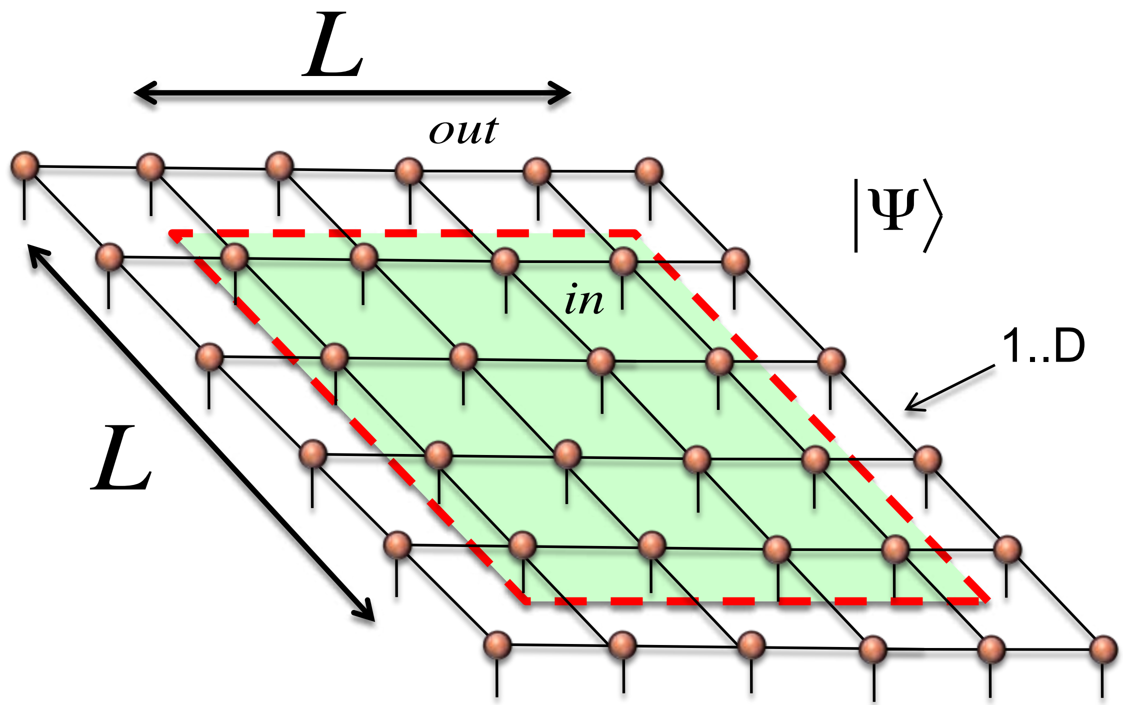
...but, after a lot of gymnastics, they can be your allies!

PEPS obey 2d area-law



e.g., F. Verstraete, I. Cirac, V. Murg, Adv. Phys. ***57****, 143 (2008)*

PEPS obey 2d area-law



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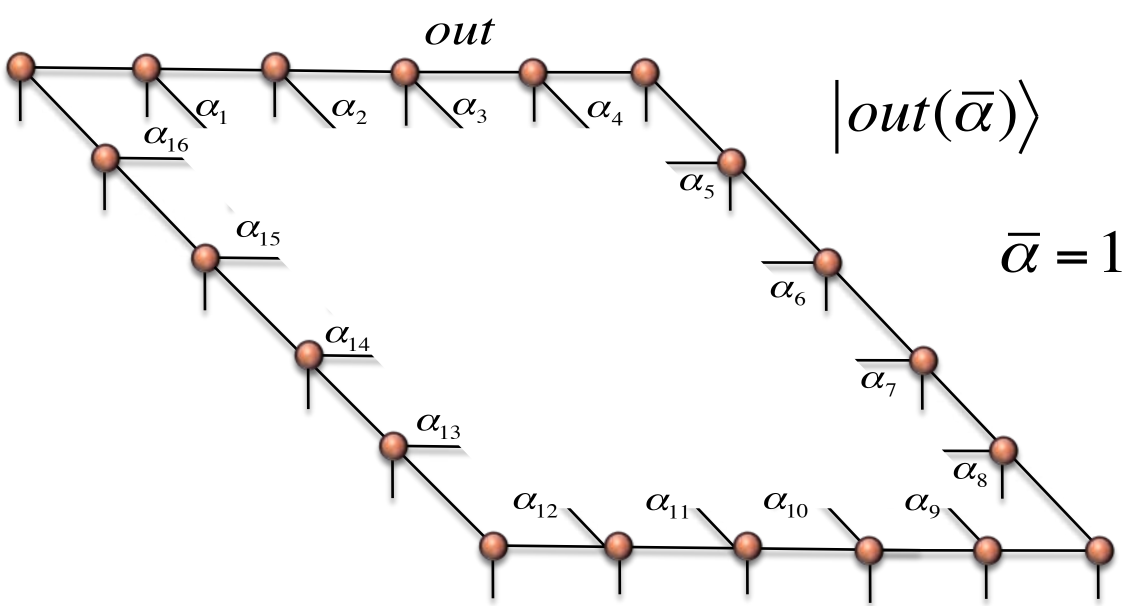
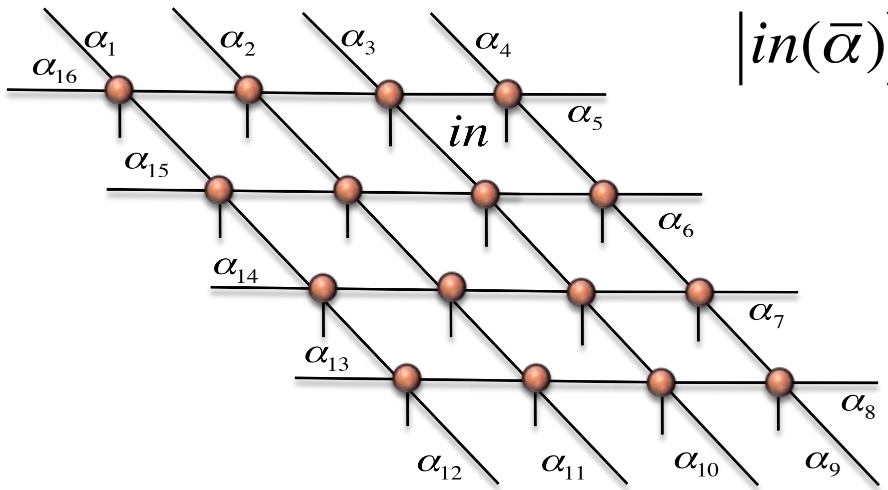


$$|out(\bar{\alpha})\rangle$$

$$\bar{\alpha} = 1, 2, \ldots, D^{4L}$$

e.g., F. Verstraete, I. Cirac, V. Murg, Adv. Phys. ***57****, 143 (2008)*

PEPS obey 2d area-law



$$|in(\bar{\alpha})\rangle$$

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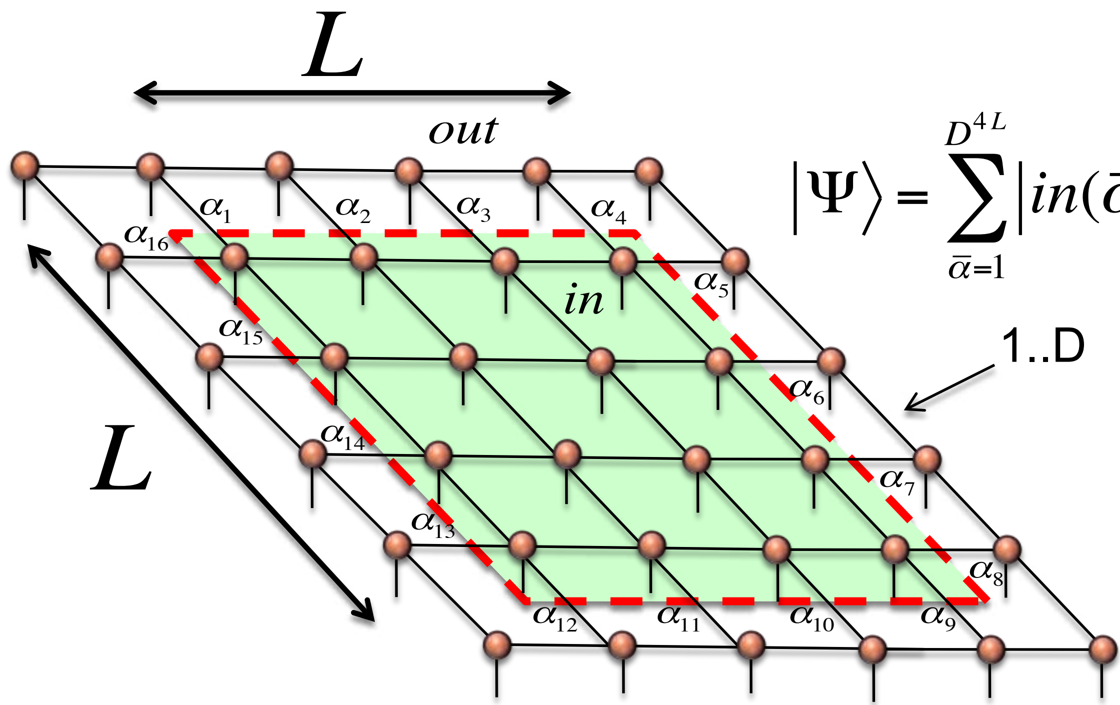
PEPS obey 2d area-law



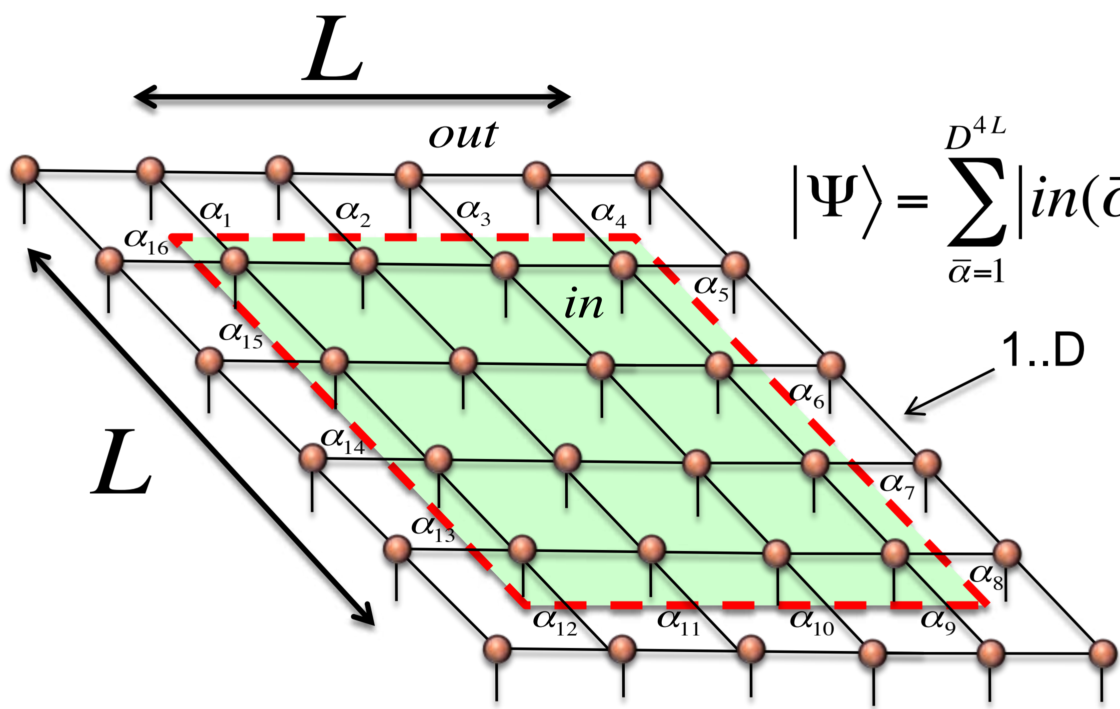
$$|\Psi\rangle = \sum_{\bar{\alpha}=1}^{D^{4L}} |in(\bar{\alpha})\rangle |out(\bar{\alpha})\rangle$$

$$\rho_{in} = \mathrm{tr}_{out}\left(|\Psi\rangle\langle\Psi|\right) = \sum_{\bar{\alpha},\bar{\alpha}'} X_{\bar{\alpha},\bar{\alpha}'} |in(\bar{\alpha})\rangle\langle in(\bar{\alpha}')| \qquad X_{\bar{\alpha},\bar{\alpha}'} = \langle out(\bar{\alpha}')|out(\bar{\alpha})\rangle$$

$$rank(\rho_{in}) \leq D^{4L} \qquad S(L) = -\mathrm{tr}\left(\rho_{in}\log\rho_{in}\right) \leq \log(D)4L$$

e.g., F. Verstraete, I. Cirac, V. Murg, Adv. Phys. ***57****, 143 (2008)*

PEPS obey 2d area-law



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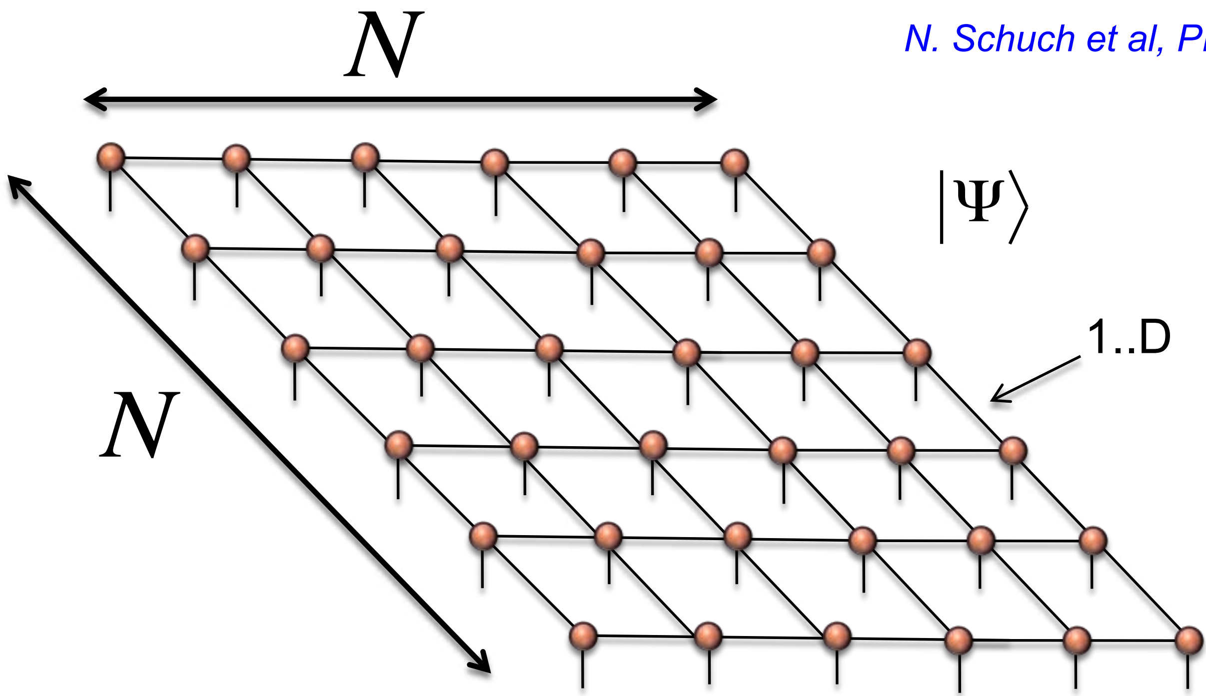
prefactor — size of the 1d boundary

e.g., F. Verstraete, I. Cirac, V. Murg, Adv. Phys. ***57****, 143 (2008)*

Exact contraction is ineficient

N. Schuch et al, PRL 98, 140506 (2007)



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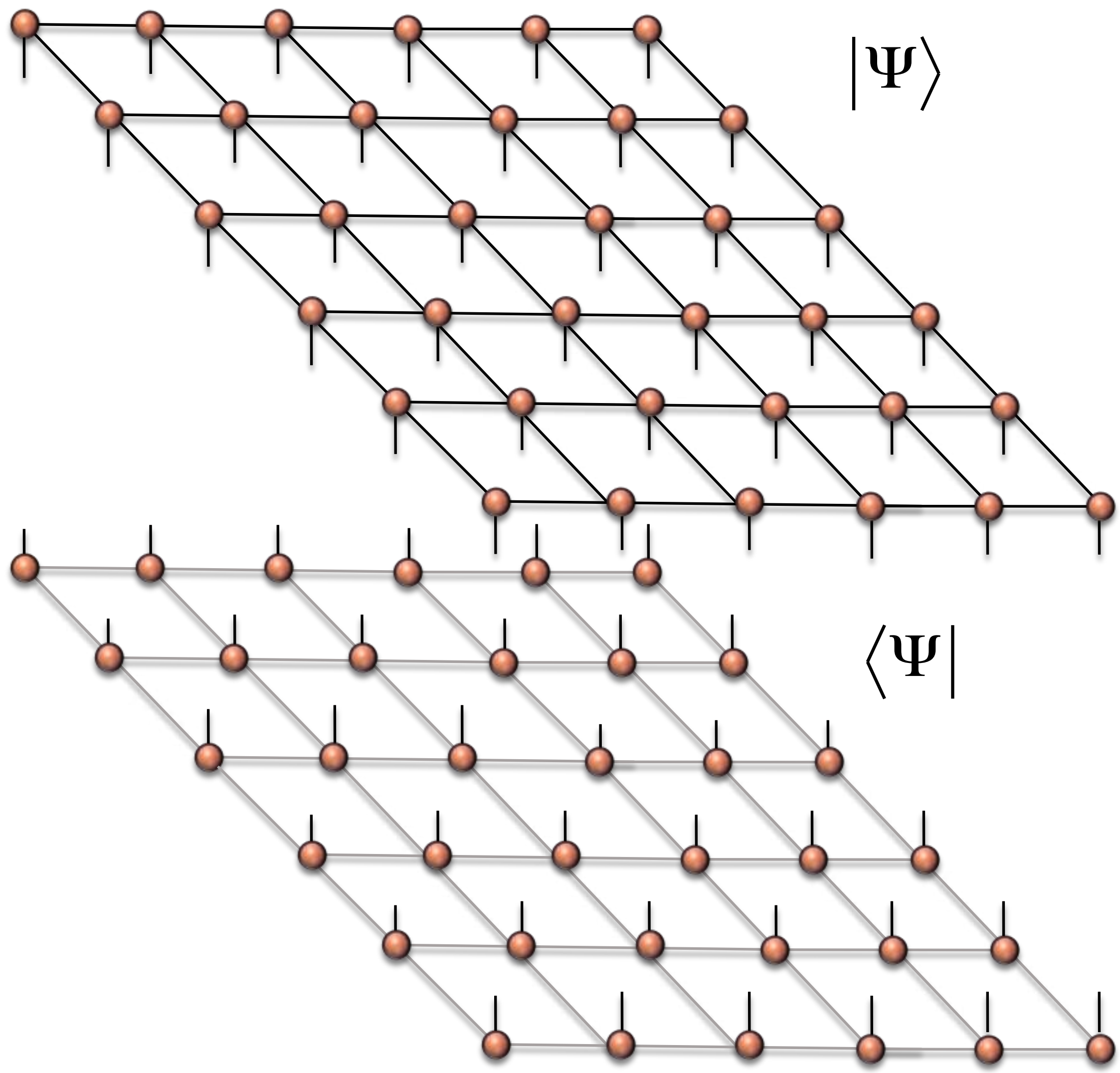

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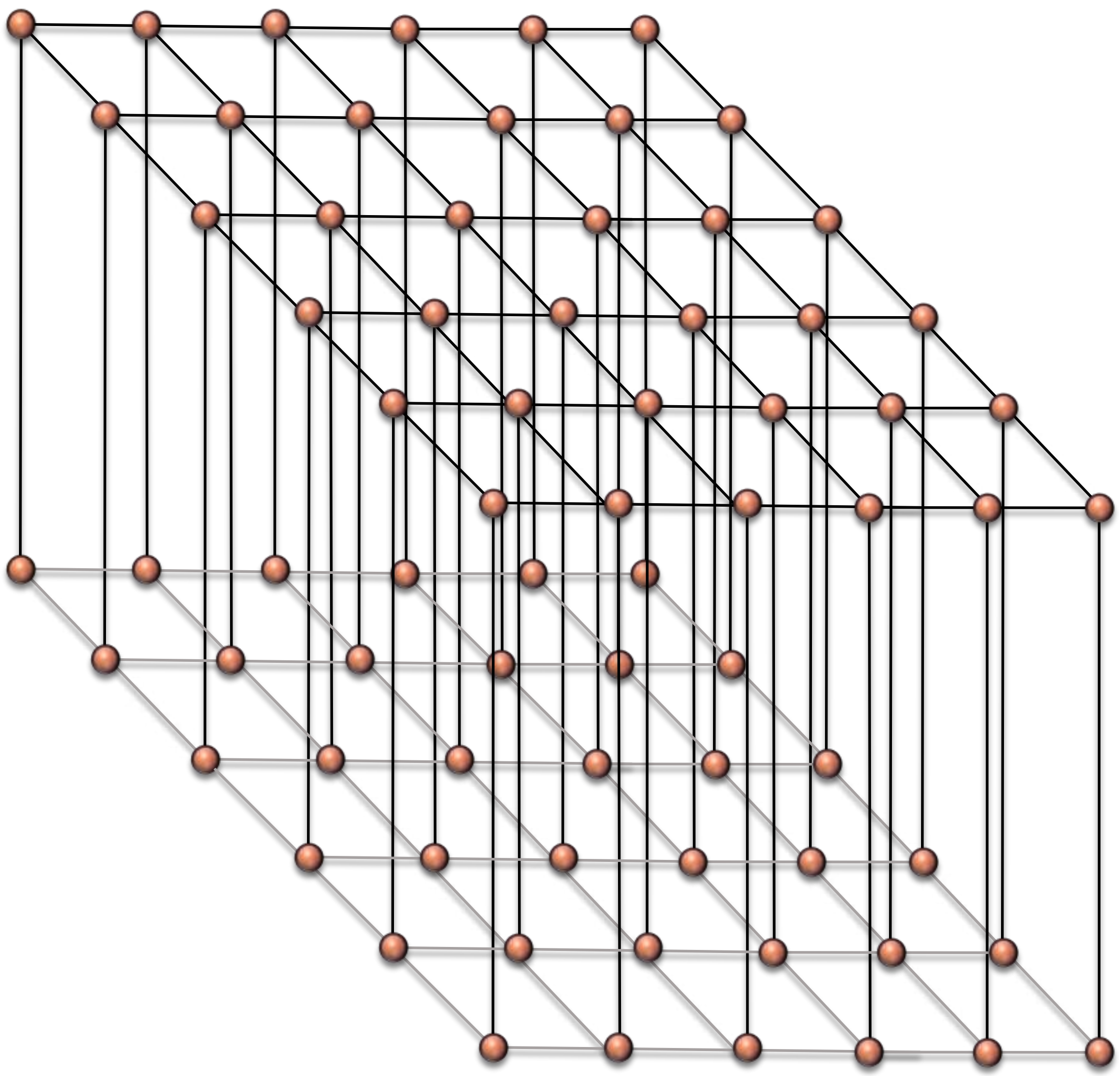

$\langle \Psi | \Psi \rangle$

Exact contraction is inneficient

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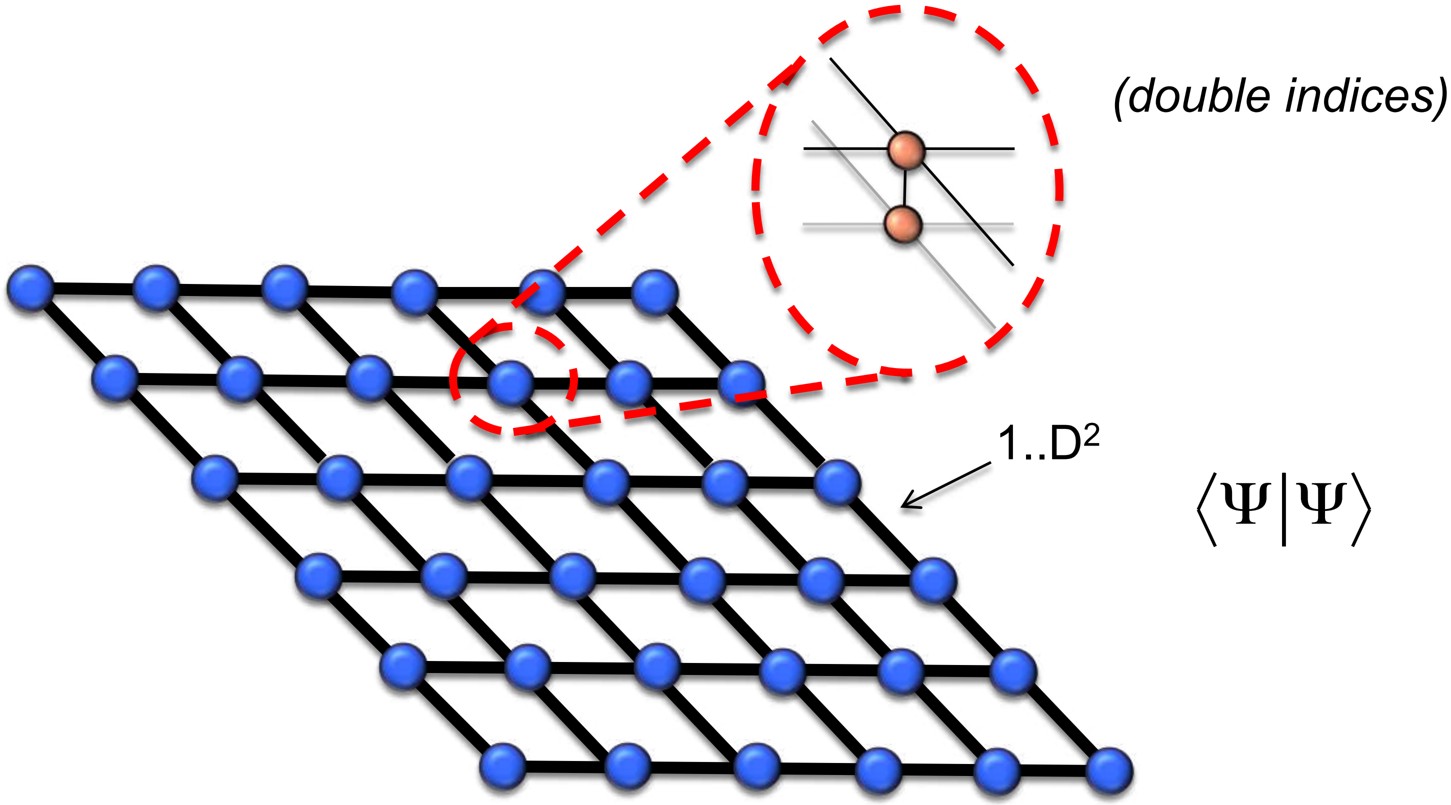

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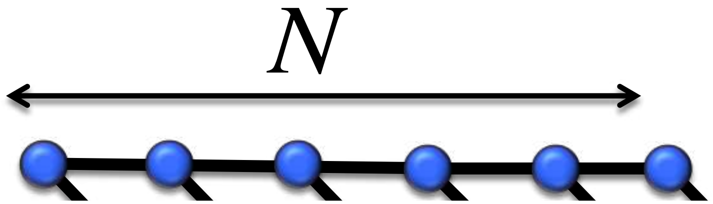

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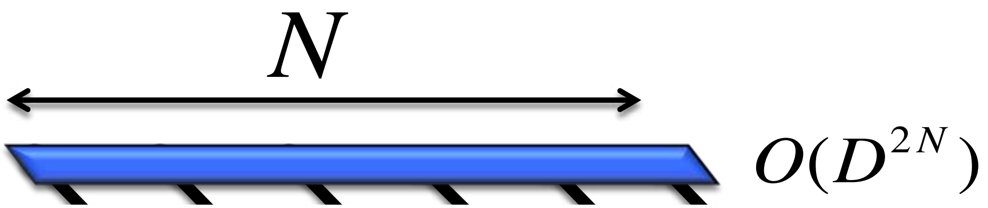

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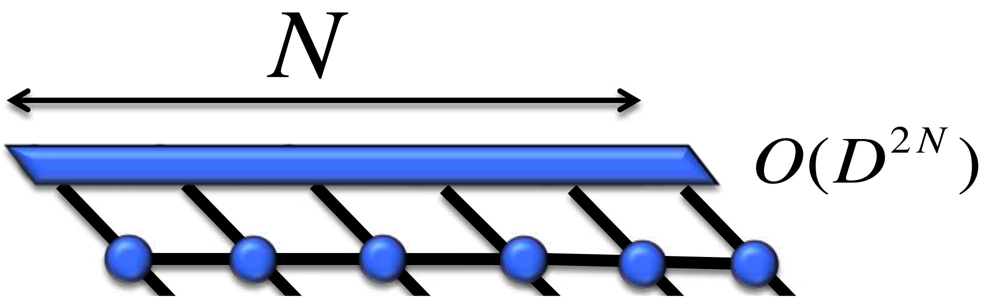

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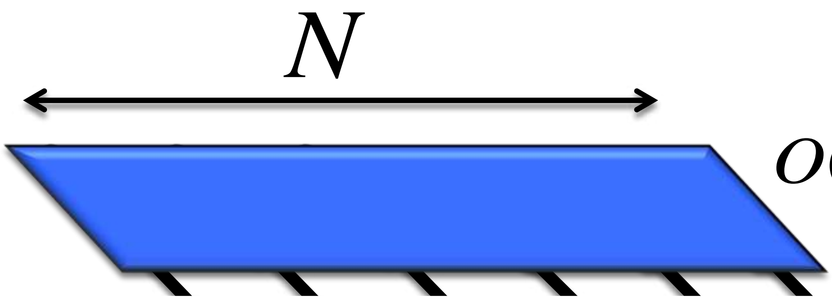
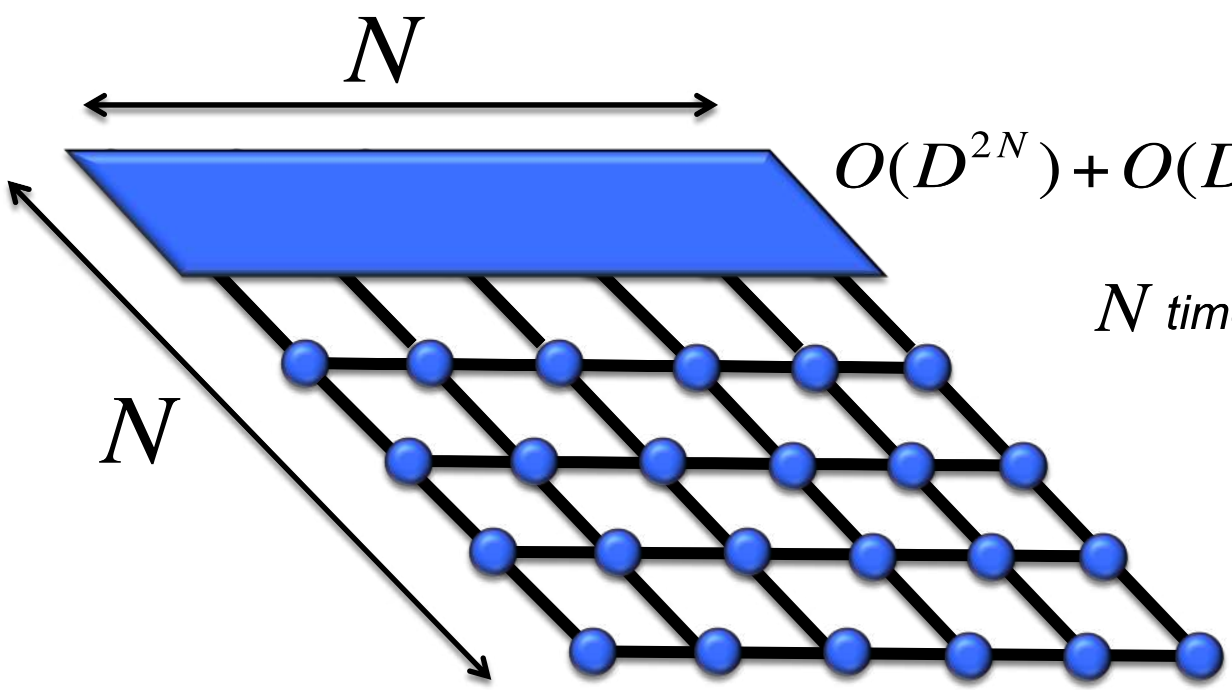


$$O(D^{2N}) + O(D^{2N})$$

Exact contraction is ineficient

N. Schuch et al, PRL 98, 140506 (2007)



$$O(D^{2N}) + O(D^{2N}) + \ldots$$

N *times*

computing time ~ $O(ND^{2N})$ Exponential amount of time!

Mathematical statement: exact contraction of a PEPS is a #P-Hard problem (harder than NP-Complete)

Applies also to expectation values of observables

Critical correlation functions

F. Verstraete et al, PRL 96, 220601 (2006)

$$\left|\Psi(\beta)\right\rangle = \frac{1}{\sqrt{Z(\beta)}} \exp\left(\frac{\beta}{2} \sum_{\langle i,j \rangle} \sigma_z^i \sigma_z^j \right) \left| +, + \ldots + \right\rangle$$

Expectation values are those of the classical 2d Ising model

$$\left\langle \sigma_z^r \sigma_z^{r'} \right\rangle_\beta = \frac{1}{Z(\beta)} \sum_{\{s\}} s^r s^{r'} \exp\left(\beta \sum_{\langle i,j \rangle} s^i s^j \right) \qquad s = \pm 1$$

Critical correlation functions

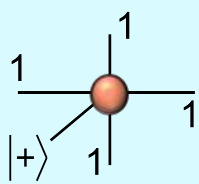
F. Verstraete et al, PRL 96, 220601 (2006)

$$|\Psi(\beta)\rangle = \frac{1}{\sqrt{Z(\beta)}} \exp\left(\frac{\beta}{2}\sum_{\langle i,j\rangle} \sigma_z^i \sigma_z^j\right) |+,+\ldots+\rangle$$

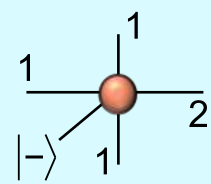
Expectation values are those of the classical 2d Ising model

$$\langle \sigma_z^r \sigma_z^{r'} \rangle_\beta = \frac{1}{Z(\beta)} \sum_{\{s\}} s^r s^{r'} \exp\left(\beta \sum_{\langle i,j\rangle} s^i s^j\right) \qquad s = \pm 1$$

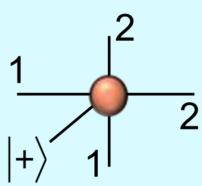
It is a PEPS with D=2:



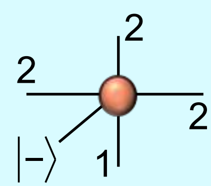
$= \left(\cosh(\beta/2)\right)^4$



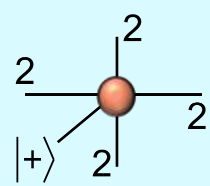
$= \left(\cosh(\beta/2)\right)^3 \left(\sinh(\beta/2)\right)$



$= \left(\cosh(\beta/2)\right)^2 \left(\sinh(\beta/2)\right)^2$



$= \left(\cosh(\beta/2)\right) \left(\sinh(\beta/2)\right)^3$



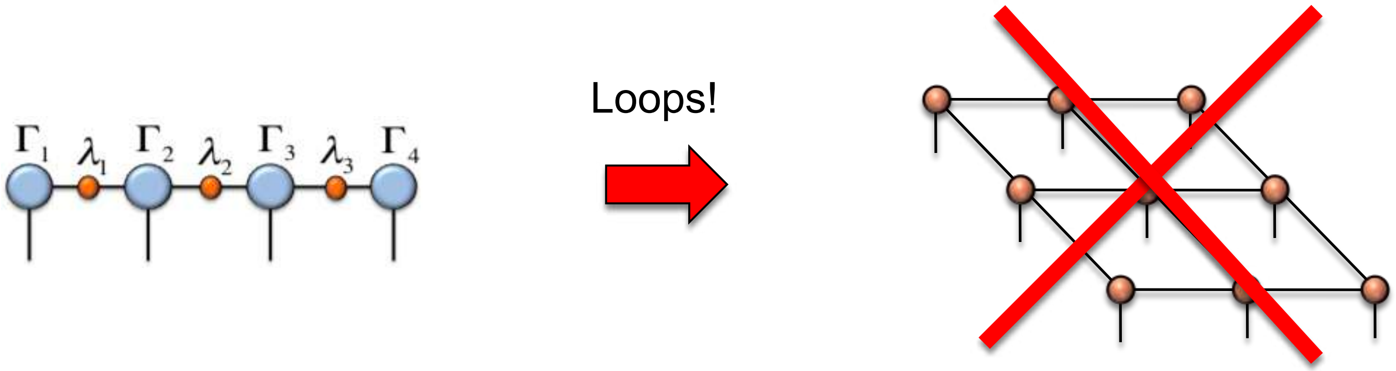
$= \left(\sinh(\beta/2)\right)^4$

+ permutations

At $\beta_c = \left(\log(1+\sqrt{2})\right)/2$ the correlation length is infinite: $\langle \sigma_z^r \sigma_z^{r'} \rangle_{\beta_c} \approx \frac{a}{|r-r'|^{1/4}}$

One more thing about PEPS

They have no exact canonical form…



…but there are approximate versions

M. P. Zelatel, F. Pollmann, arXiv:1902.05100
R. Haghshenas, M. J. O'Rourke, G. K.-Lic Chan, arXiv:1903.03843

Outline

1) Basics

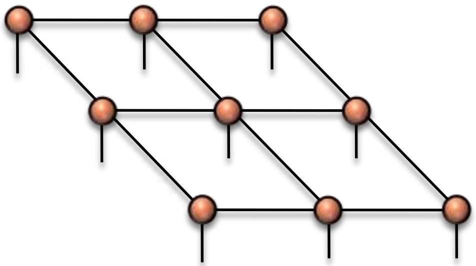
2) 1d MPS

3) 2d PEPS

4) Numerical algorithms

5) MERA

6) Extras



Outline

1) Basics

2) 1d MPS

3) 2d PEPS

4) Numerical algorithms

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6) Extras

Tensor Networks as an ansatz

Variational optimization

(e.g. DMRG)

$$\min_{|\Psi\rangle \in TN} \langle \Psi | H | \Psi \rangle / \langle \Psi | \Psi \rangle$$

ground states

*e.g., S. White, PRL **69**, 2863 (1992)*

Real/Imaginary time evolution

(e.g. TEBD)

$$e^{-iHt} |\Psi\rangle \qquad e^{-Ht} |\Psi\rangle$$

dynamics ***ground states***

*e.g., G. Vidal, PRL **91**, 147902 (2003)*

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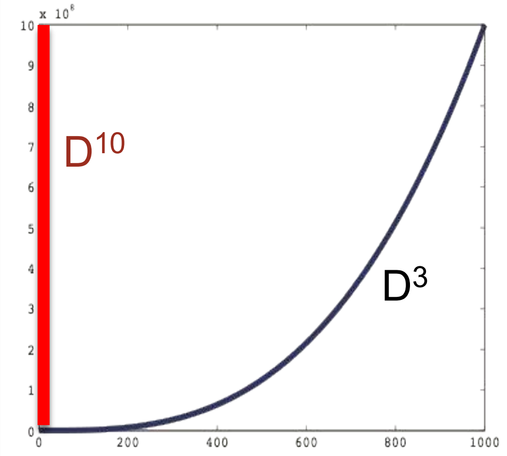
- MPS methods for **1d are very efficient** (e.g. DMRG obc → D^3)
- **2d PEPS** ~ D^{10}. **But low D expected** because of high connectivity, or entanglement monogamy**. D=2 can be critical.**
- **Infinite lattices** for translation-invariant systems (thermodynamic limit)
- **Internal symmetries, fermionic systems, continuum limit** (cMPS → quantum field theories)
- **Limitation:** amount and structure of **entanglement**



Examples: DMRG and TEBD

VOLUME 69, NUMBER 19 PHYSICAL REVIEW LETTERS 9 NOVEMBER 1992

Density Matrix Formulation for Quantum Renormalization Groups

Steven R. White

Department of Physics, University of California, Irvine, California 92717

(Received 22 May 1992)

A generalization of the numerical renormalization-group procedure used first by Wilson for the Kondo problem is presented. It is shown that this formulation is optimal in a certain sense. As a demonstration of the effectiveness of this approach, results from numerical real-space renormalization-group calculations for Heisenberg chains are presented.

PACS numbers: 75.10.Jm, 02.70.+d, 05.30.−d

Efficient simulation of one-dimensional quantum many-body systems

Guifré Vidal[1]

[1]*Institute for Quantum Information, California Institute of Technology, Pasadena, CA 91125, USA*

(Dated: February 1, 2008)

We present a numerical method to simulate the time evolution, according to a Hamiltonian made of local interactions, of quantum spin chains and systems alike. The efficiency of the scheme depends on the amount of the entanglement involved in the simulated evolution. Numerical analysis indicate that this method can be used, for instance, to efficiently compute time-dependent properties of low-energy dynamics of sufficiently regular but otherwise arbitrary one-dimensional quantum many-body systems.

PACS numbers: 03.67.-a, 03.65.Ud, 03.67.Hk

Variational optimization

$$\min\left(\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}\right)$$

Optimize over each tensor individually and sweep over the entire system (e.g., DMRG)

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$$|\Psi\rangle \xrightarrow{\min_{A^1}} |\Psi^1\rangle \xrightarrow{\min_{A^2}} \cdots \xrightarrow{\min_{A^N}} |\Psi^N\rangle \xrightarrow{\min_{A^{N-1}}} |\Psi^{N+1}\rangle \xrightarrow{\min_{A^{N-2}}} \cdots$$

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$$A^{*i}$$

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle\right) = 0$$

$$A^{i}$$

Minimization of quadratic function

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Minimization of quadratic function

$\mathbf{H}^i_{eff} \vec{A}^i = \lambda \mathbf{N}^i \vec{A}^i$ Generalized eigenvalue problem

Once $\mathbf{H}^i_{eff}$ and $\mathbf{N}^i$ are known, we can solve this problem efficiently

$\mathbf{H}^i_{eff}$ and $\mathbf{N}^i$ are exact in 1d, and approximate in 2d

e.g. 2d calculation of $\mathbf{N}^i \vec{A}^i$

F. Verstraete, I. Cirac, cond-mat/0407066

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle\right) = 0 \quad \Rightarrow \quad \mathbf{H}^i_{eff}\vec{A}^i = \lambda \mathbf{N}^i \vec{A}^i$$

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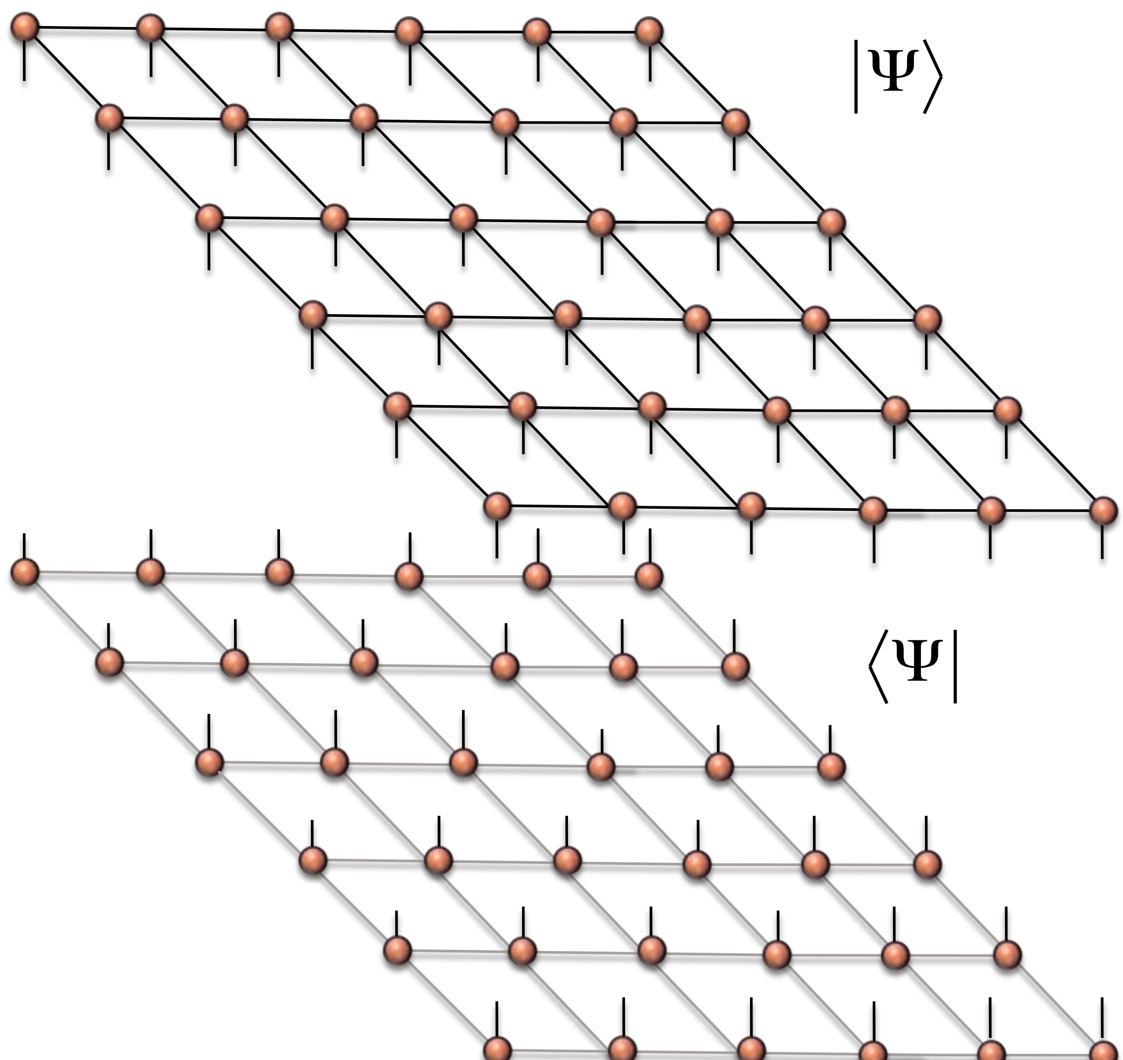

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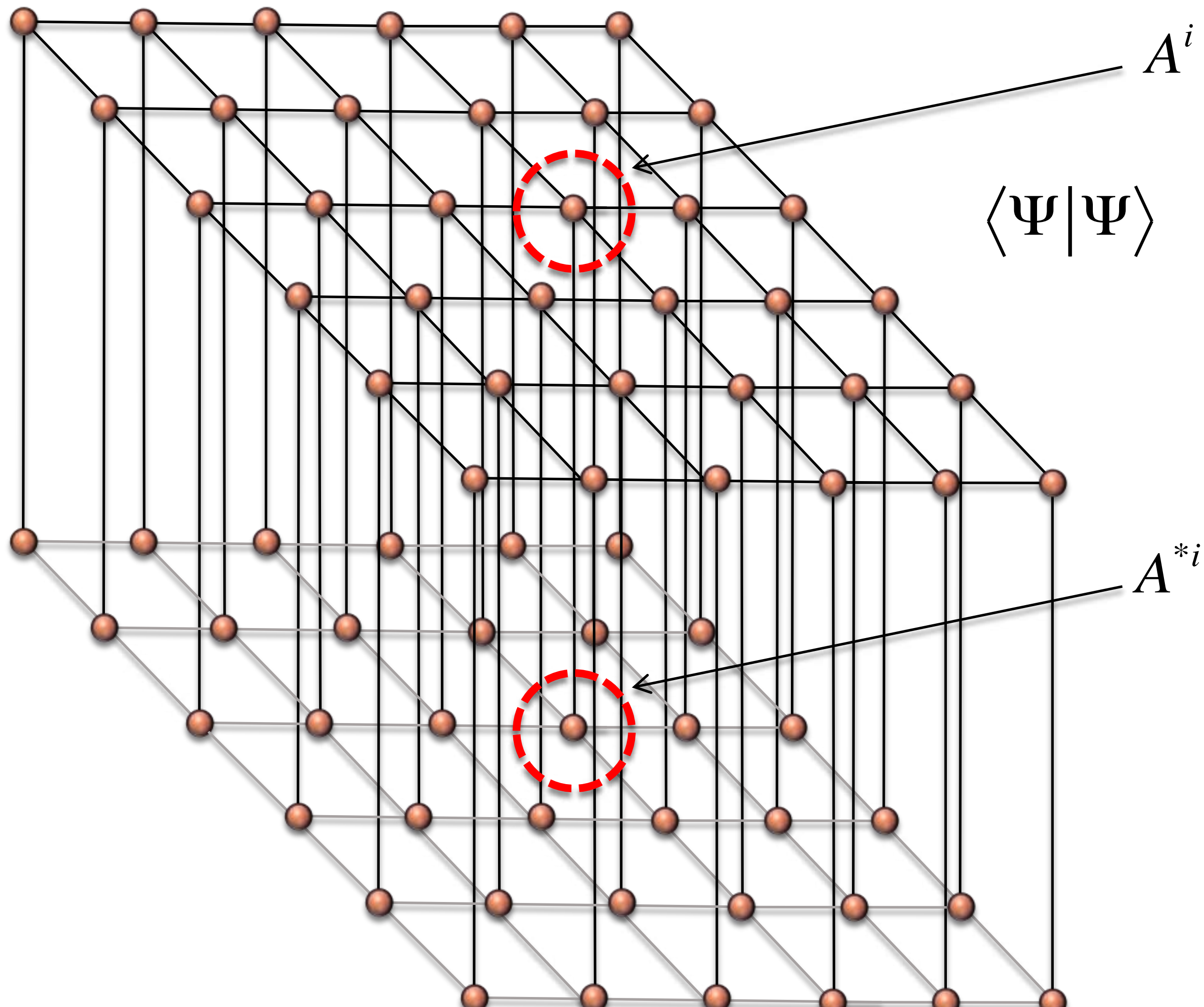

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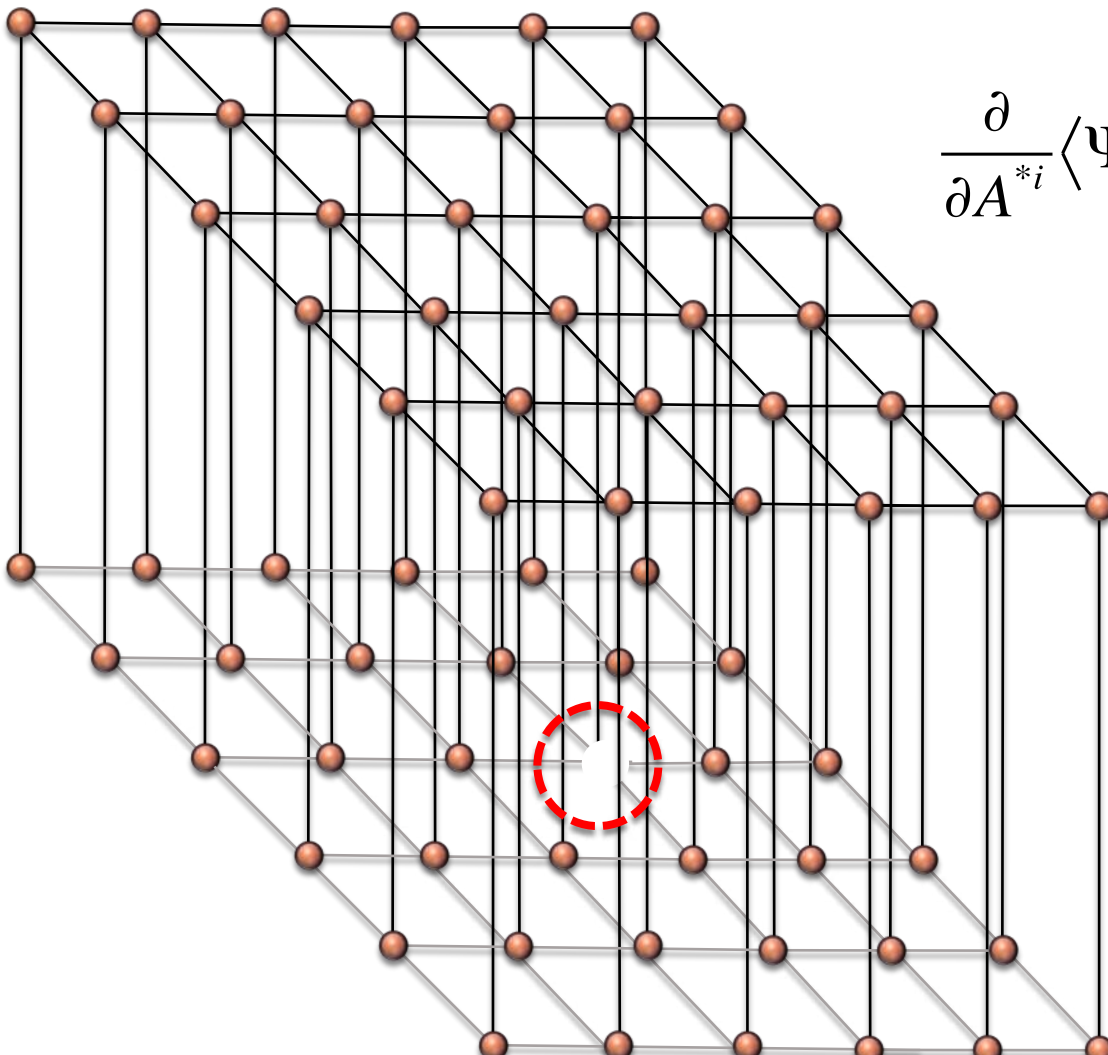

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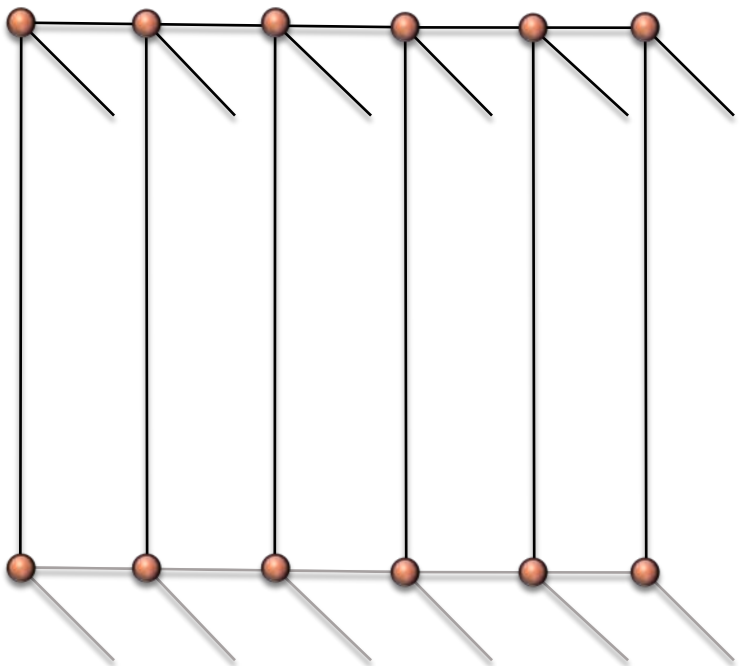

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MPS 

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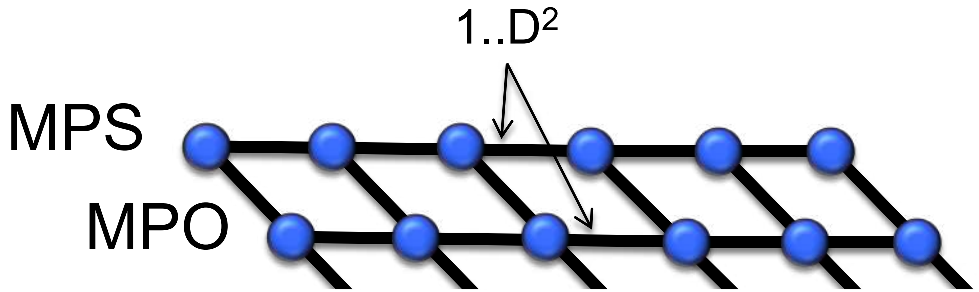

$$\frac{\partial}{\partial A^{*i}}\langle\Psi|\Psi\rangle = \mathbf{N}^i \vec{A}^i$$

1d problem: use a 1d method for MPS
(e.g., DMRG or TEBD)

e.g. 2d calculation of $\mathbf{N}^i \vec{A}^i$

F. Verstraete, I. Cirac, cond-mat/0407066

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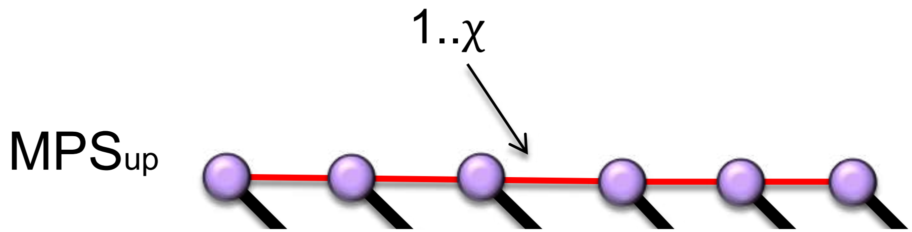

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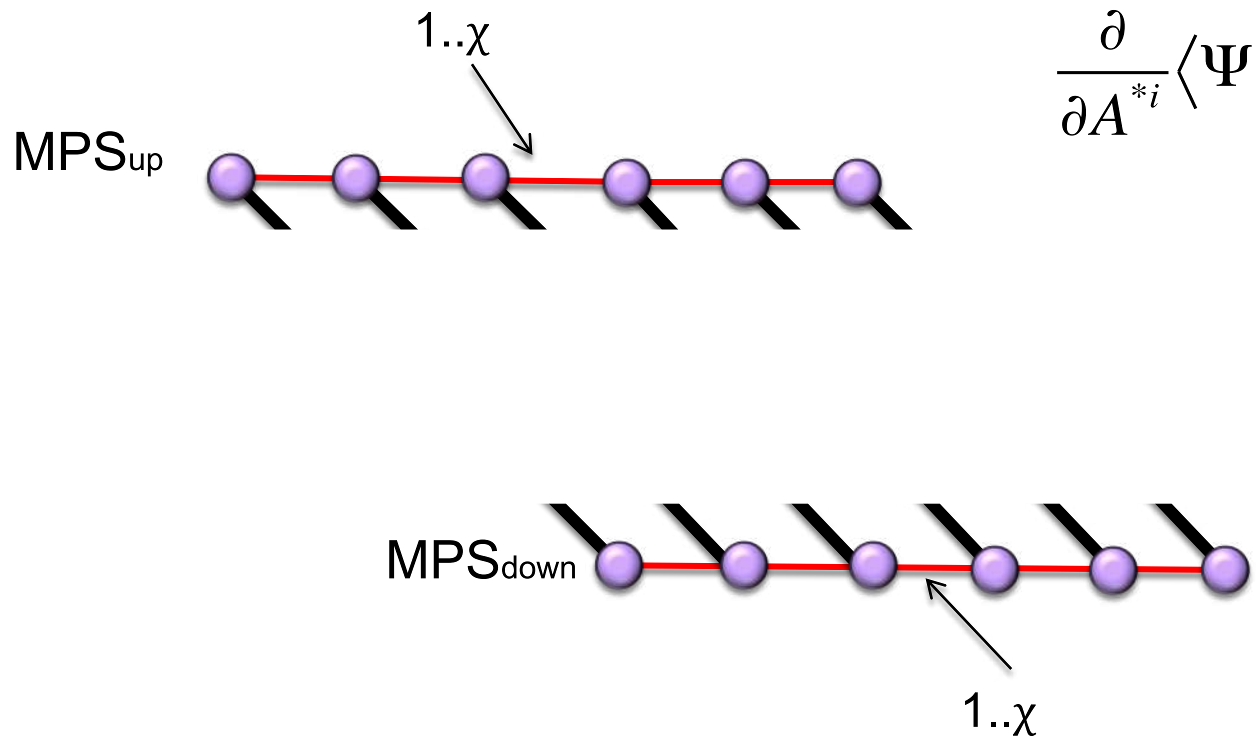

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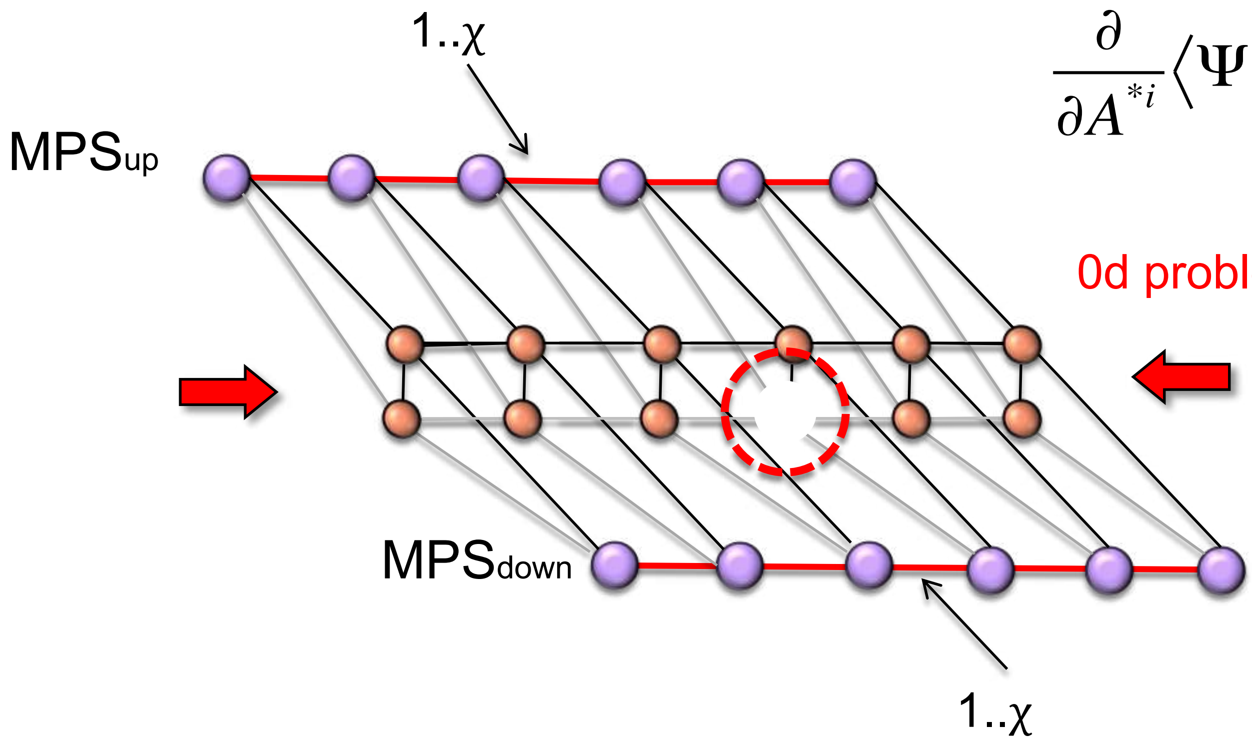

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0d problem: exact!

e.g. 2d calculation of $\mathbf{N}^i \vec{A}^i$

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Dimensional reduction

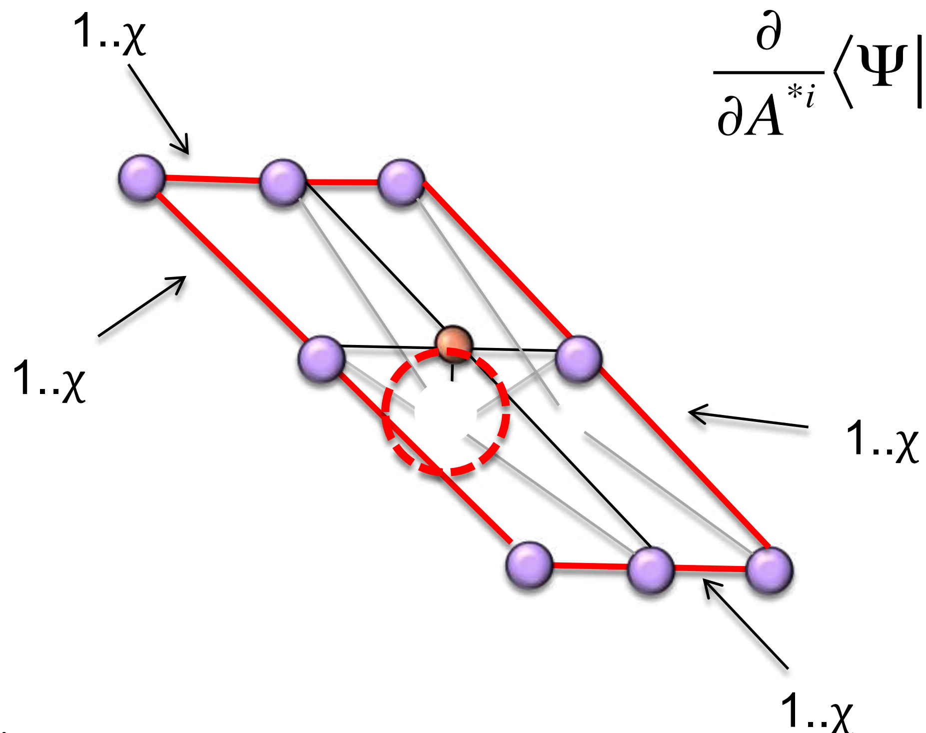
2d problem

↓

1d problem: use DMRG!

↓

0d problem: exact!



$$\frac{\partial}{\partial A^{*i}}\langle\Psi|\Psi\rangle = \mathbf{N}^i \vec{A}^i$$

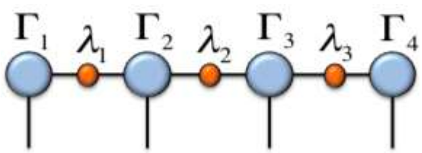
$\mathbf{N}^i \vec{A}^i$ is the environment of tensor A^{*i}

$\mathbf{H}^i_{eff}\vec{A}^i$ is computed similarly, but sandwitching with the Hamiltonian

Valid also for any expectation value

Canonical form helps!

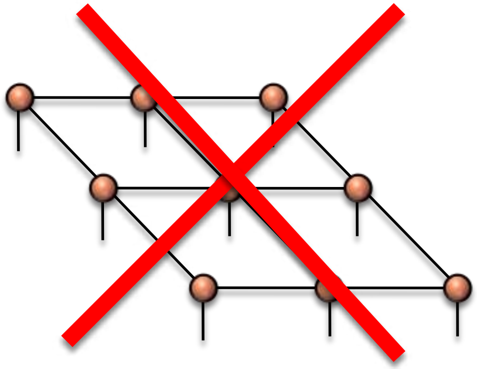
MPS with obc
There is canonical form



$$\mathbf{H}^i_{eff}\vec{A}^i = \lambda\vec{A}^i$$

Normal eigenvalue problem.
Very stable.

PEPS, and TNs with loops
There is no canonical form



$$\mathbf{H}^i_{eff}\vec{A}^i = \lambda\mathbf{N}^i\vec{A}^i$$

Generalized eigenvalue problem.
Less stable.
Approximate canonical forms.

Time evolution

(real, imaginary)

Time evolution (e.g. imaginary)

e.g. J. Jordan et al, PRL 101, 250602 (2008)

$$|\Psi_0\rangle = \lim_{\tau\to\infty} \frac{e^{-\tau H}|\Psi\rangle}{\left\| e^{-\tau H}|\Psi\rangle \right\|}$$

Divide into small time-steps $\delta\tau << 1$

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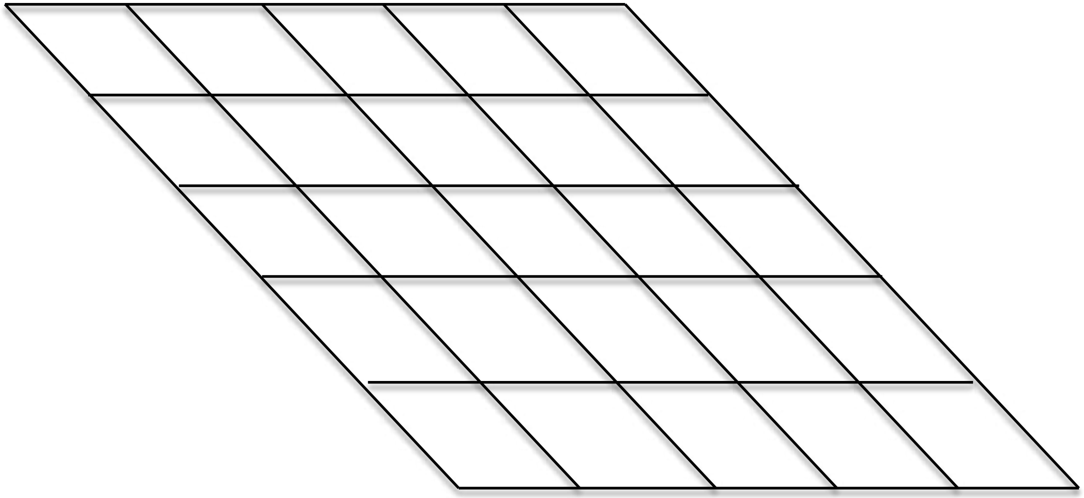
Split the Hamiltonian
(e.g. 2-body n.n.)

$$H = H_{hor}^{even} + H_{hor}^{odd} + H_{ver}^{even} + H_{ver}^{odd}$$



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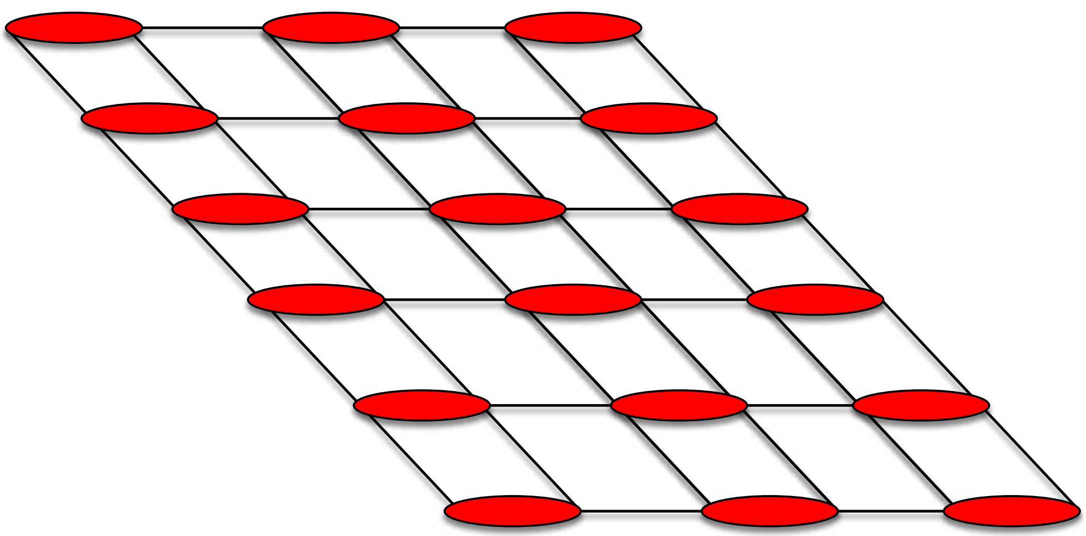

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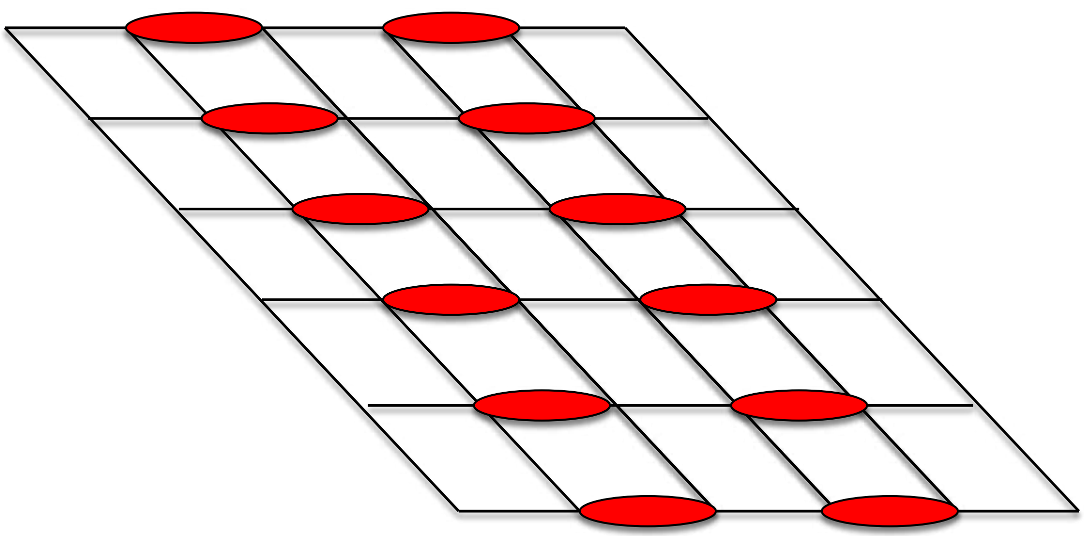

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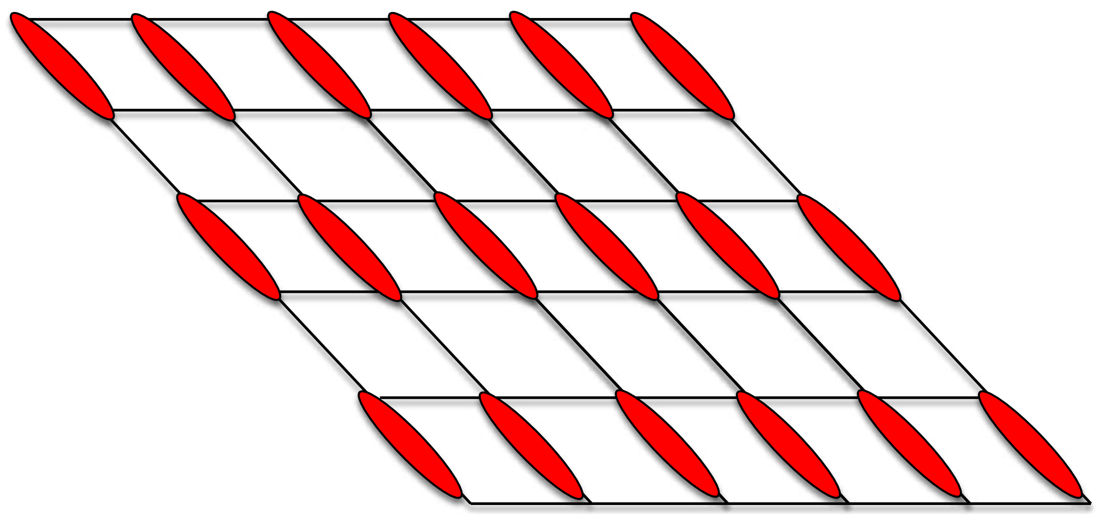

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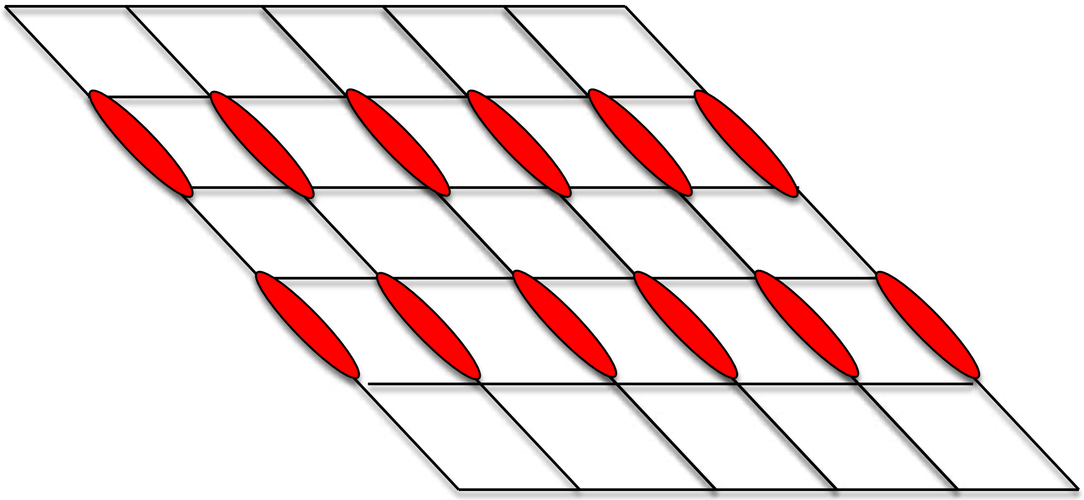

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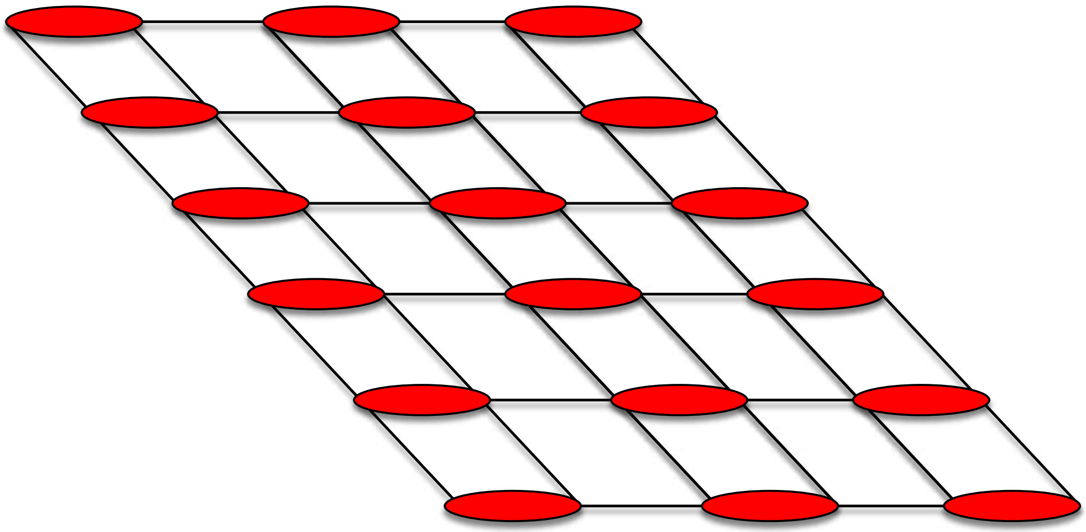
$$e^{-\delta\tau H} \approx e^{-\delta\tau H_{hor}^{even}} e^{-\delta\tau H_{hor}^{odd}} e^{-\delta\tau H_{ver}^{even}} e^{-\delta\tau H_{ver}^{odd}} + O(\delta\tau^2)$$

$$e^{-\delta\tau H_{hor}^{even}}$$



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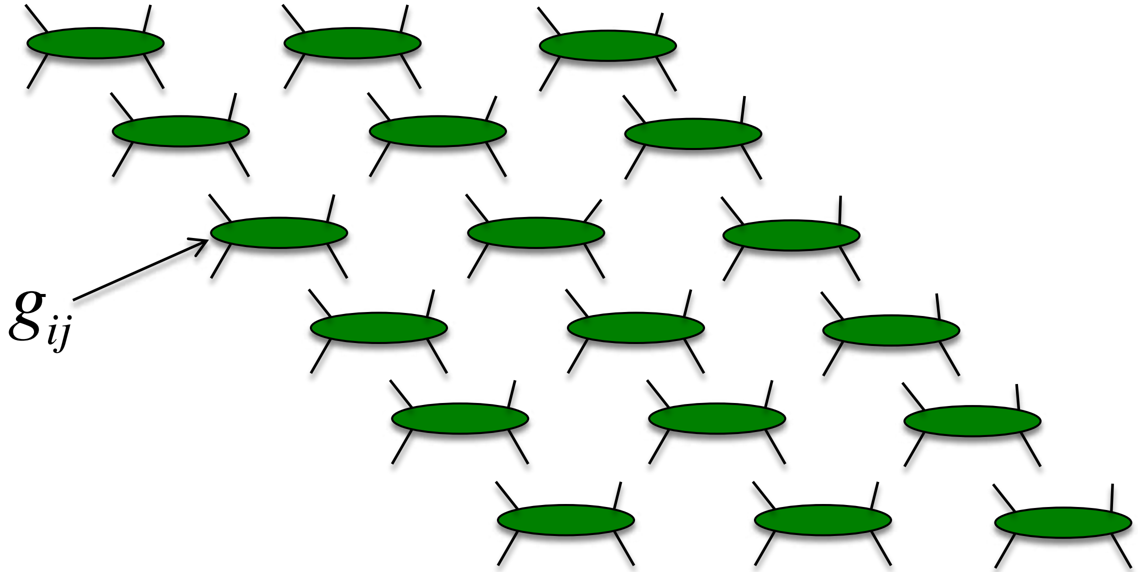
$$e^{-\delta\tau H_{hor}^{even}} = \bigotimes_{<i,j>} e^{-\delta\tau h_{ij}} = \bigotimes_{<i,j>} g_{ij}$$

2-body gates



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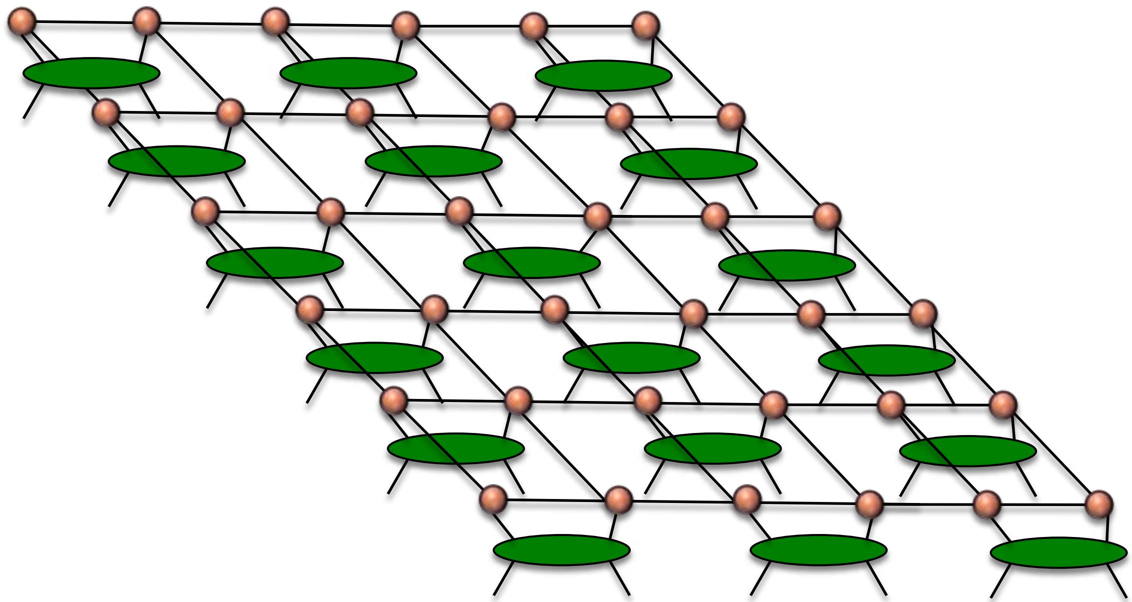
$$e^{-\delta\tau H_{hor}^{even}} \left|\Psi\right\rangle = \left|\tilde{\Psi}\right\rangle$$ evolved state



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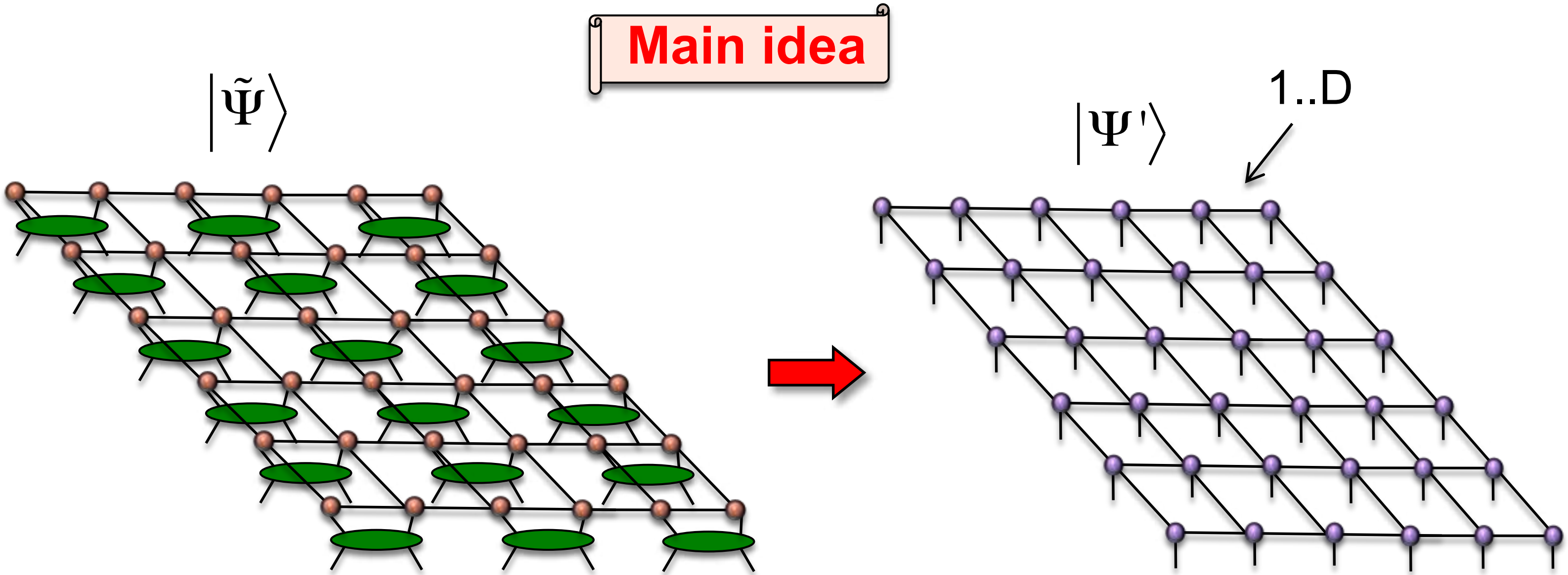


Different approaches to this problem: (fast) full update, simplified update, TPVA...

Full update: $\min \left\| |\tilde{\Psi}\rangle - |\Psi'\rangle \right\|^2$

Finite systems: optimize over all tensors in the PEPS (as before)

Infinite systems: optimize over tensors in the PEPS unit cell (iPEPS)

Require ***calculations of environments***, like the one shown before.

But… does it work?

But... does it work?

YES, it does

+ lots of other examples

J. Jordan, RO, G. Vidal, F. Verstraete, I. Cirac, PRL **101** 250602 (2008)
P. Corboz, RO, B. Bauer, G. Vidal, PRB **81** 165104 (2010)

P. Corboz, PRB **93** 045116 (2016)

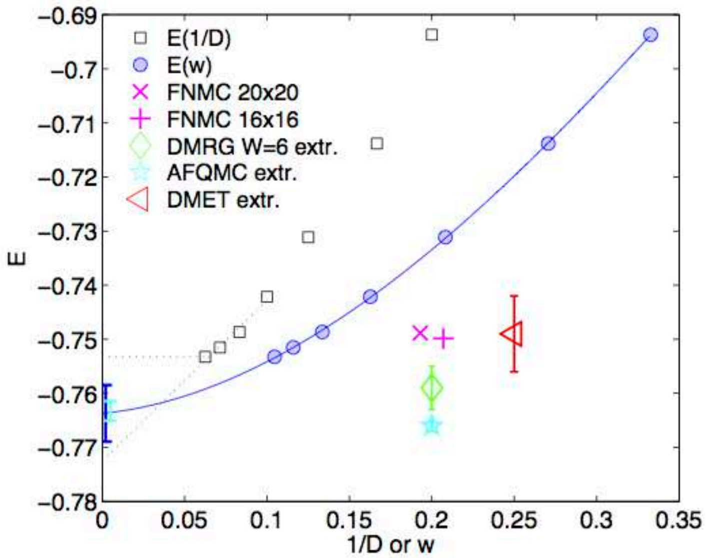


FIG. 4. (Color online) iPEPS energy of a period-5 stripe in the doped case in the strongly correlated regime ($U/t = 8$, $n = 0.875$) in comparison with other methods.

Simulating a Quantum Computer

With MPS

Efficient classical simulation of slightly entangled quantum computations

Guifré Vidal[1]

[1] *Institute for Quantum Information, California Institute of Technology, Pasadena, CA 91125, USA*

(Dated: February 1, 2008)

We present a scheme to efficiently simulate, with a classical computer, the dynamics of multipartite quantum systems on which the amount of entanglement (or of correlations in the case of mixed-state dynamics) is conveniently restricted. The evolution of a pure state of n qubits can be simulated by using computational resources that grow linearly in n and exponentially in the entanglement. We show that a pure-state quantum computation can only yield an exponential speed-up with respect to classical computations if the entanglement increases with the size n of the computation, and gives a lower bound on the required growth.

PACS numbers: 03.67.-a, 03.65.Ud, 03.67.Hk

Simulating a Quantum Computer

With PEPS

PHYSICAL REVIEW LETTERS **123,** 190501 (2019)

General-Purpose Quantum Circuit Simulator with Projected Entangled-Pair States and the Quantum Supremacy Frontier

Chu Guo,[1,*] Yong Liu,[2,*] Min Xiong,[2] Shichuan Xue,[2] Xiang Fu,[2] Anqi Huang,[2] Xiaogang Qiang,[2]
Ping Xu,[2] Junhua Liu,[3,4] Shenggen Zheng,[5] He-Liang Huang,[1,6,7] Mingtang Deng,[2]
Dario Poletti,[8,†] Wan-Su Bao,[1,7,‡] and Junjie Wu[2,§]
[1]*Henan Key Laboratory of Quantum Information and Cryptography, IEU, Zhengzhou 450001, China*
[2]*Institute for Quantum Information & State Key Laboratory of High Performance Computing, College of Computer, National University of Defense Technology, Changsha 410073, China*
[3]*Information Systems Technology and Design, Singapore University of Technology and Design, 8 Somapah Road, 487372 Singapore*
[4]*Quantum Intelligence Lab (QI-Lab), Supremacy Future Technologies (SFT), Guangzhou 511340, China*
[5]*Center for Quantum Computing, Peng Cheng Laboratory, Shenzhen 518055, China*
[6]*Hefei National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*
[7]*CAS Centre for Excellence and Synergetic Innovation Centre in Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*
[8]*Science and Math Cluster and EPD Pillar, Singapore University of Technology and Design, 8 Somapah Road, 487372 Singapore*

(Received 30 July 2019; published 4 November 2019)

Recent advances on quantum computing hardware have pushed quantum computing to the verge of quantum supremacy. Here, we bring together many-body quantum physics and quantum computing by using a method for strongly interacting two-dimensional systems, the projected entangled-pair states, to realize an effective general-purpose simulator of quantum algorithms. The classical computing complexity of this simulator is directly related to the entanglement generation of the underlying quantum circuit rather than the number of qubits or gate operations. We apply our method to study random quantum circuits, which allows us to quantify precisely the memory usage and the time requirements of random quantum circuits. We demonstrate our method by computing one amplitude for a 7×7 lattice of qubits with depth $(1 + 40 + 1)$ on the Tianhe-2 supercomputer.

DOI: 10.1103/PhysRevLett.123.190501

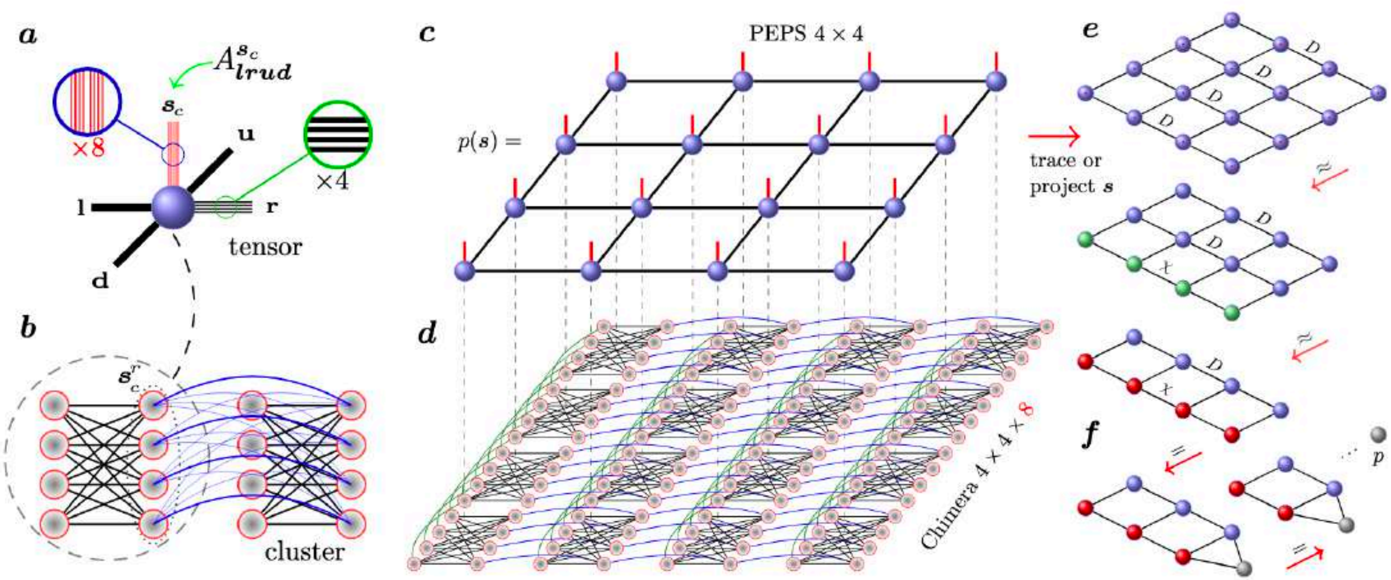
Simulating a Quantum Annealer

With PEPS

Heuristic optimization and sampling with tensor networks

Marek M. Rams,[1] Masoud Mohseni,[2] and Bartłomiej Gardas[1,3]

[1] *Jagiellonian University, Marian Smoluchowski Institute of Physics, Łojasiewicza 11, 30-348 Kraków, Poland*
[2] *Google AI Quantum, Venice, CA 90291*
[3] *University of Silesia, Institute of Physics, Bankowa 12, 40-007 Katowice, Poland*



Outline

1) Basics

2) 1d MPS

3) 2d PEPS

4) Numerical algorithms

5) MERA

6) Extras

Outline

1) Basics

2) 1d MPS

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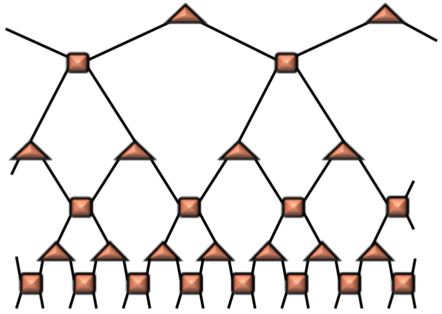
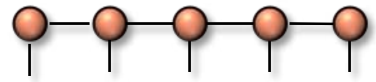


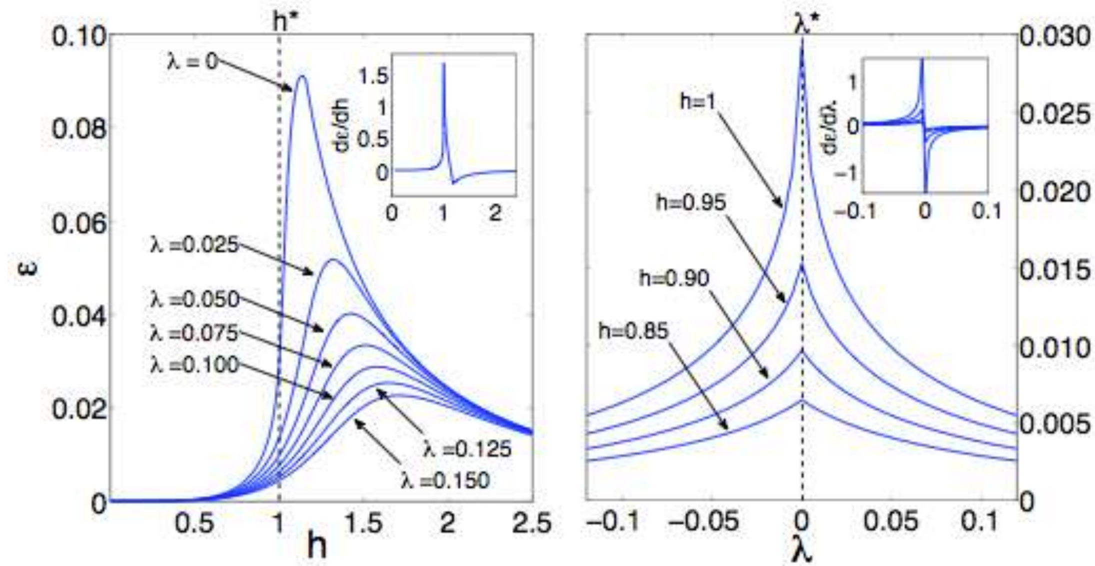
From MPS to MERA

Matrix Product States (MPS)



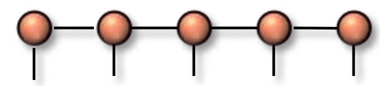
1d systems

But we want to do critical systems!!!

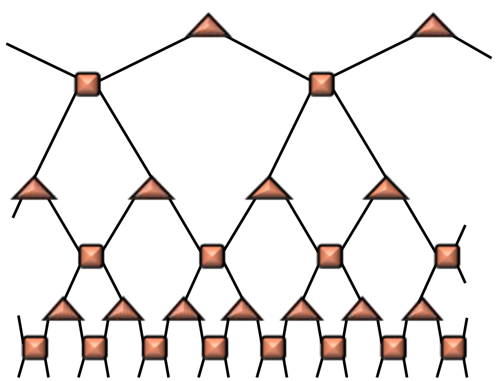


Also very painful for DMRG...

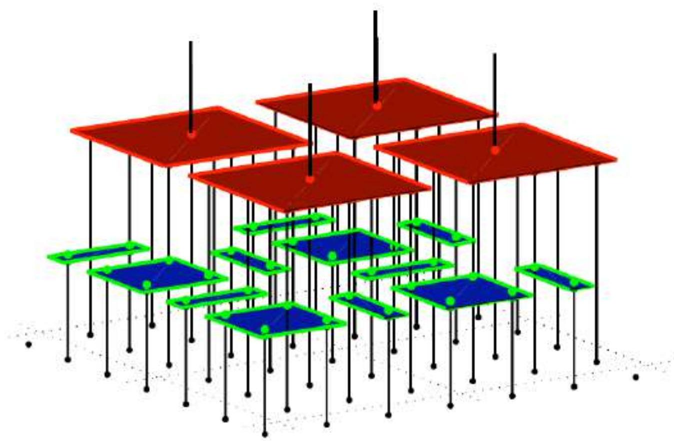
From MPS to MERA

Matrix Product States (MPS)  *1d systems*

Multiscale Entanglement Renormalization Ansatz (MERA)



1d systems

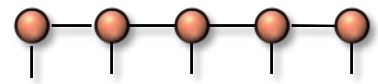


2d systems

and so on…

From MPS to MERA

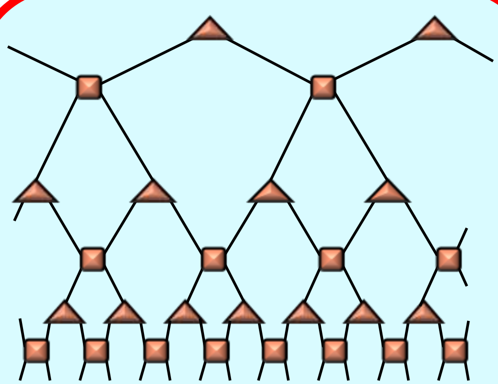
Matrix Product States (MPS)



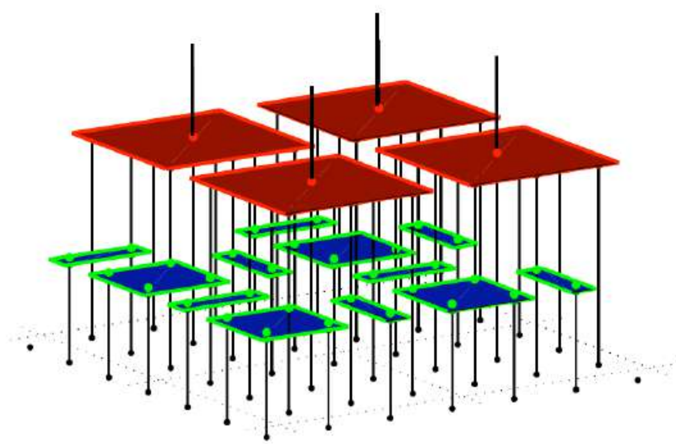
1d systems

Multiscale Entanglement Renormalization Ansatz (MERA)

This lecture



1d systems

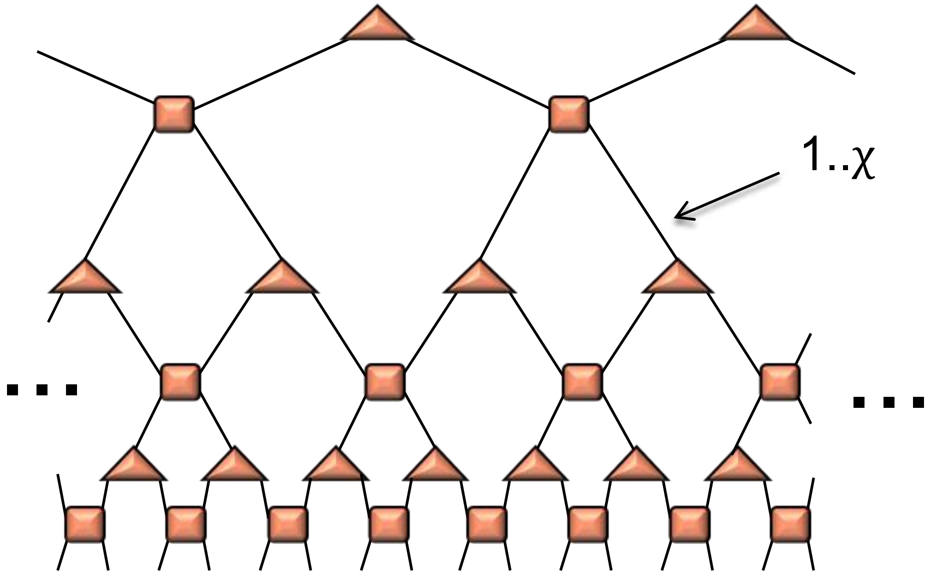


2d systems

and so on…

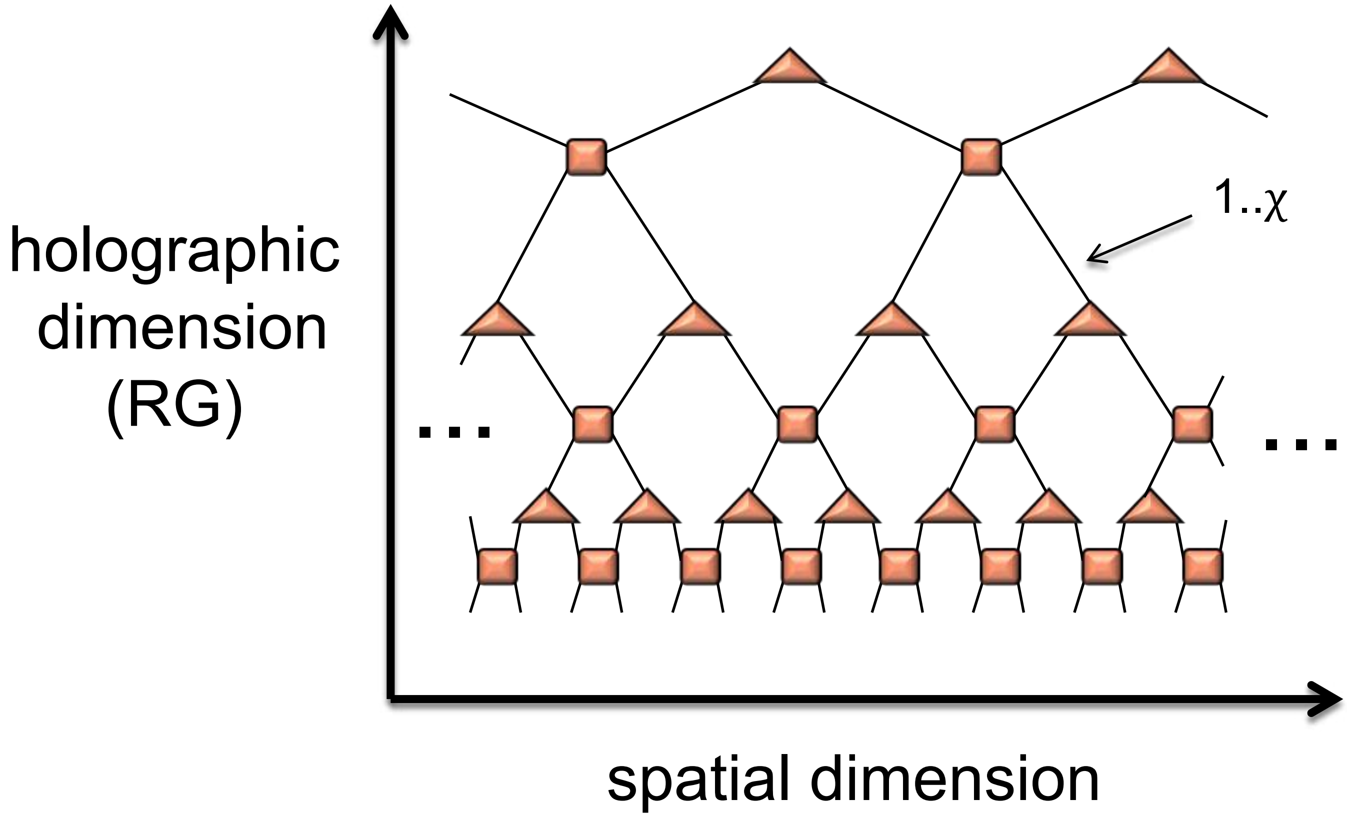
1d MERA

*e.g., G. Vidal, PRL **99**, 220405 (2007)*

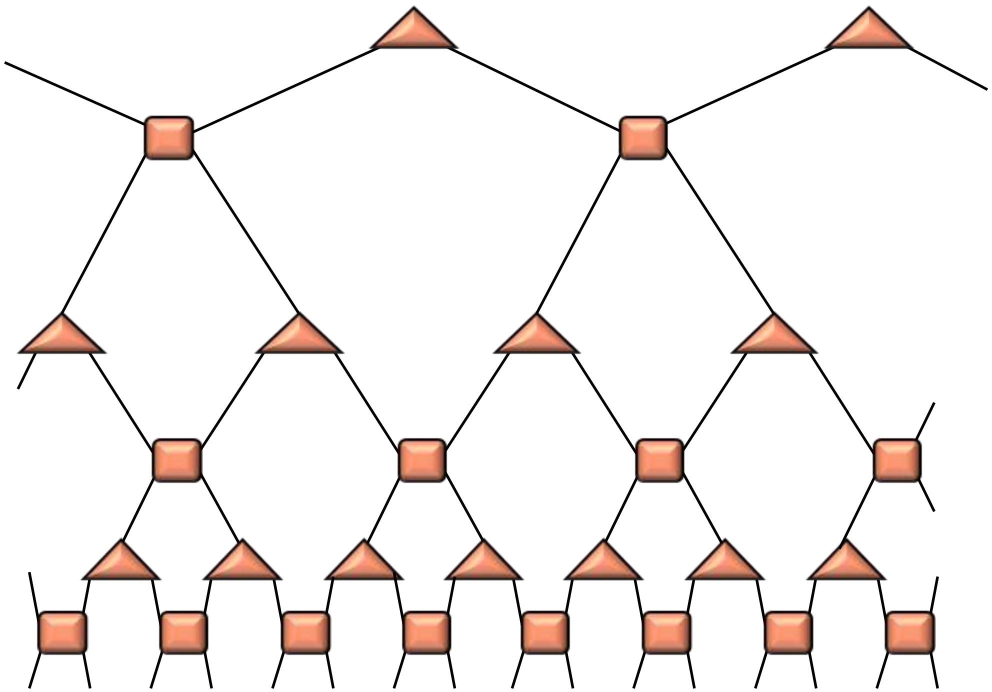


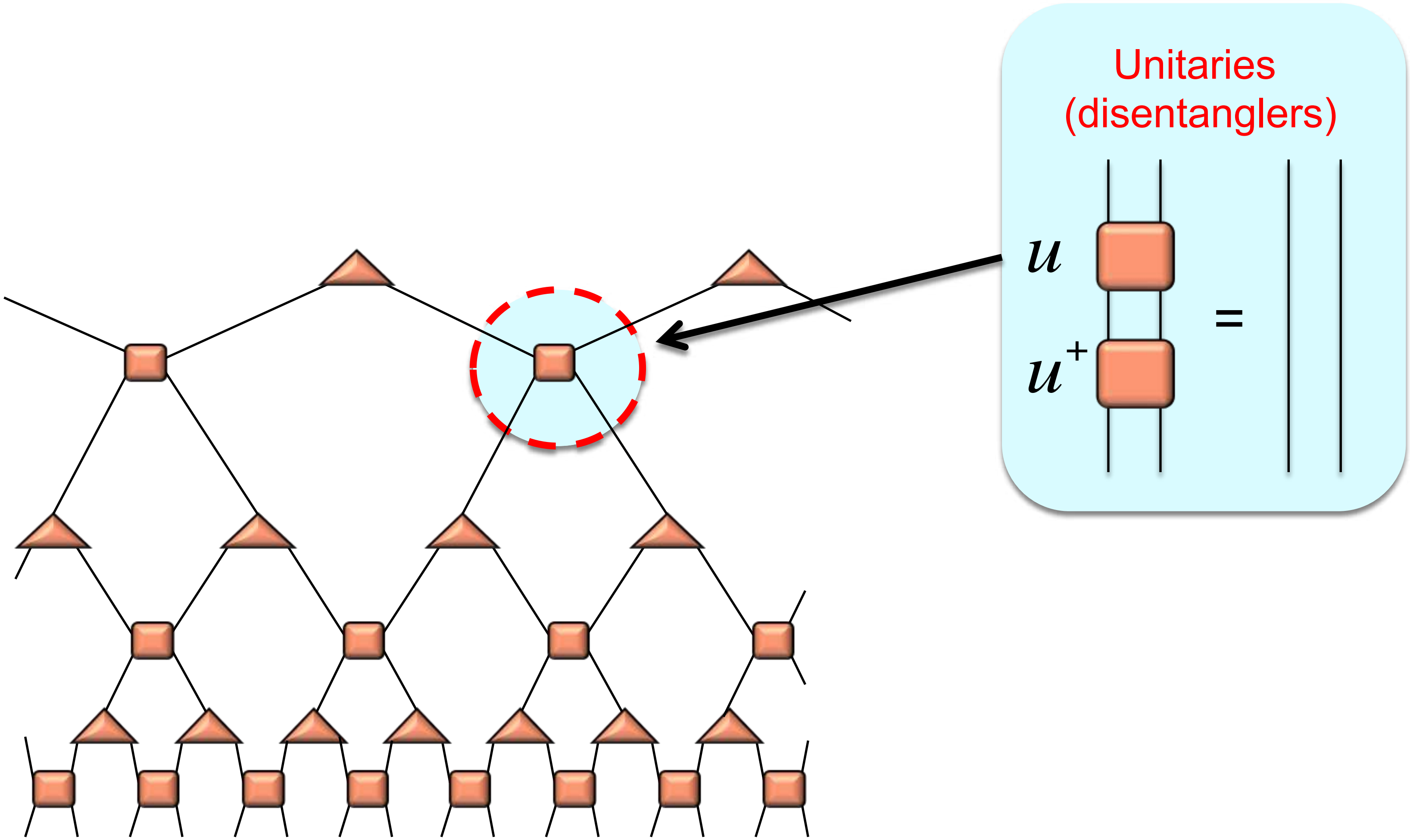
1d MERA

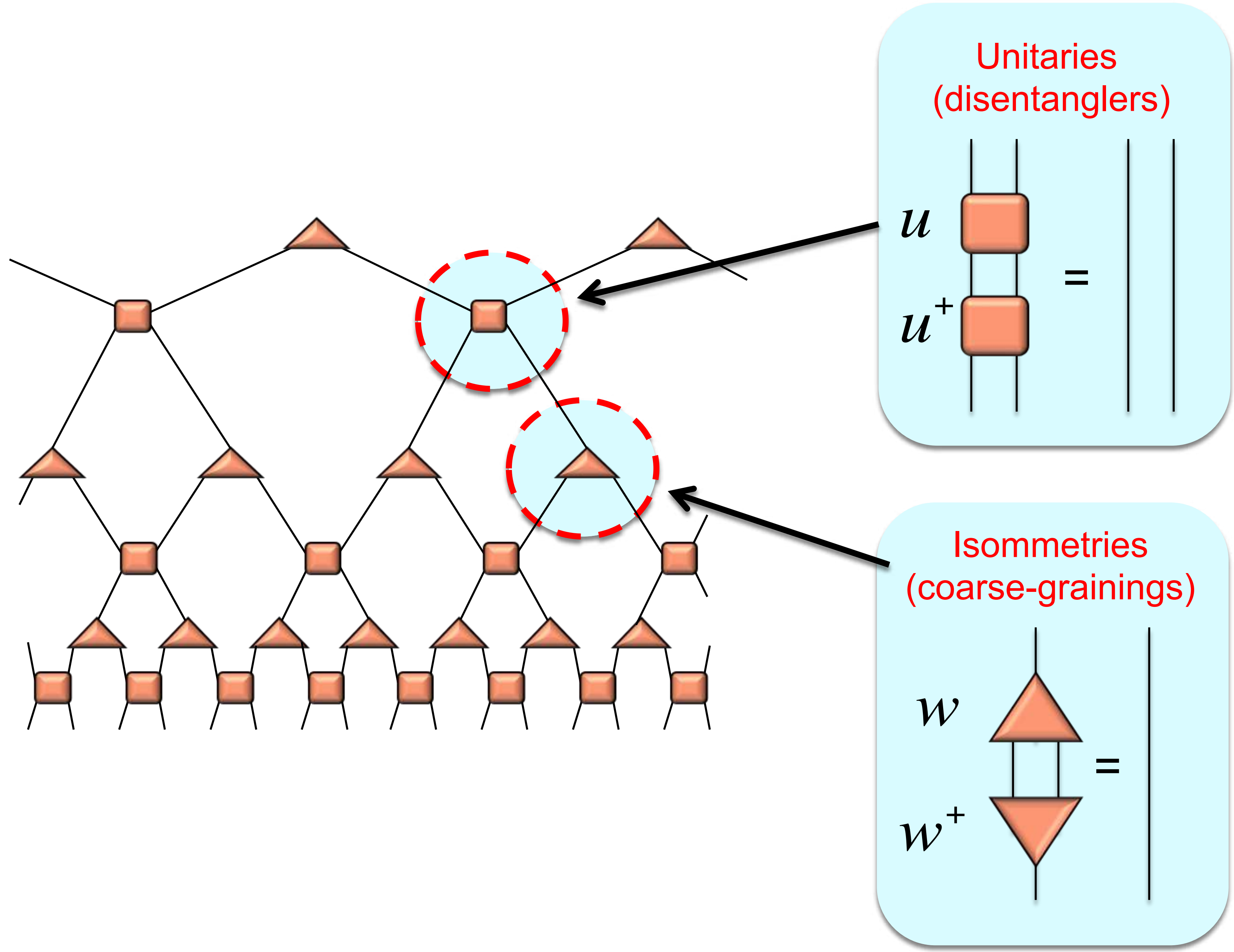
e.g., G. Vidal, PRL ***99****, 220405 (2007)*



Tensors obey constraints




Unitaries
(disentanglers)
u
u^+
=

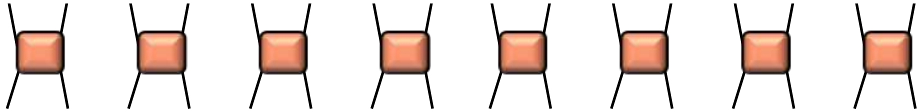


Unitaries
(disentanglers)
u
u^+
=
Isommetries
(coarse-grainings)
w
w^+
=

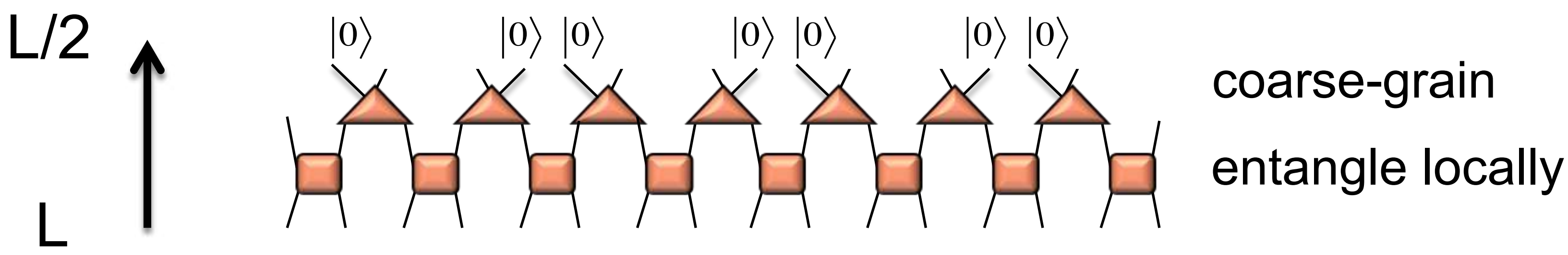
Reason:

entanglement is built locally
at all length scales

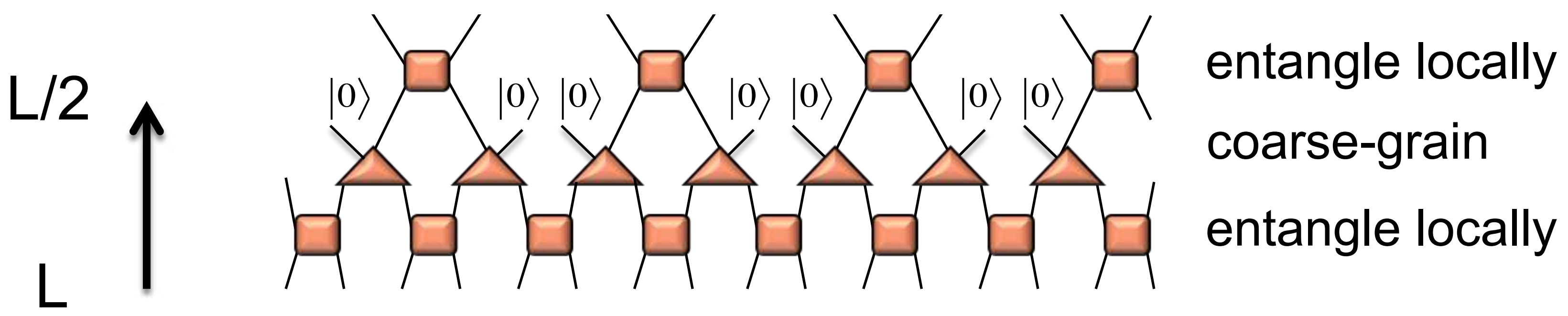
L



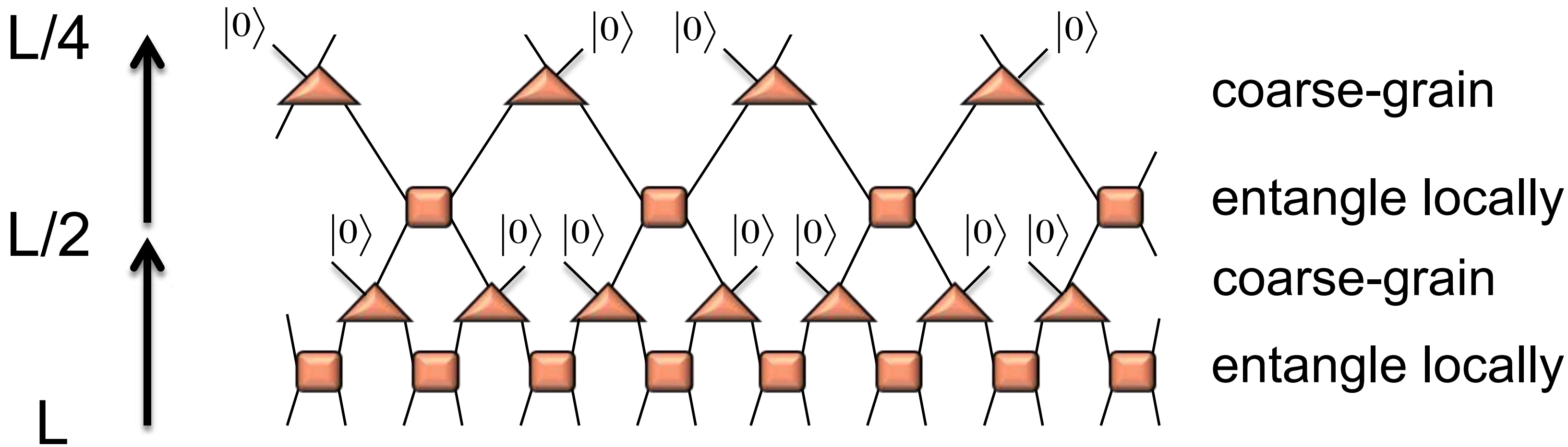
entangle locally

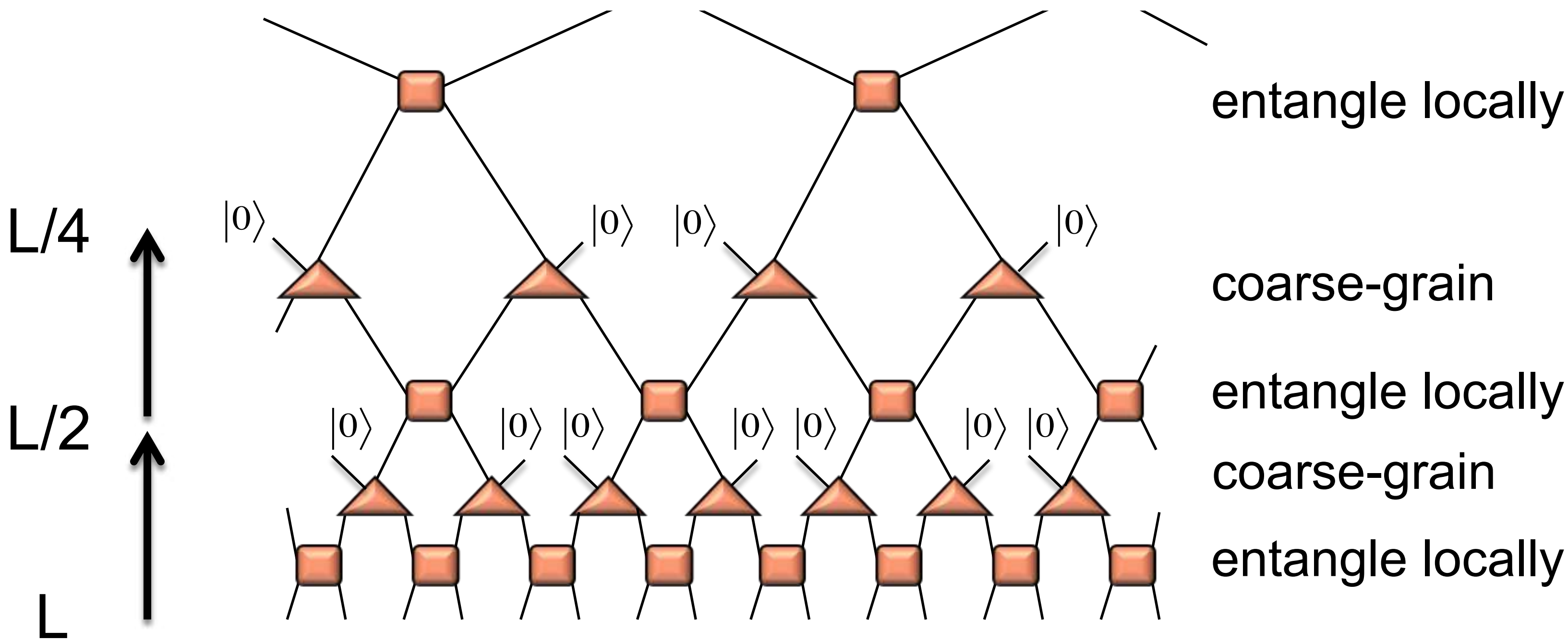


L/2
L
$|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$
coarse-grain
entangle locally

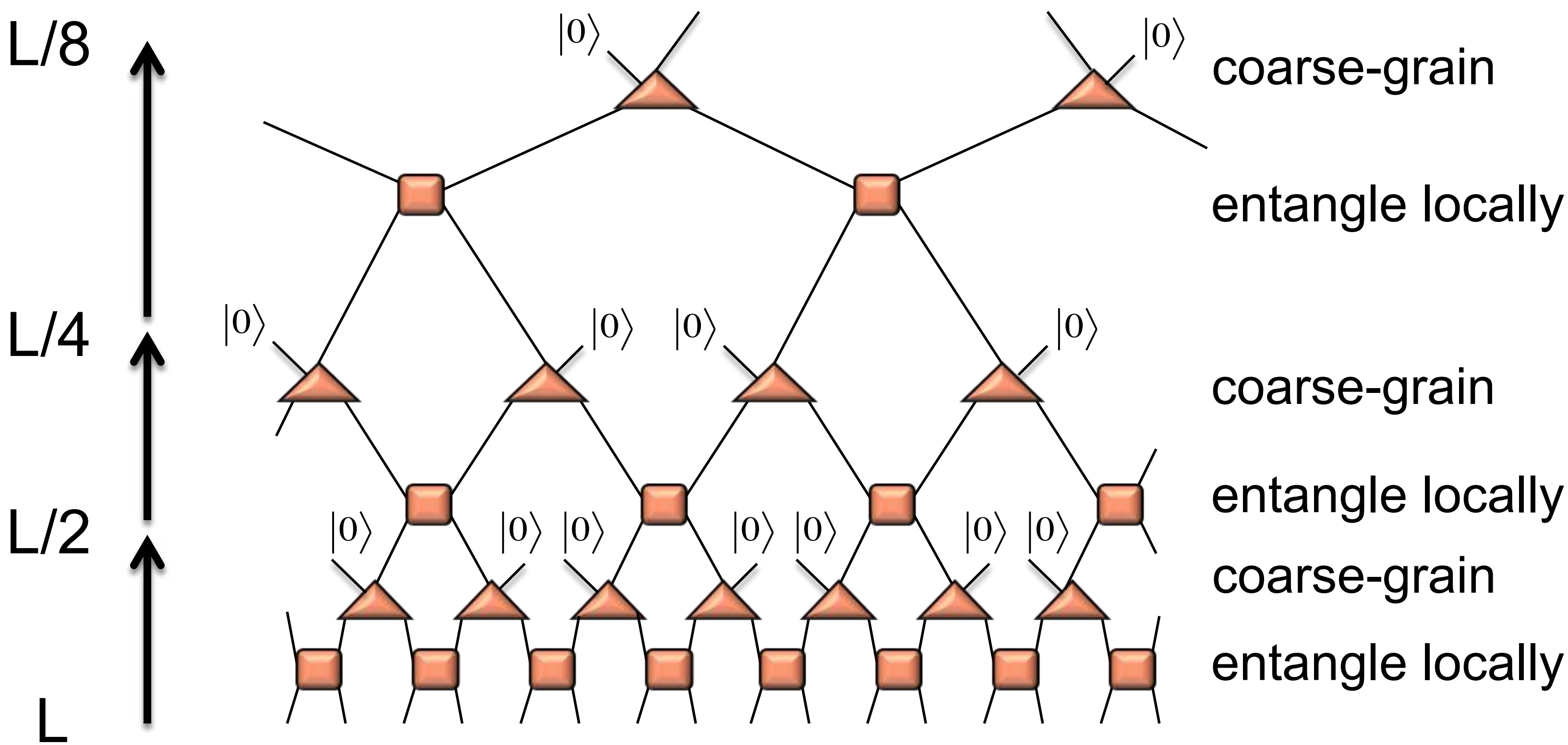


L/2
L
|0⟩
entangle locally
coarse-grain
entangle locally

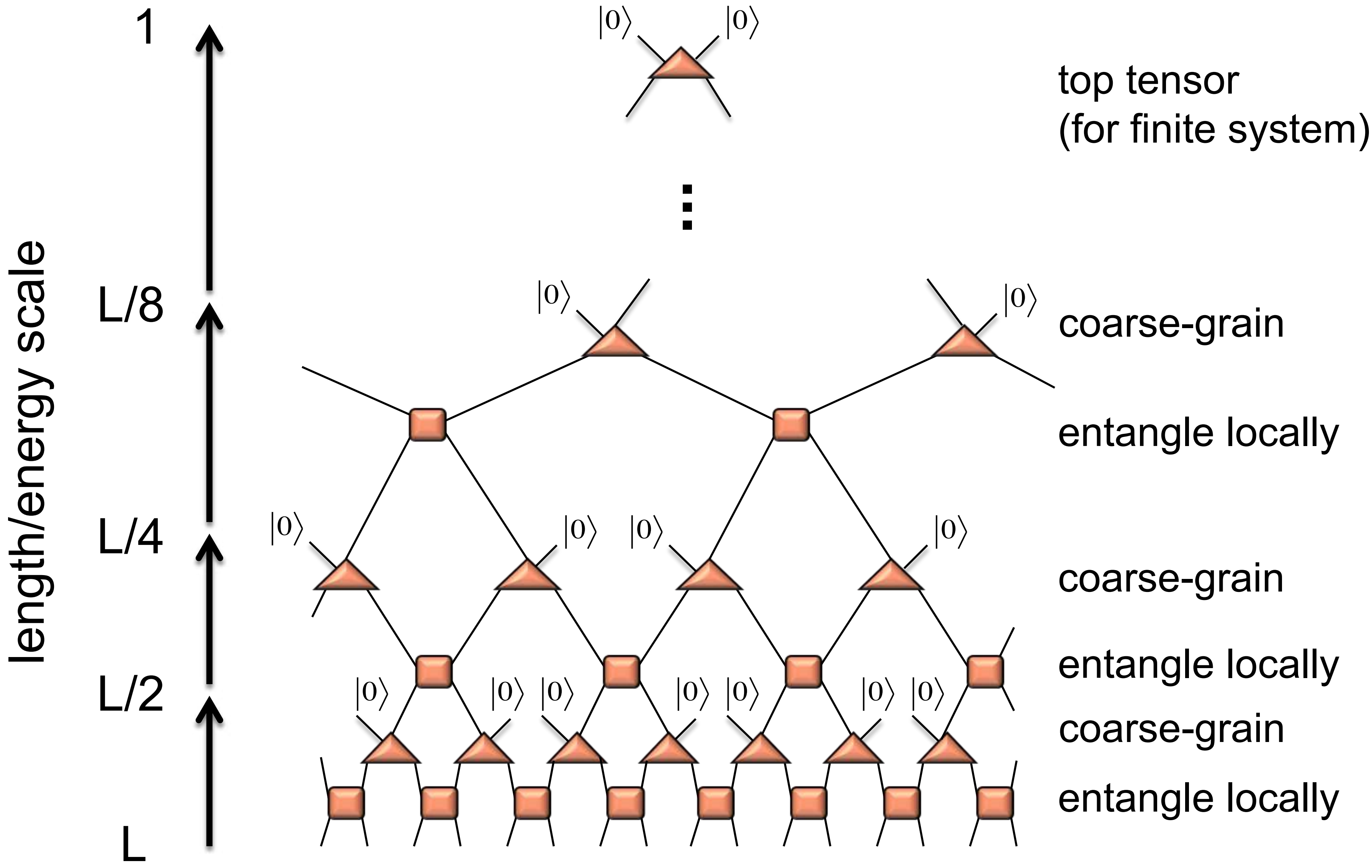

L/4
L/2
L
|0⟩
coarse-grain
entangle locally
coarse-grain
entangle locally



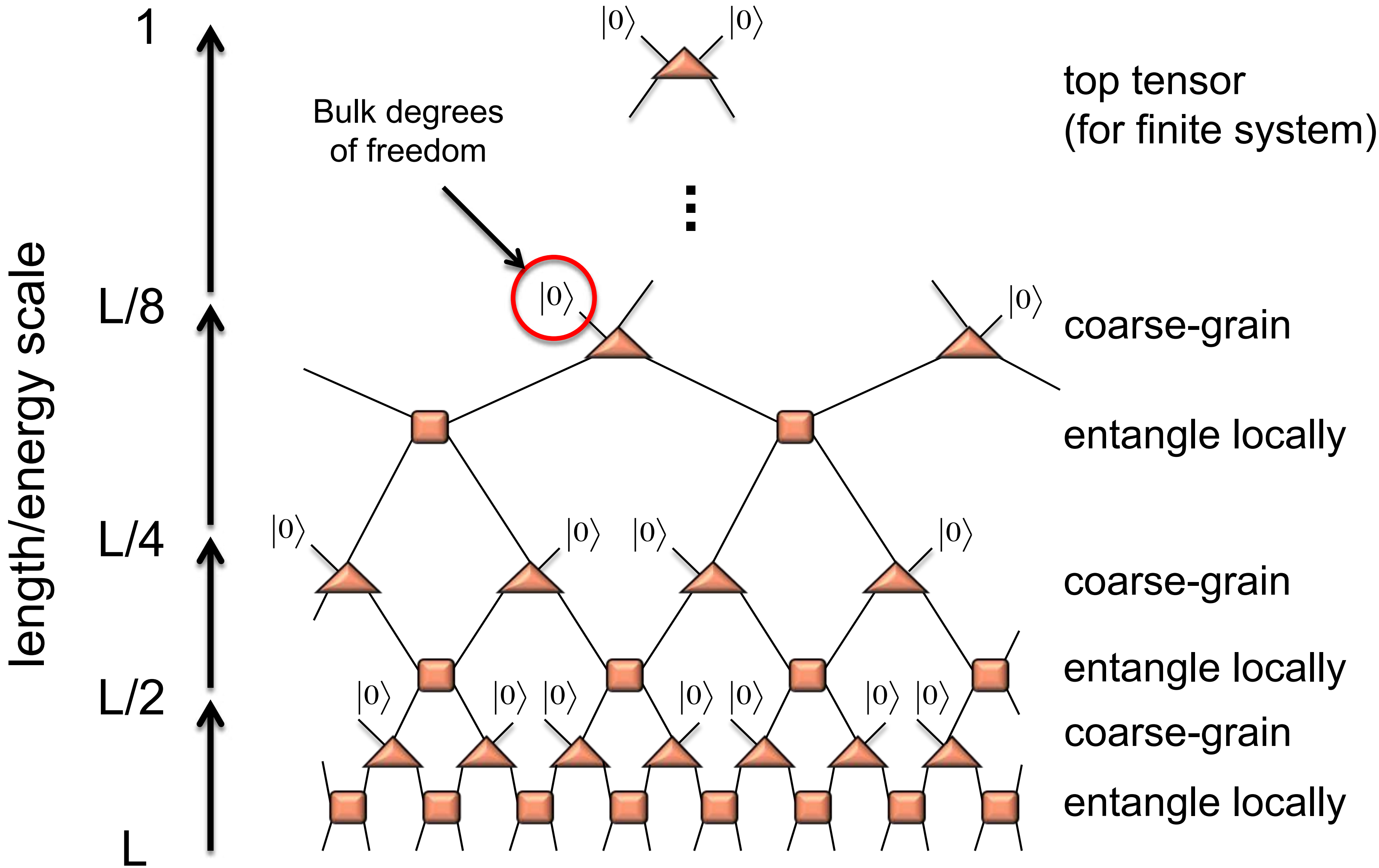
entangle locally
L/4
coarse-grain
entangle locally
L/2
coarse-grain
entangle locally
L
$|0\rangle$



L/8
L/4
L/2
L
|0⟩
coarse-grain
entangle locally
coarse-grain
entangle locally
coarse-grain
entangle locally



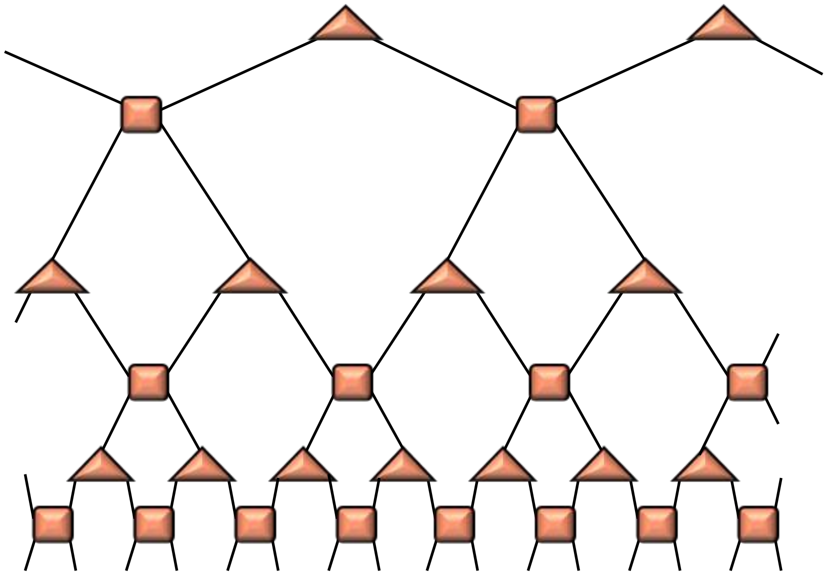
Extra dimension defines an RG flow: Entanglement Renormalization



Extra dimension defines an RG flow: Entanglement Renormalization

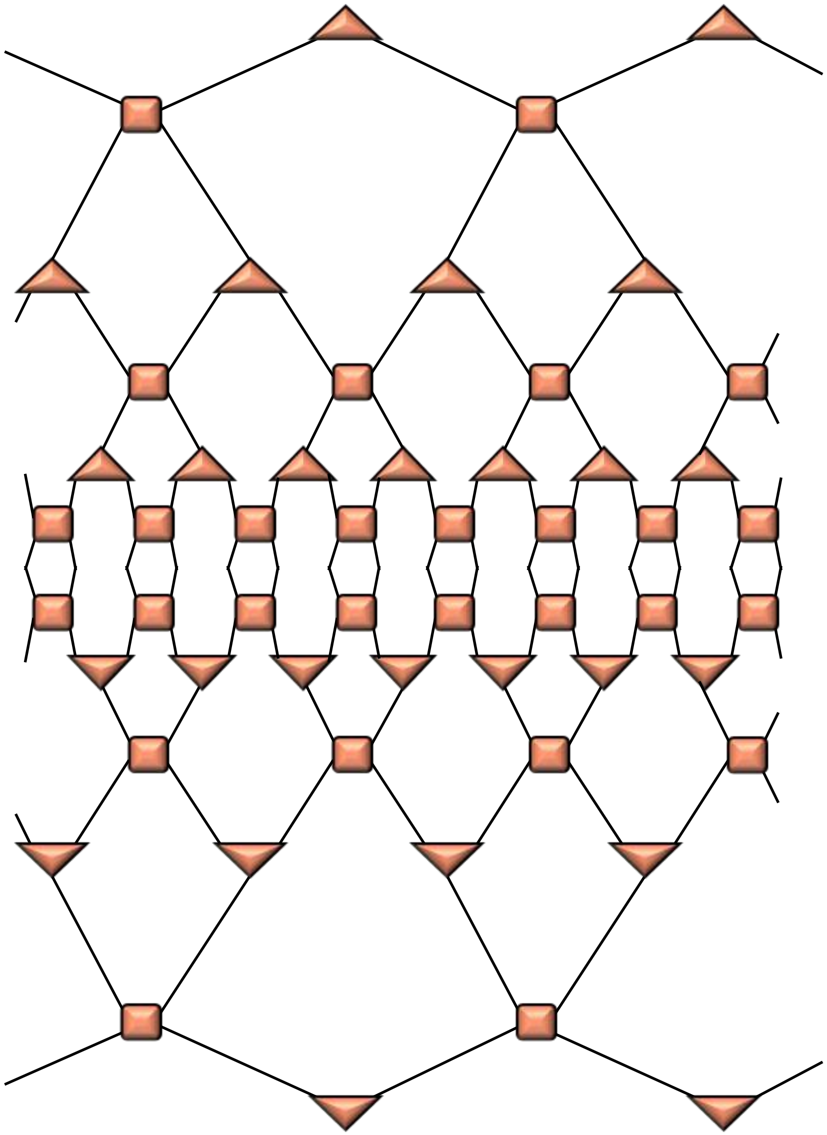
Norm of MERA

Norm of MERA



$|\Psi\rangle$

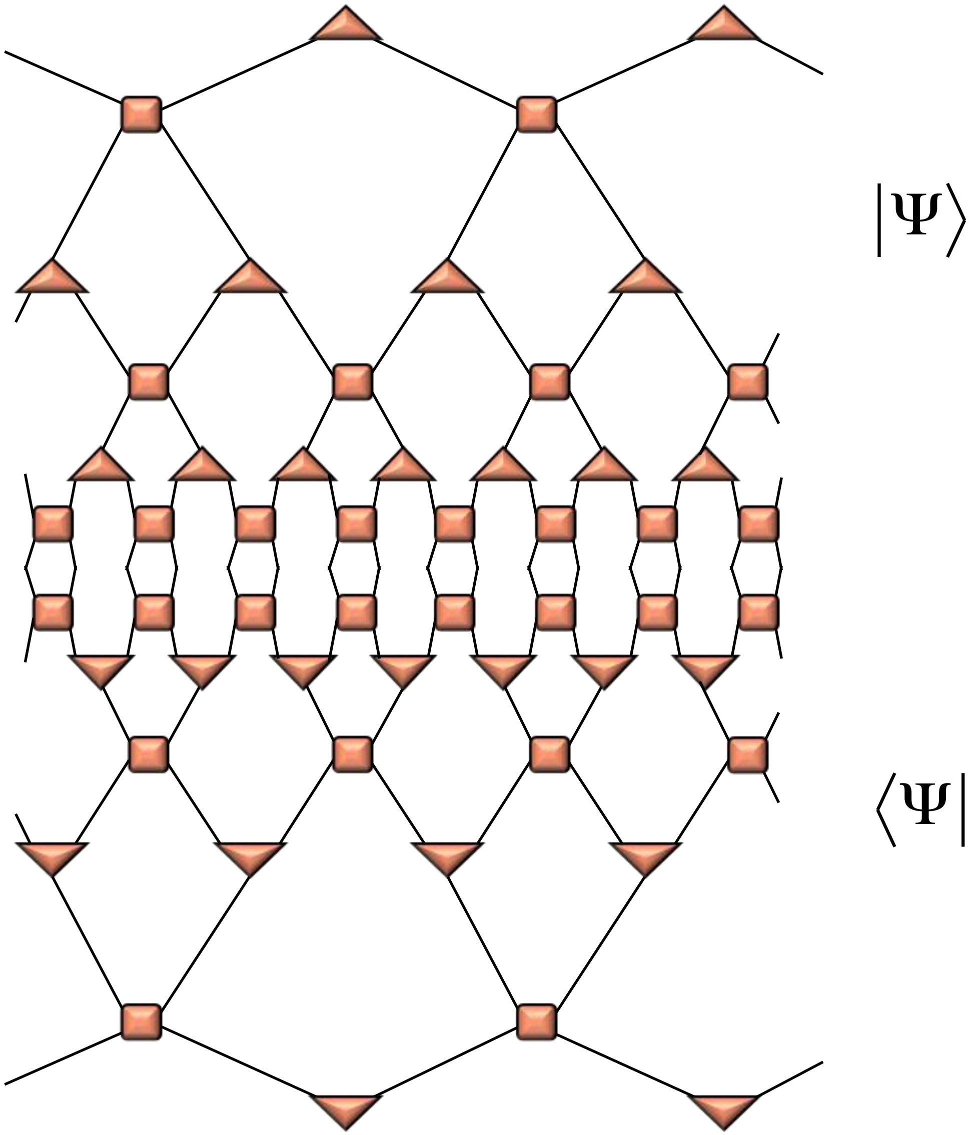
Norm of MERA

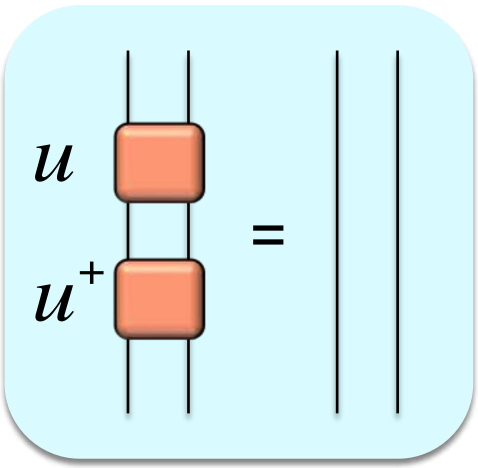


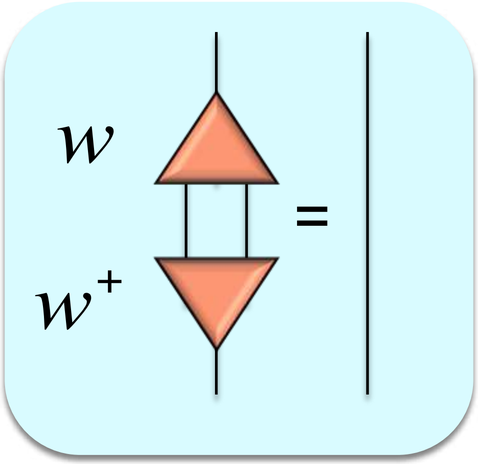
$|\Psi\rangle$

$\langle\Psi|$

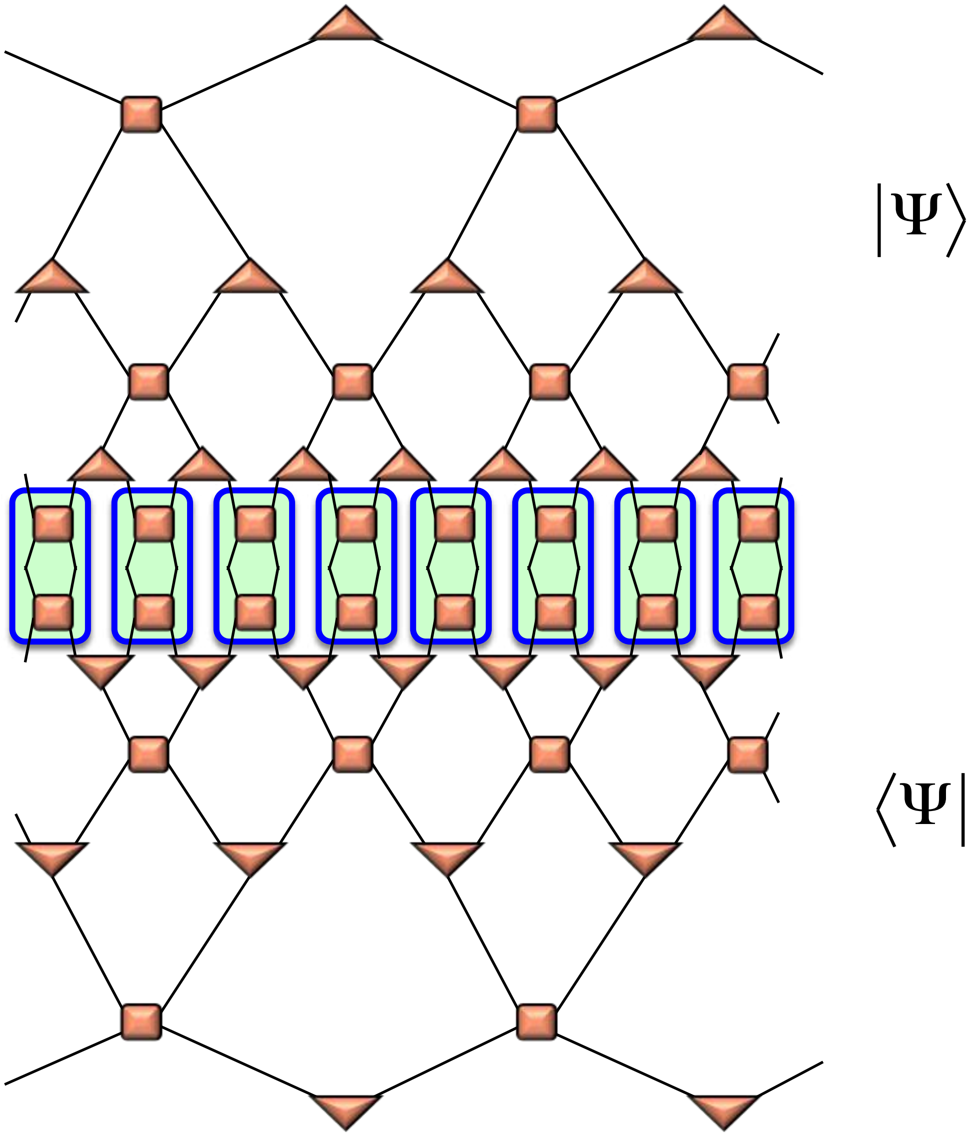
Norm of MERA

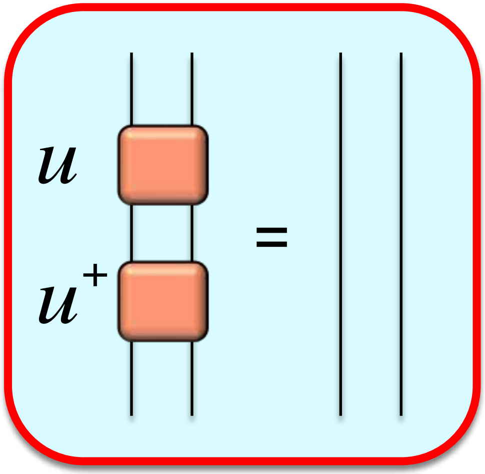


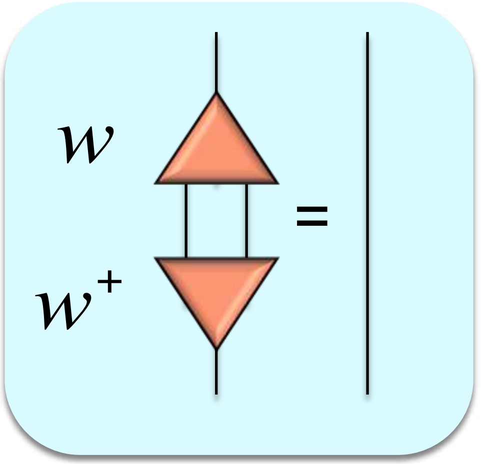




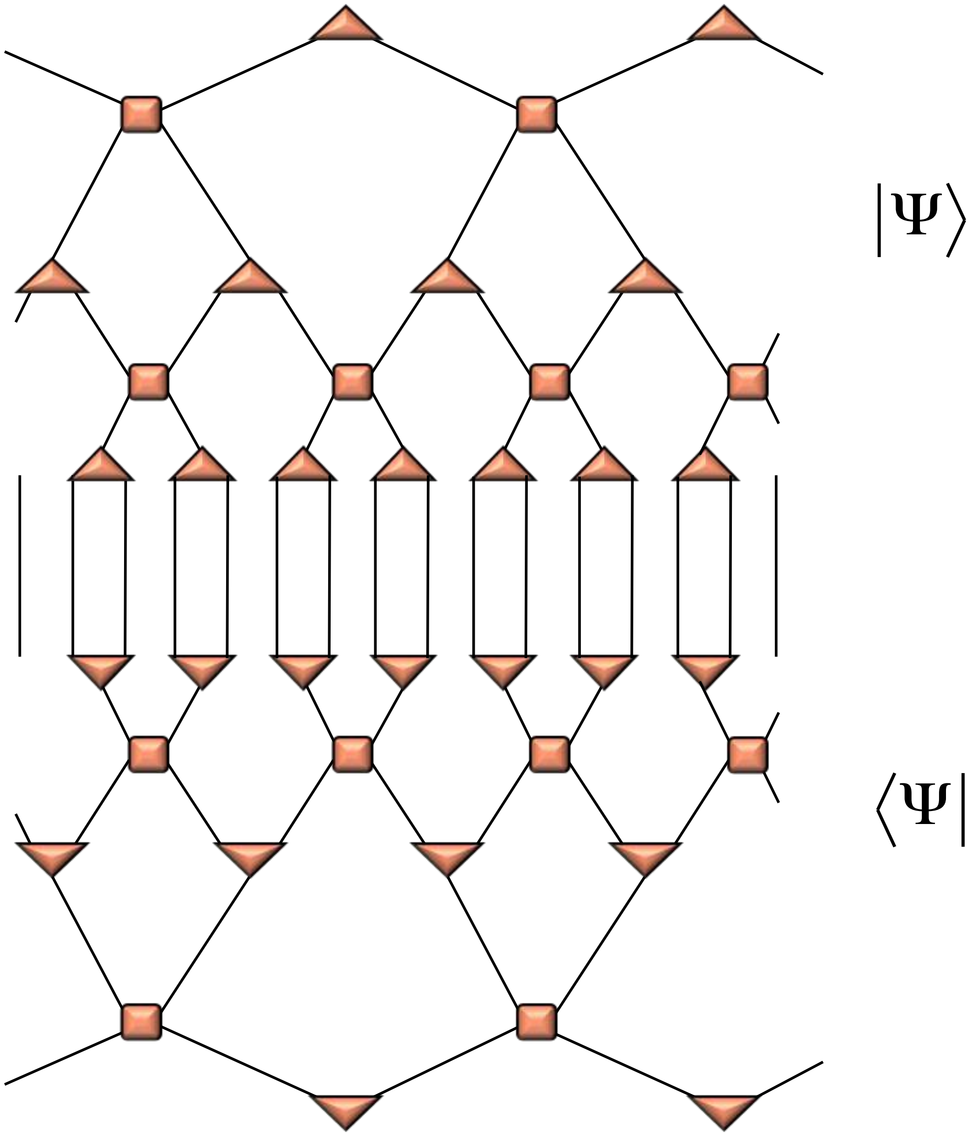
Norm of MERA

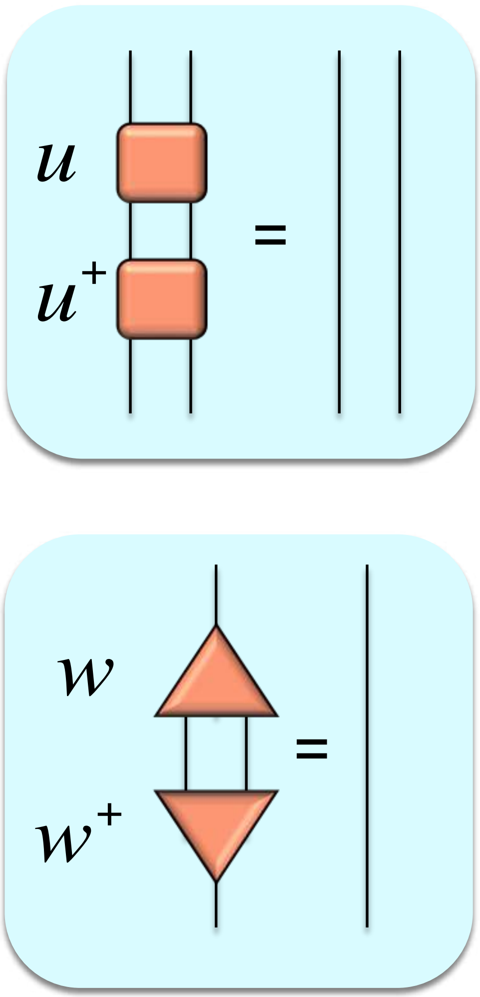




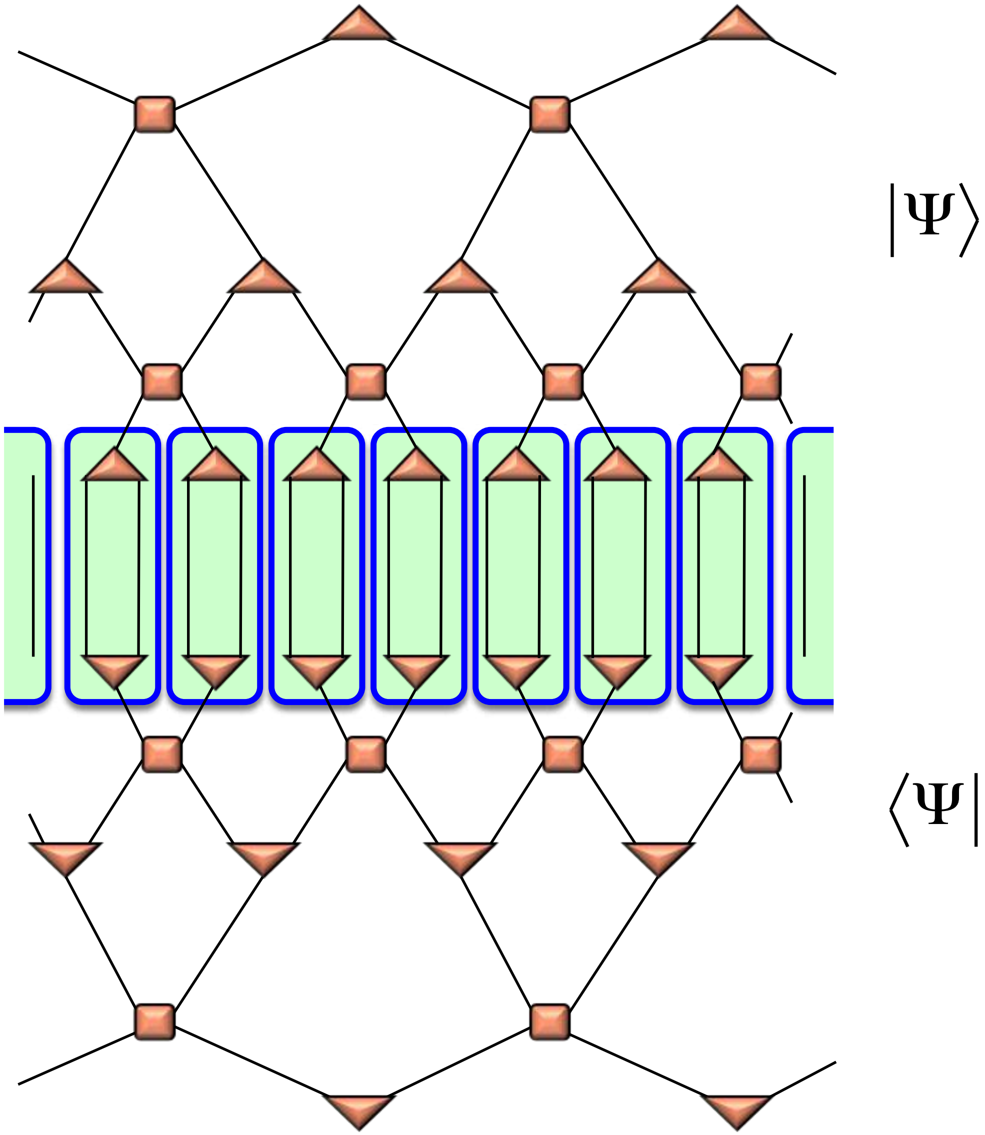


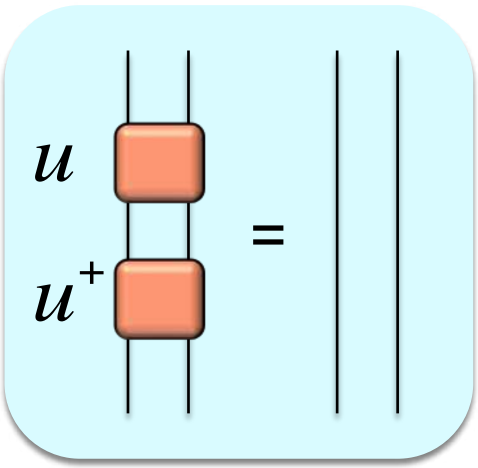
Norm of MERA

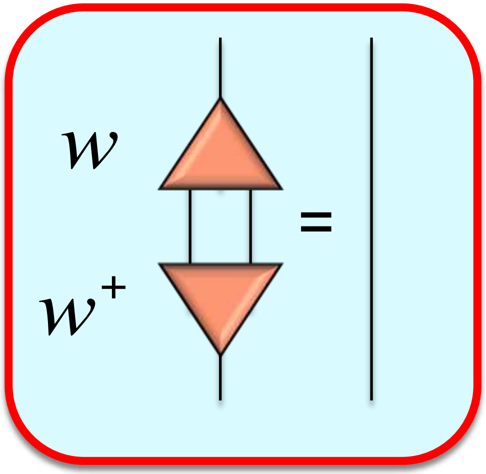




Norm of MERA







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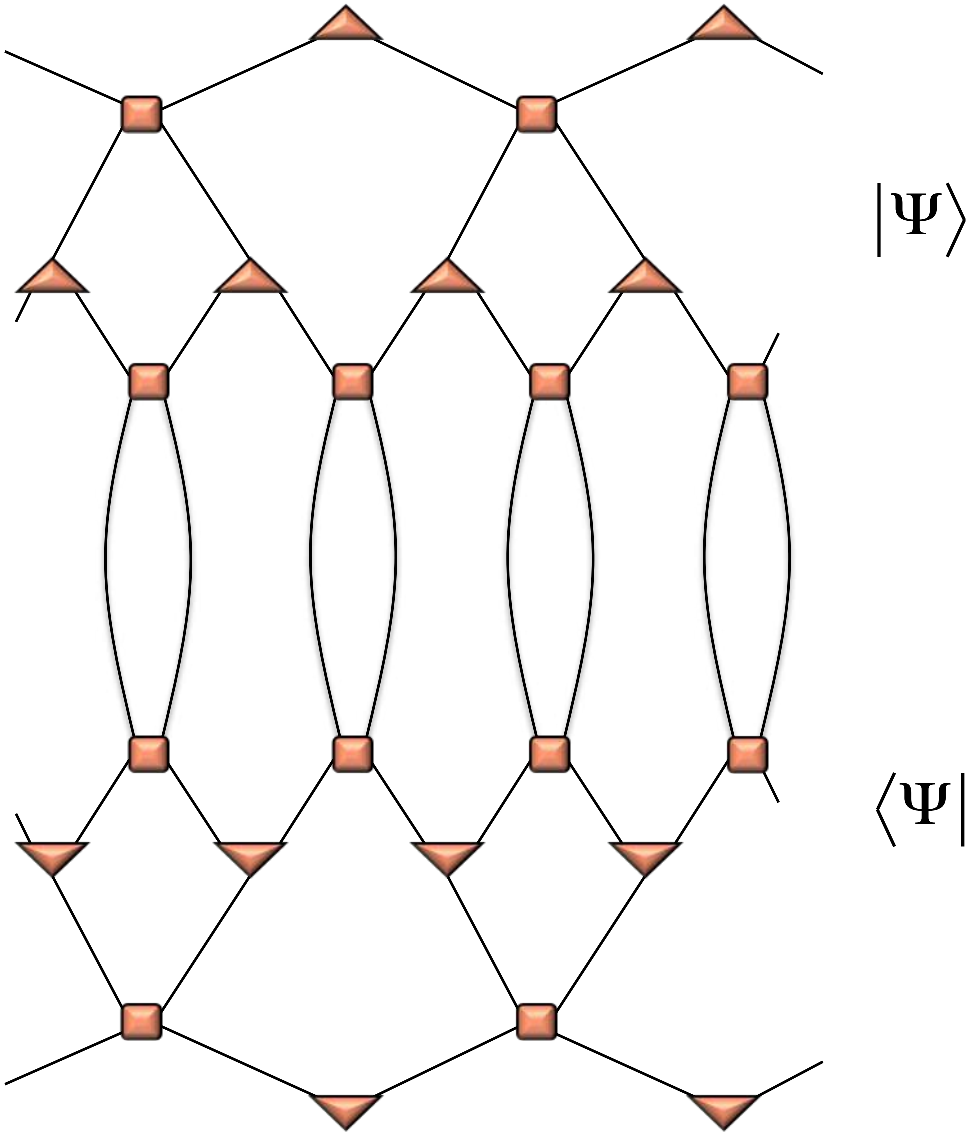

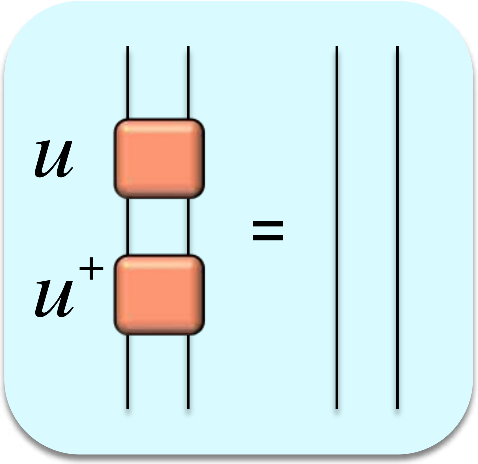

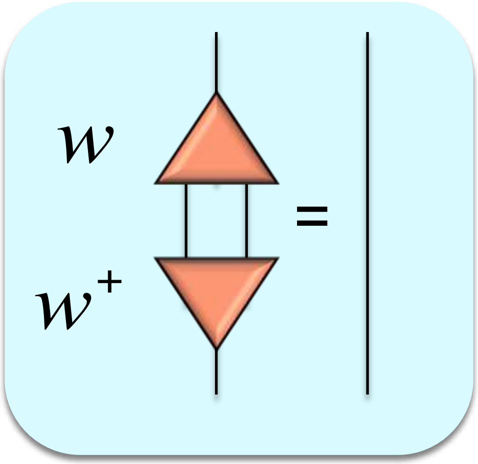

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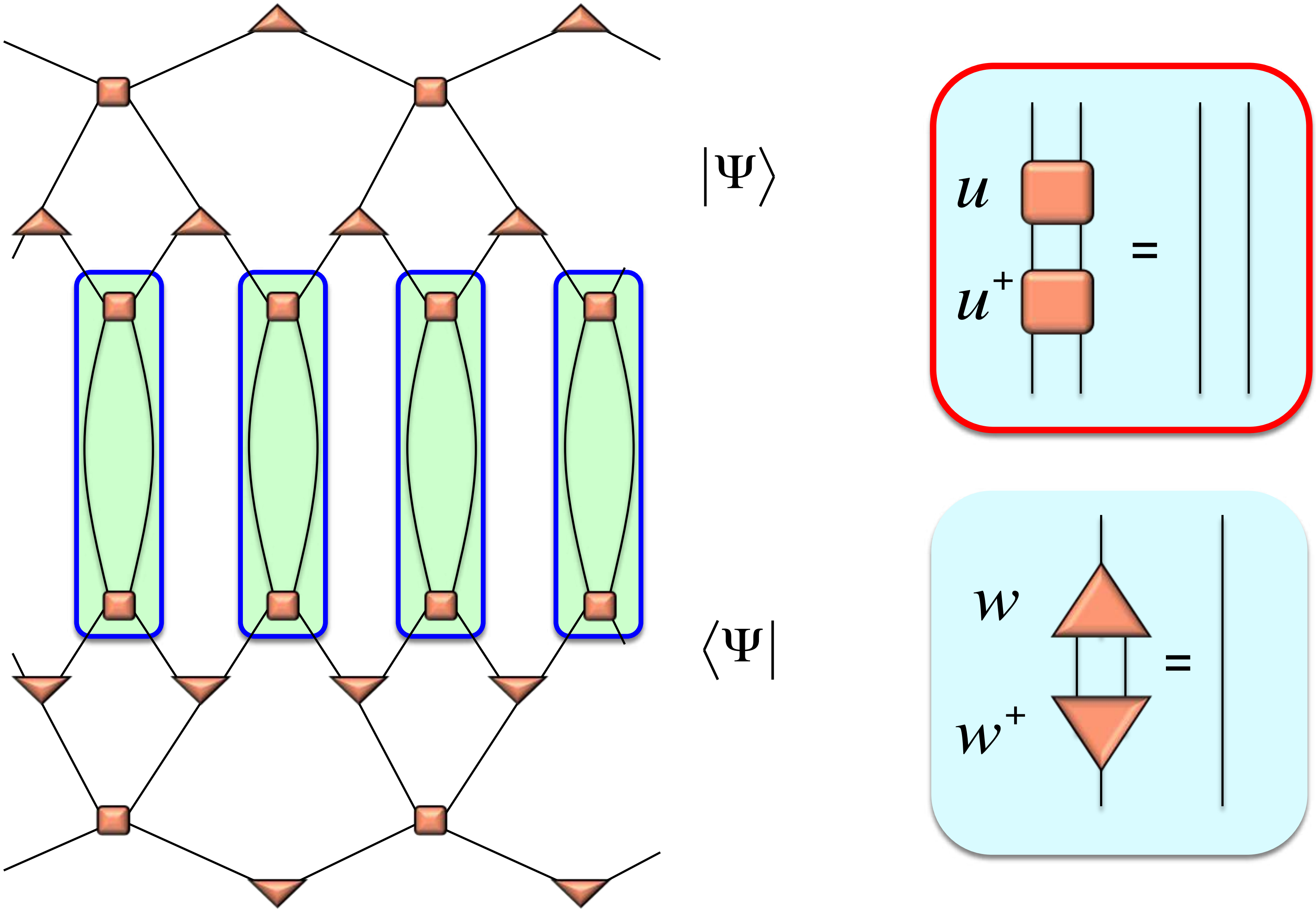

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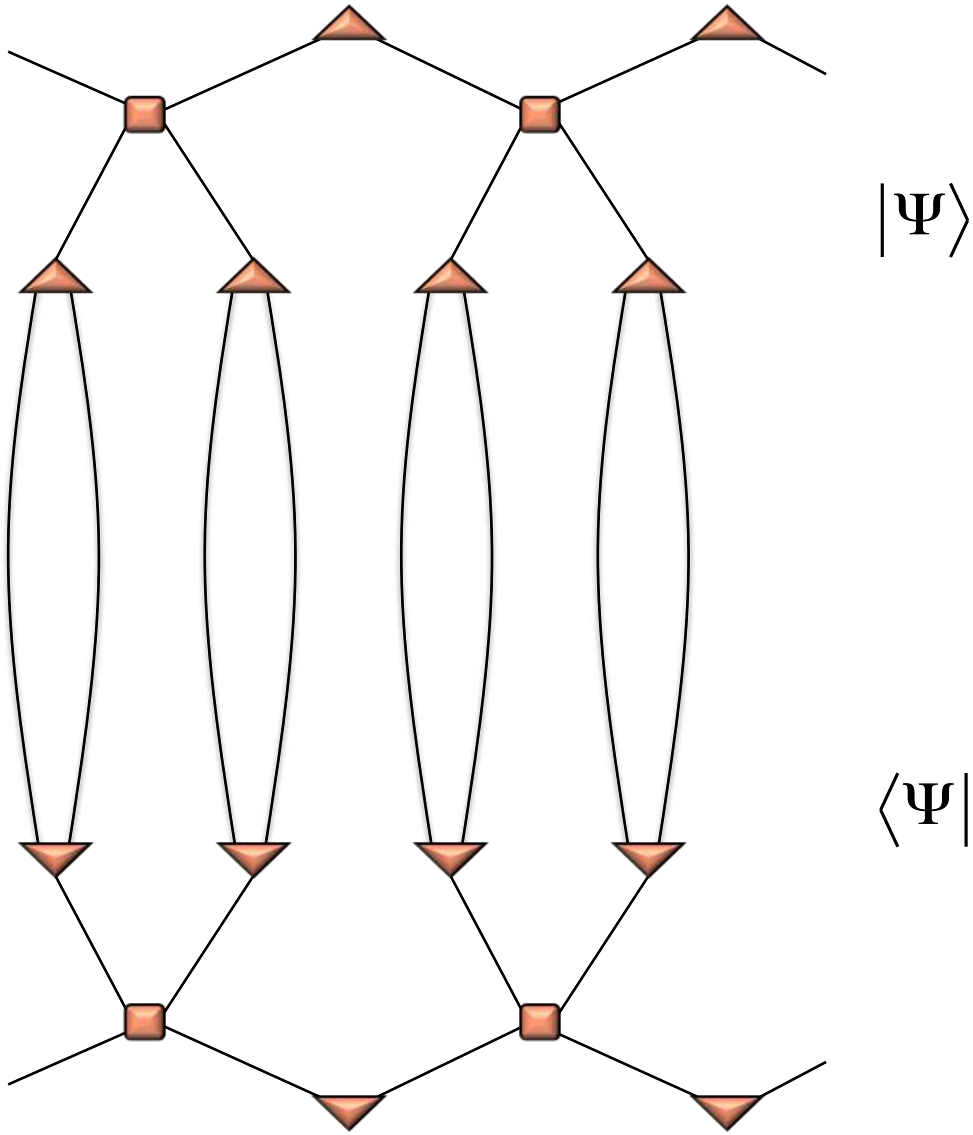

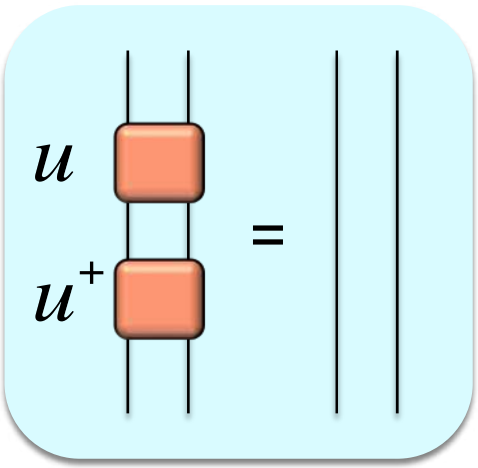

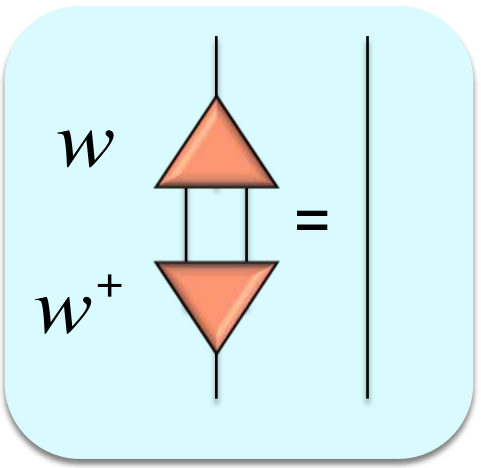

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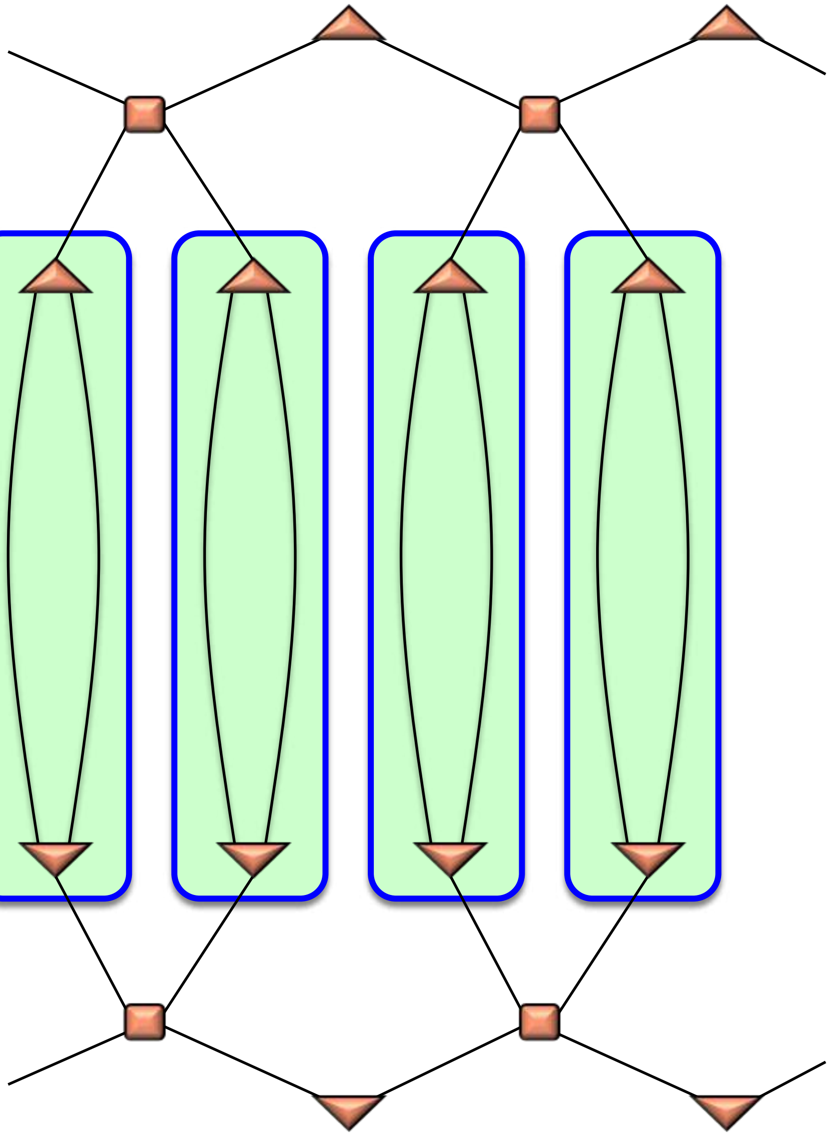
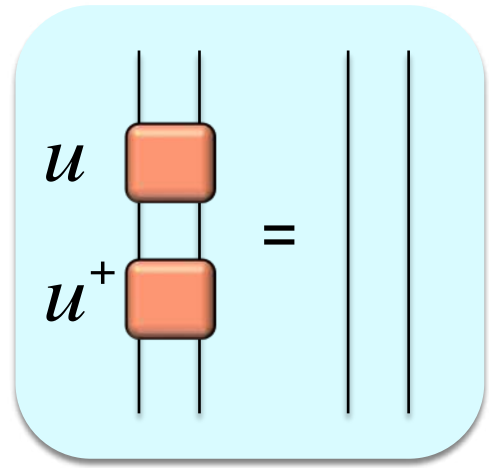
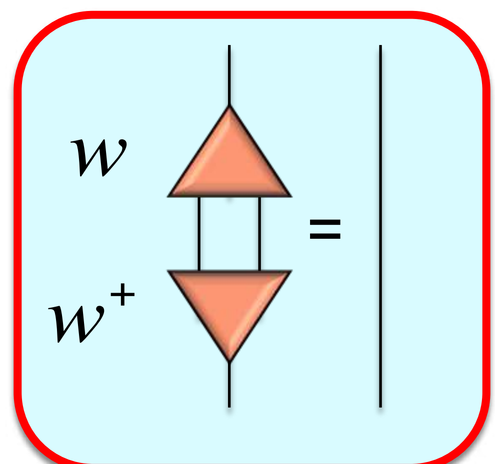


$|\Psi\rangle$

$\langle\Psi|$





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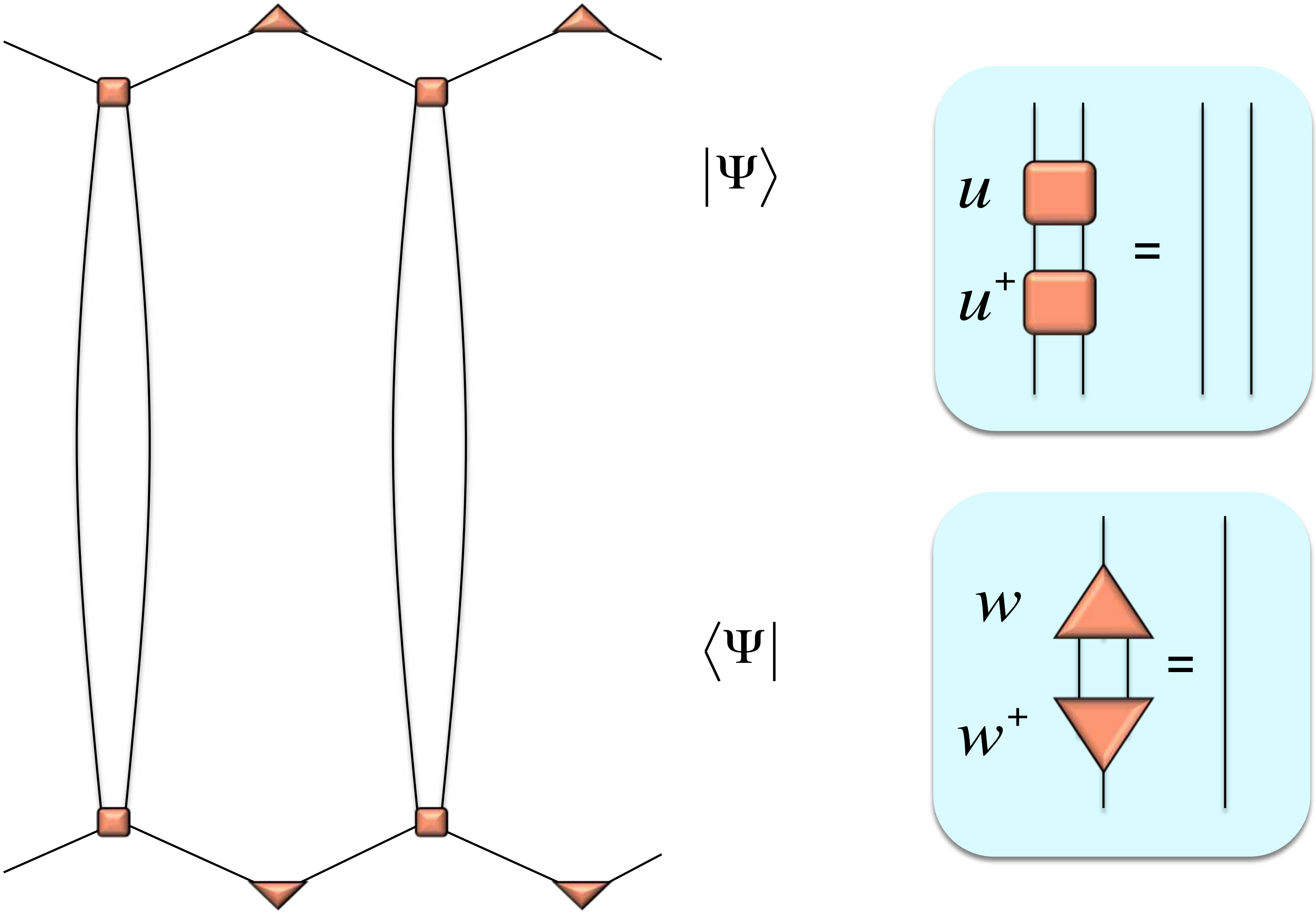
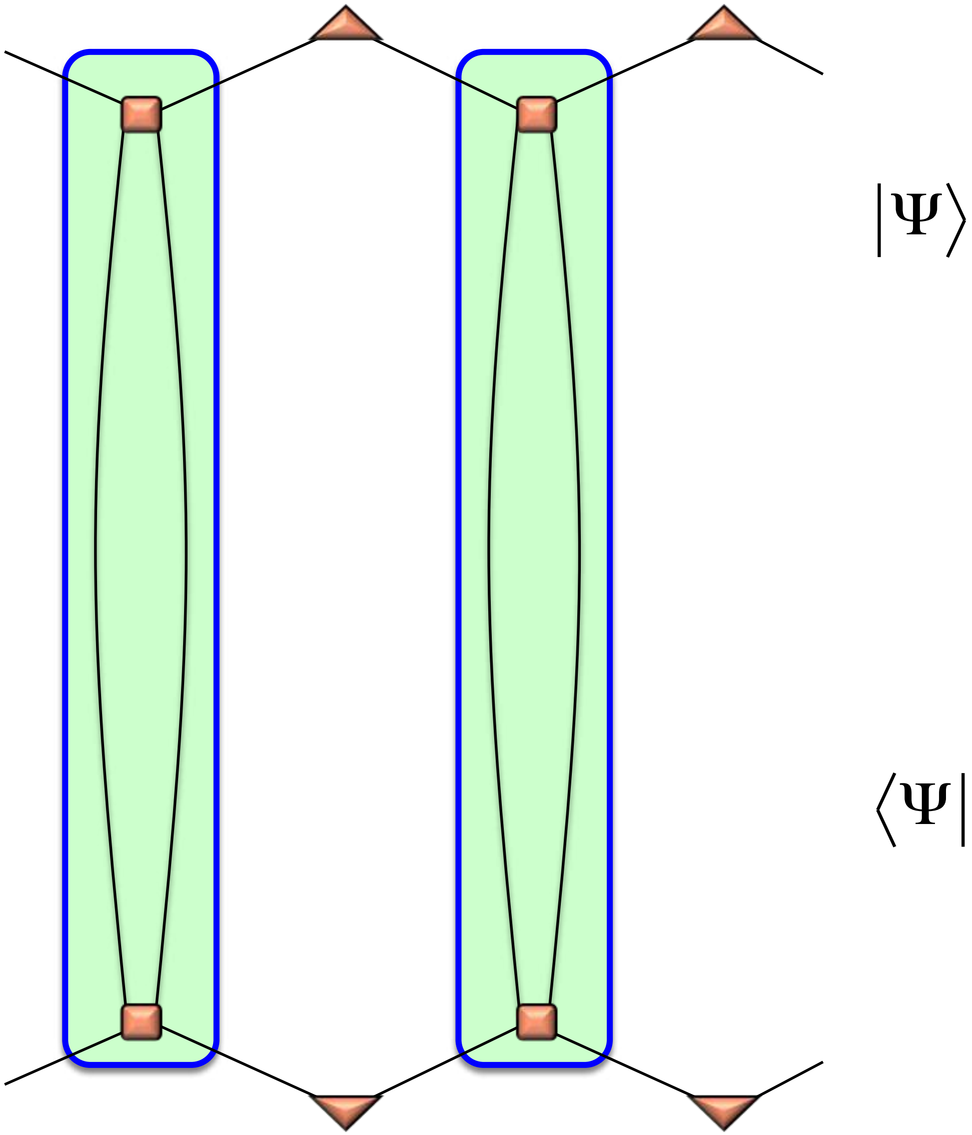
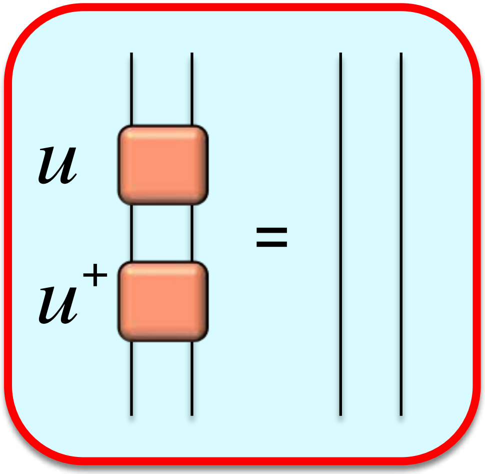
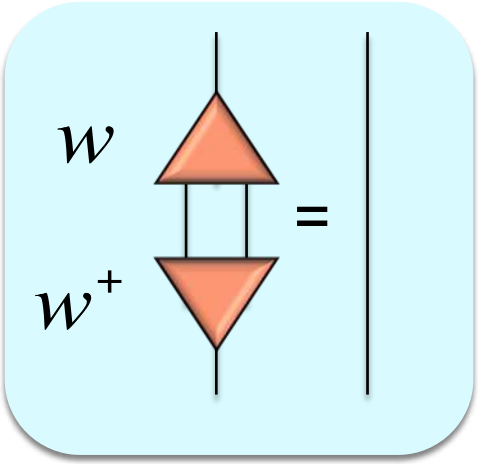


Norm of MERA







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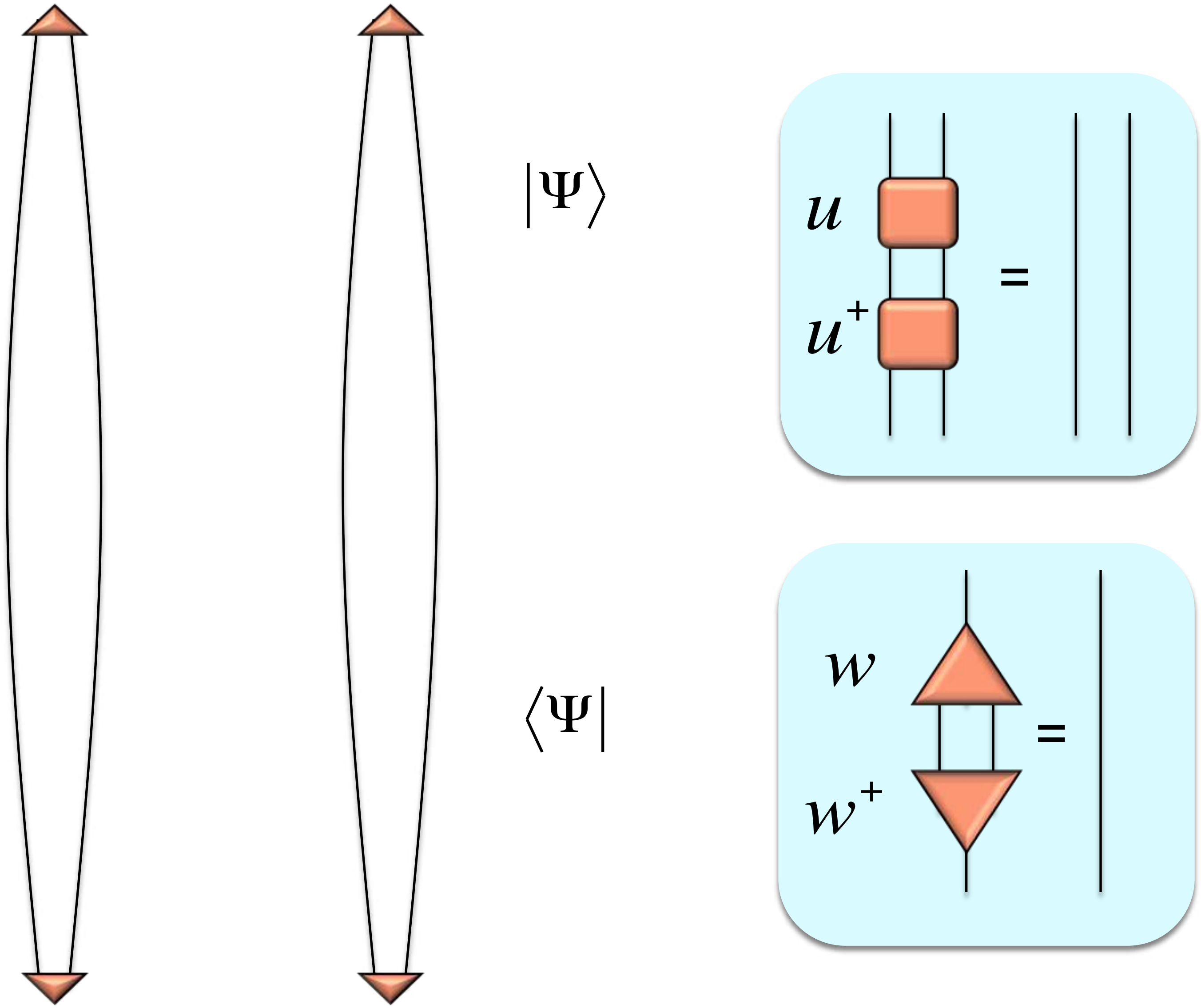

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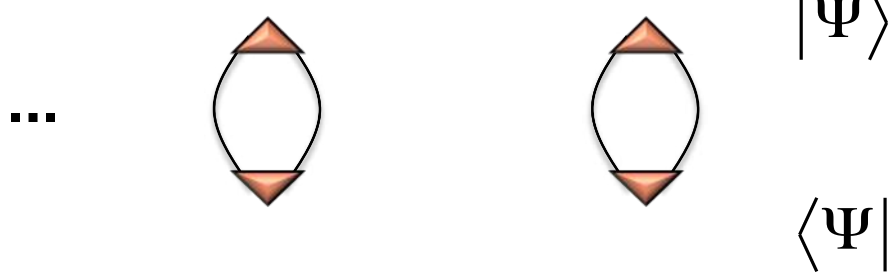
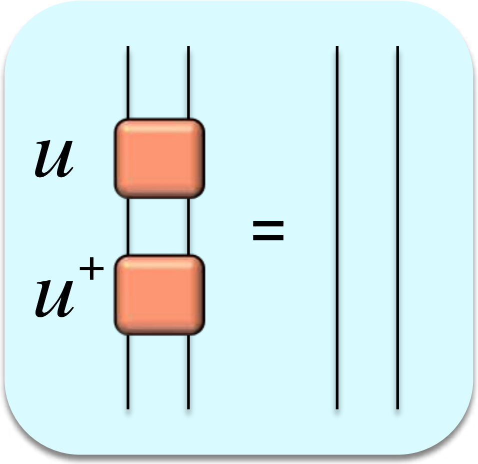
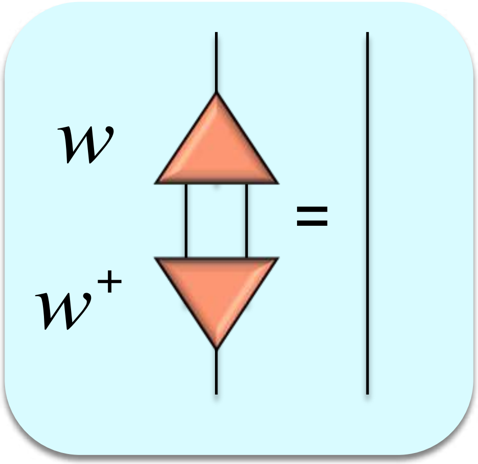
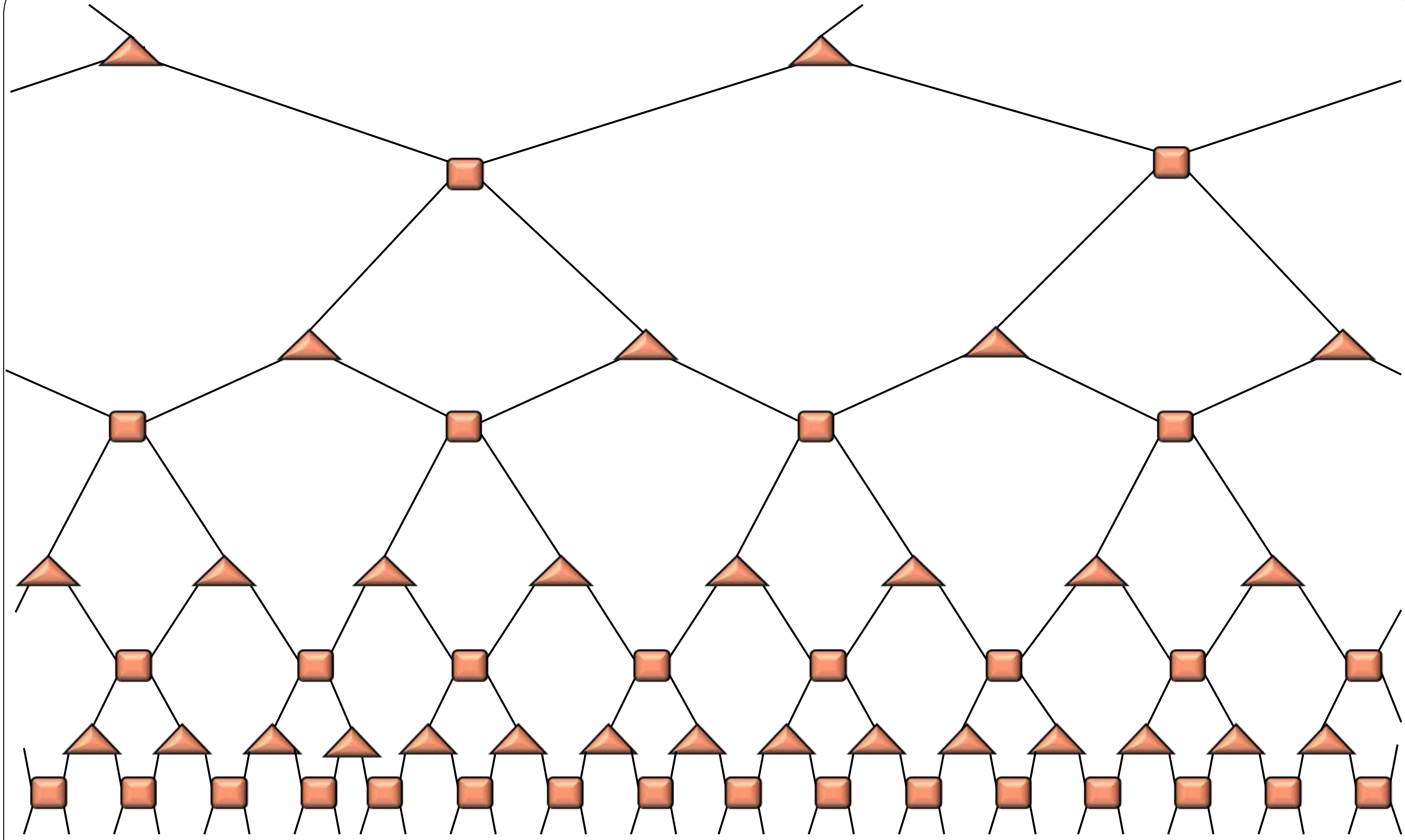


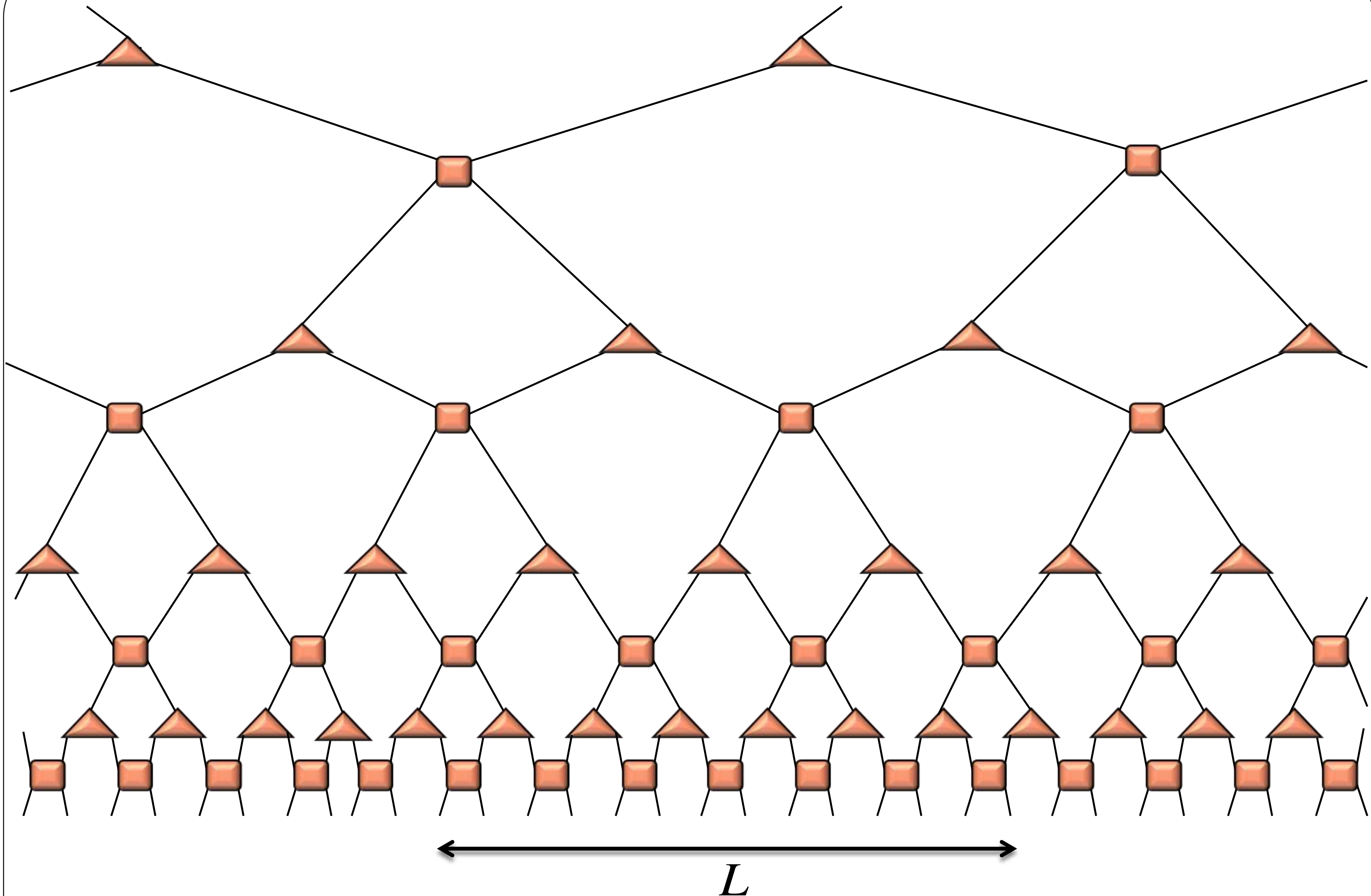




The norm is just the contraction of the top tensors

Entropy of 1d MERA

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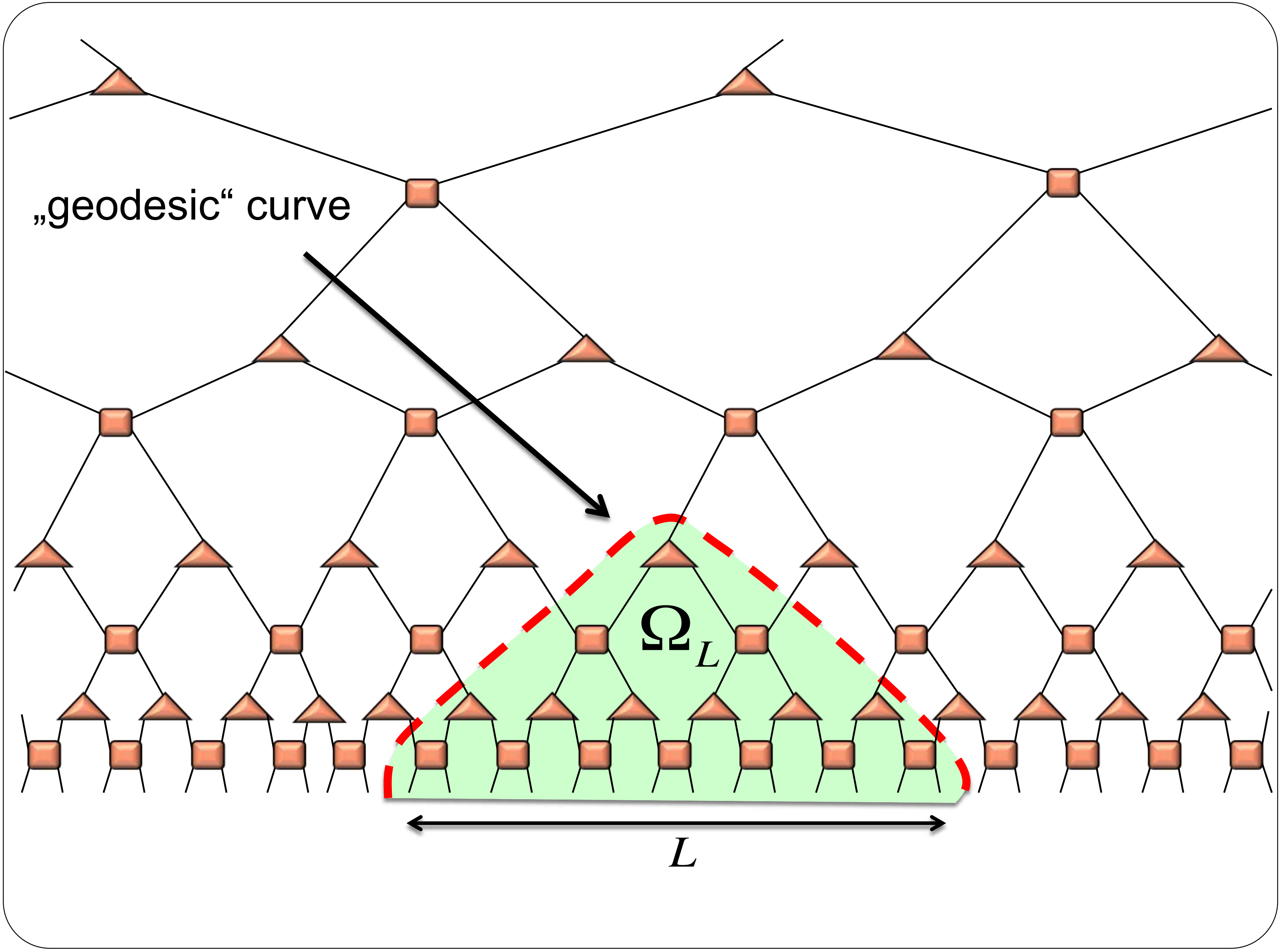
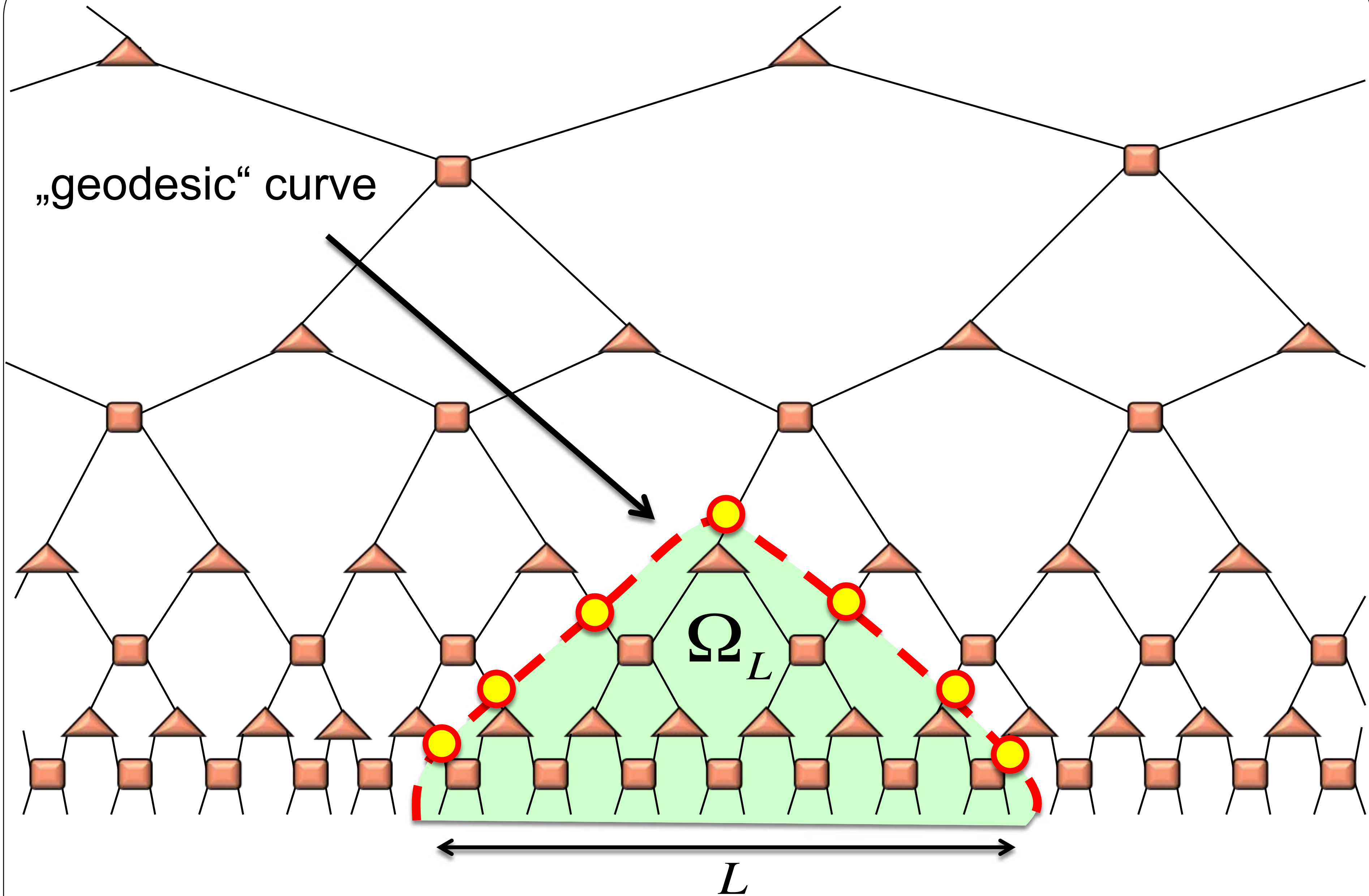
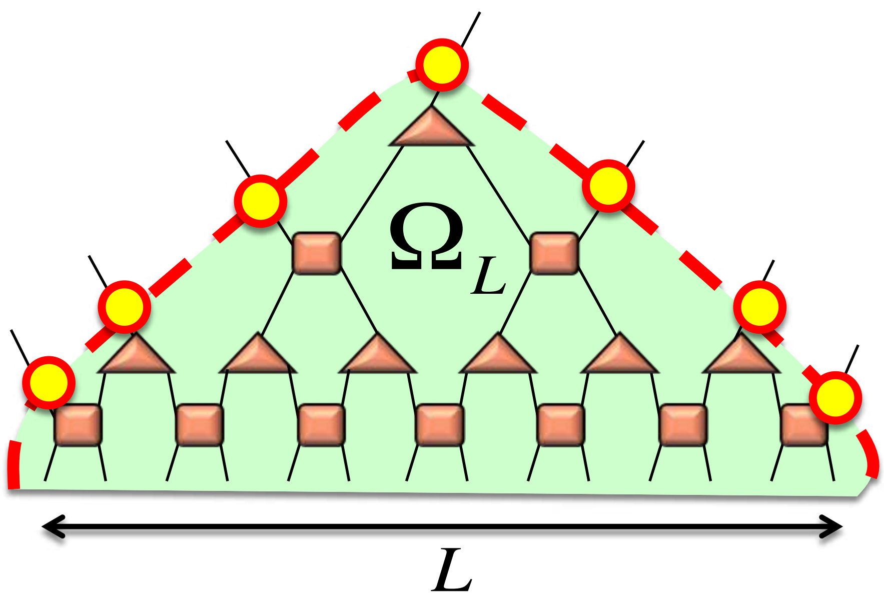
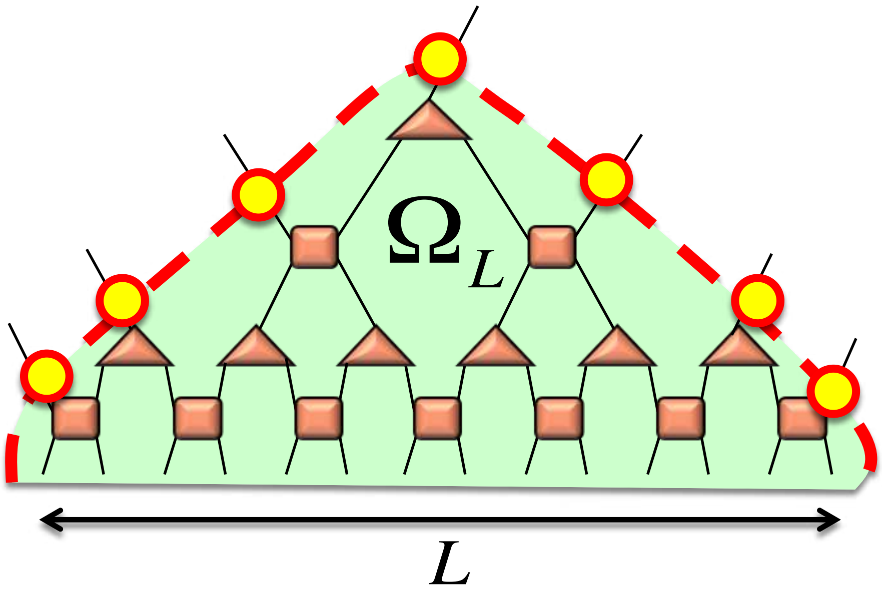

„geodesic“ curve
Ω_L
L



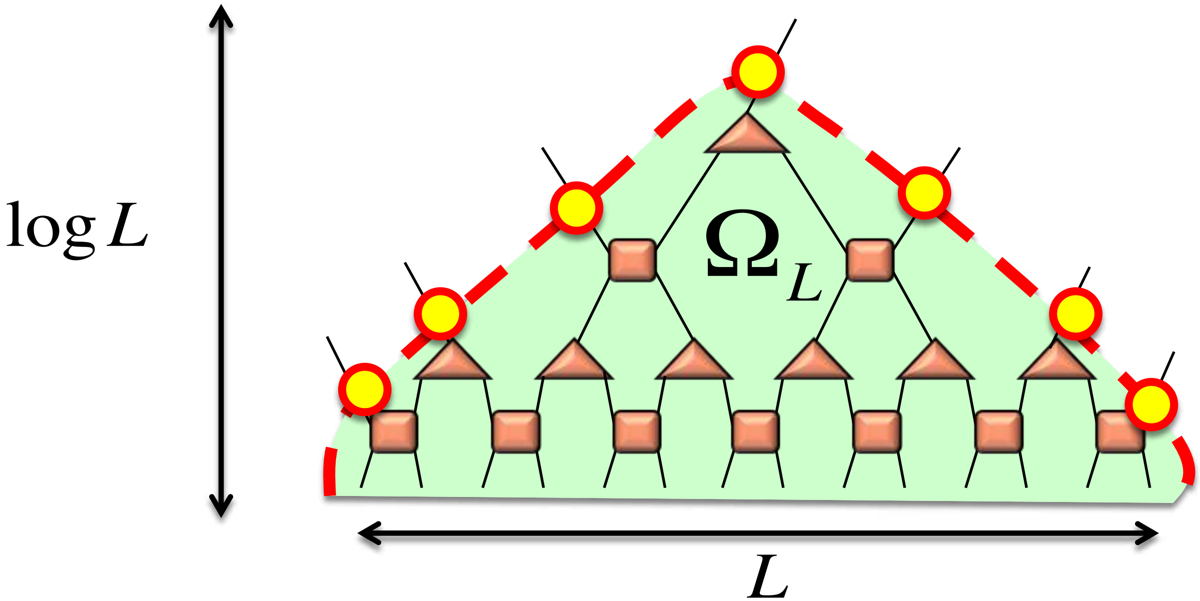
Entanglement as boundary in holographic geometry: $S(L) \leq \log(\chi)\left|\partial\Omega_L\right|$



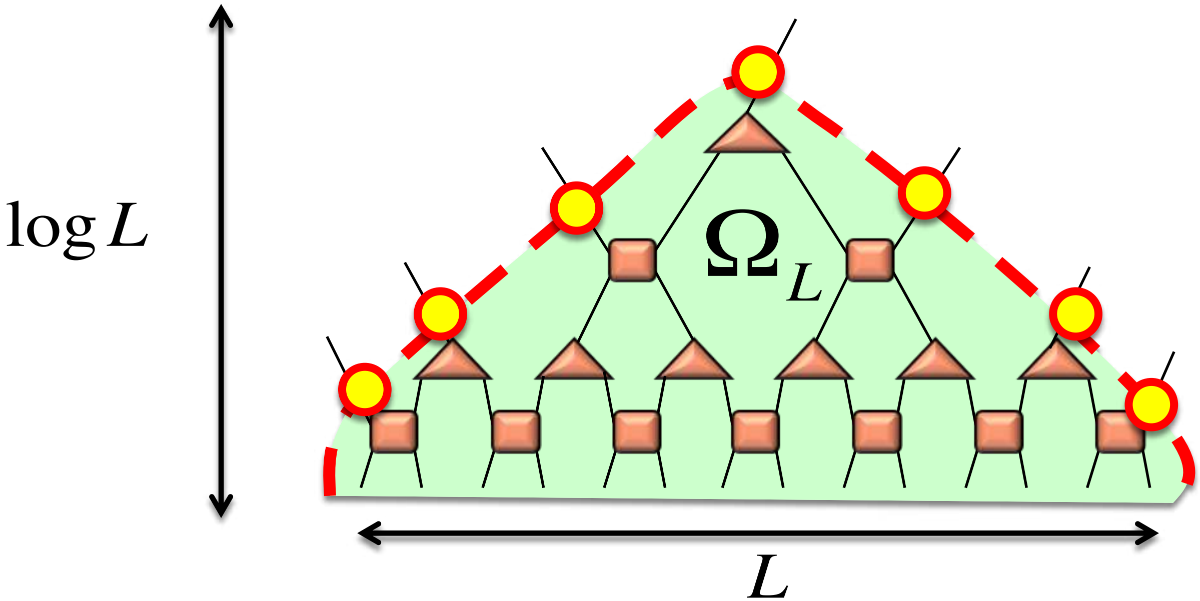
Entanglement as boundary in holographic geometry: $S(L) \leq \log(\chi)\left|\partial\Omega_L\right|$



Entanglement as boundary in holographic geometry: $S(L) \leq \log(\chi)\left|\partial\Omega_L\right|$



Entanglement as boundary in holographic geometry: $S(L) \leq \log(\chi)\left|\partial\Omega_L\right|$



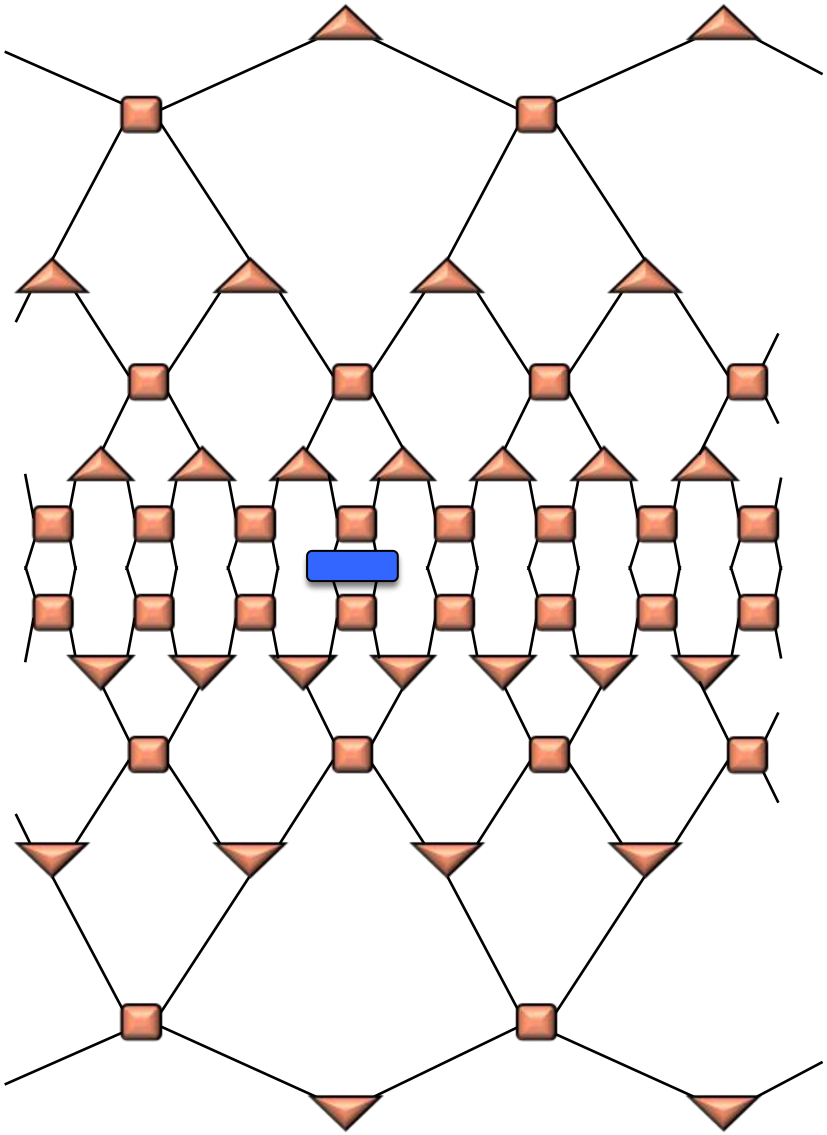
Entanglement as boundary in holographic geometry: $S(L) \le \log(\chi)\left|\partial\Omega_L\right|$

Constant contribution at every layer

1d MERA can produce logarithmic violations to the area-law: $S(L) \approx \log L$

(like 1d critical systems!)

Expectation values

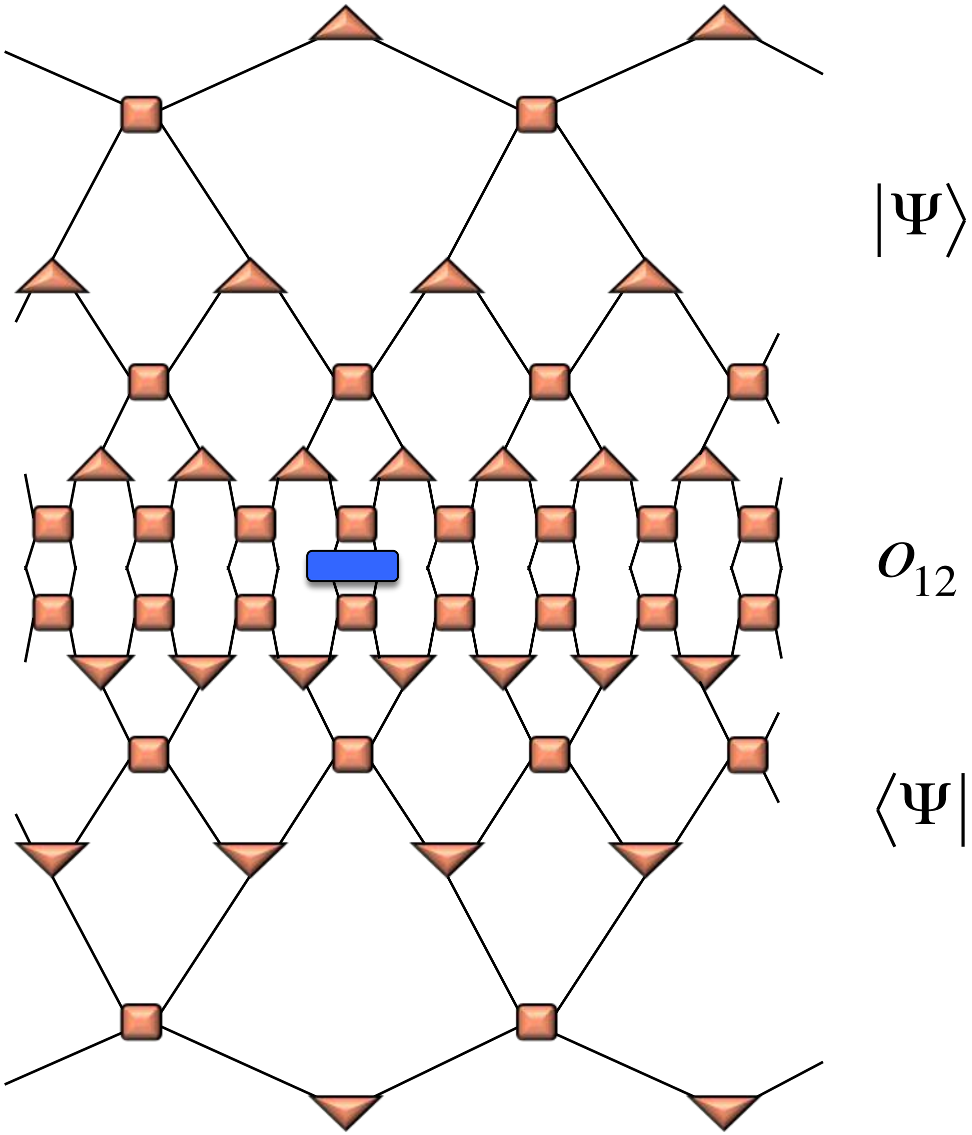


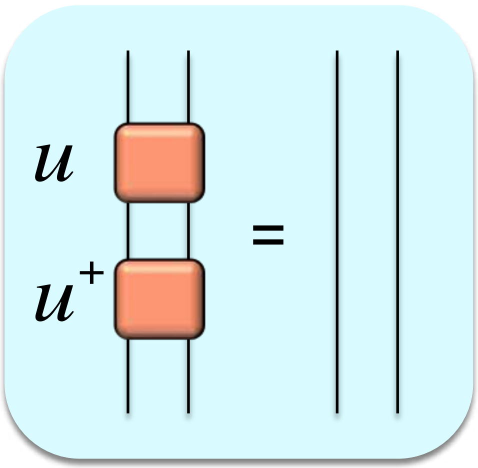
$|\Psi\rangle$

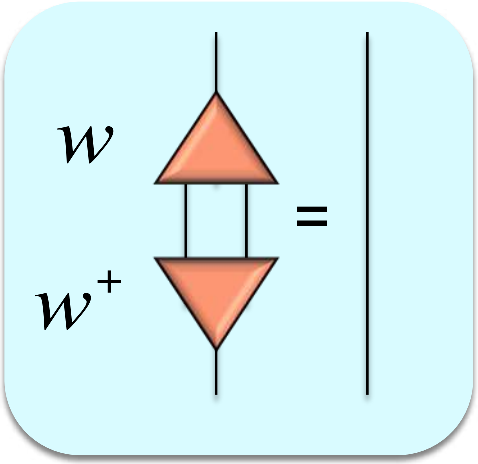
o_{12}

$\langle\Psi|$

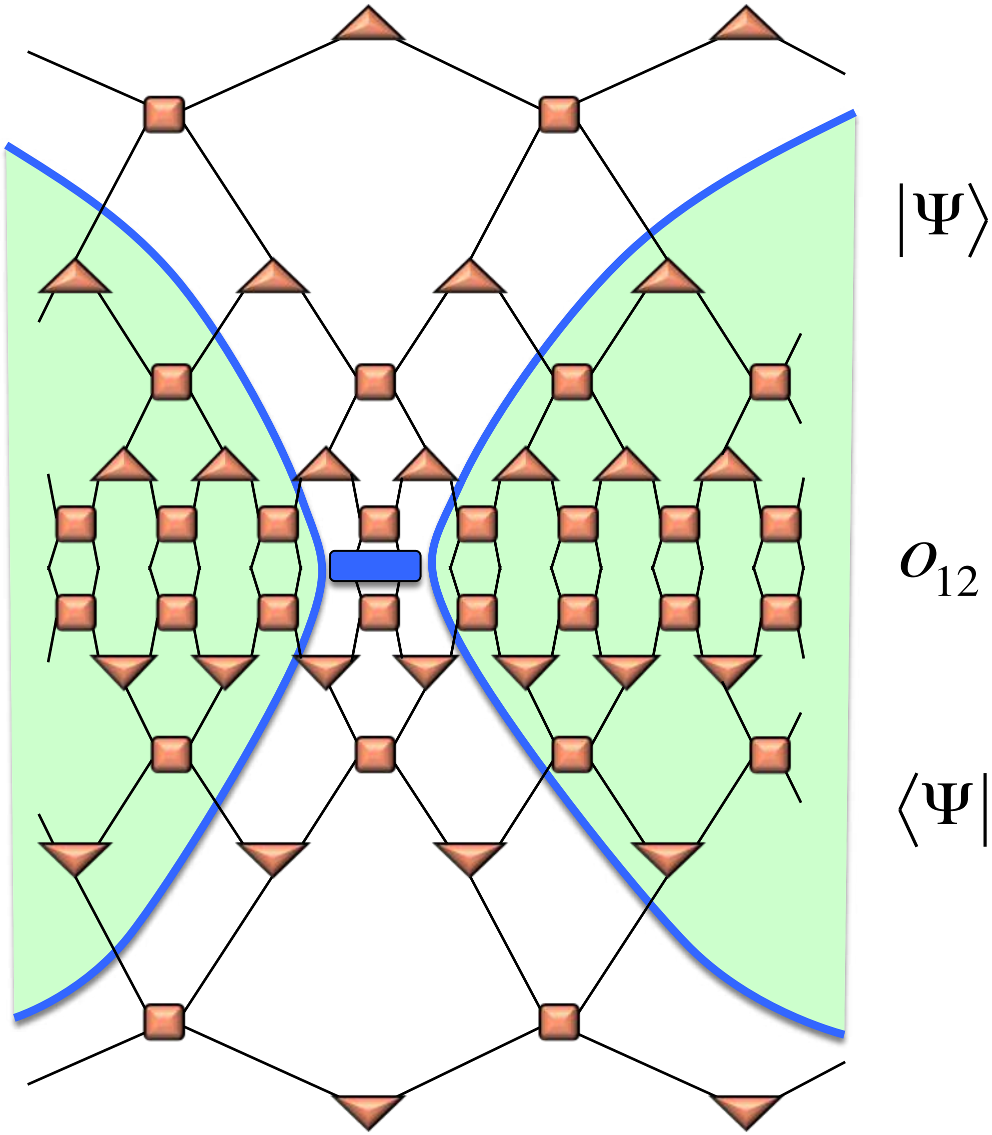
Expectation values

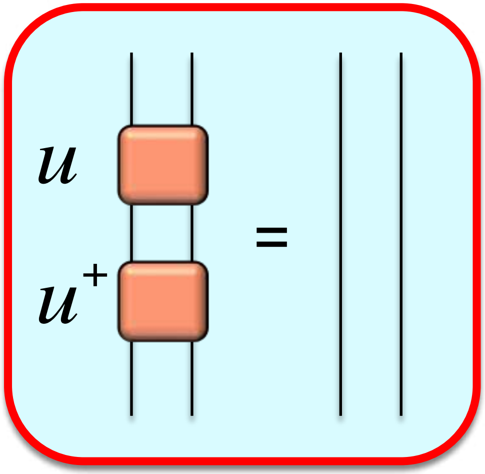


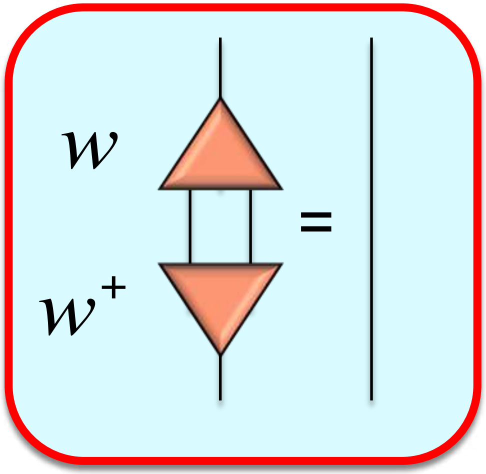




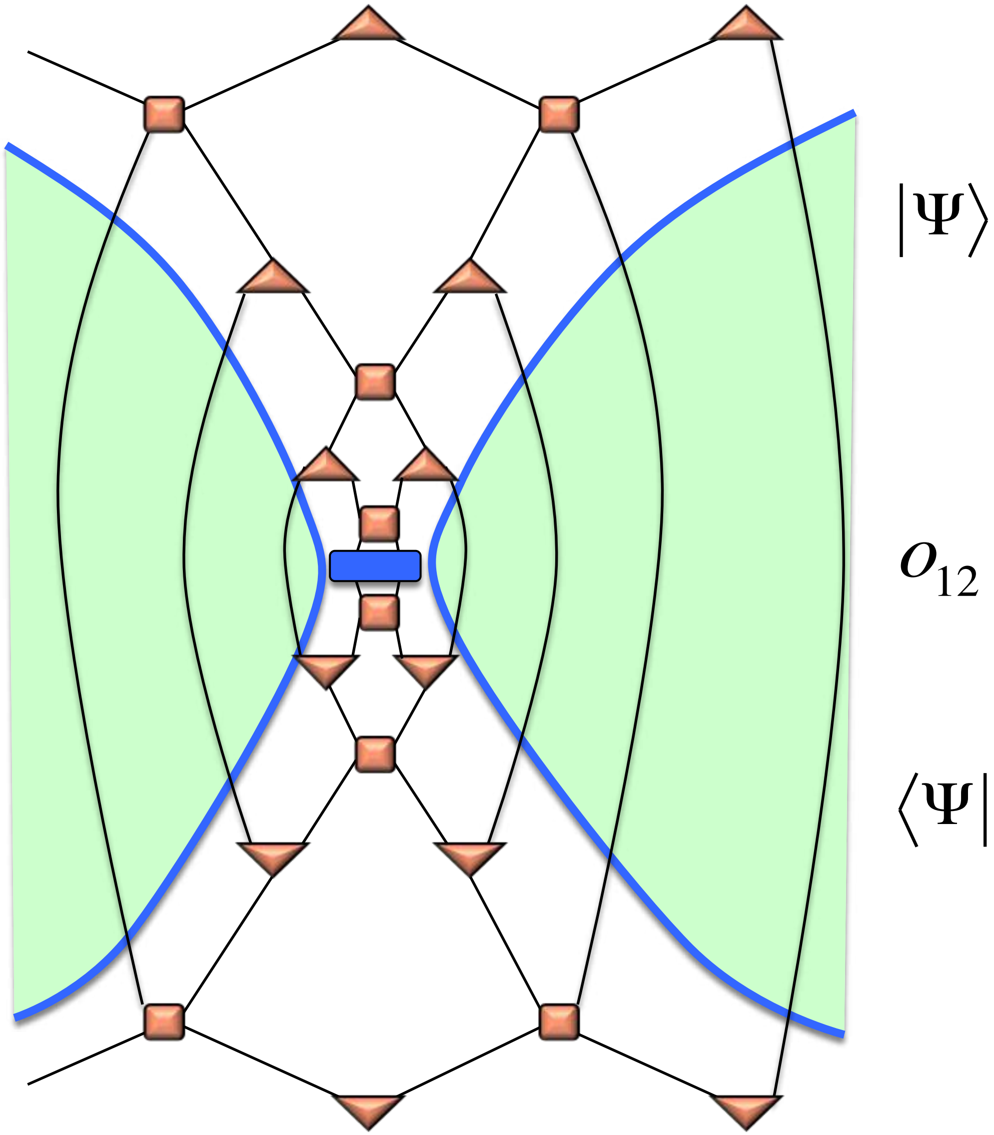
Expectation values

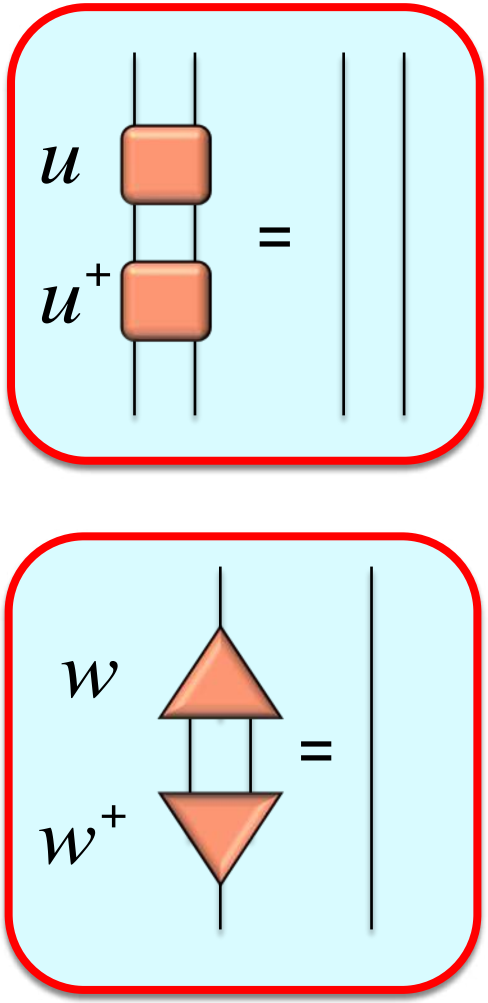




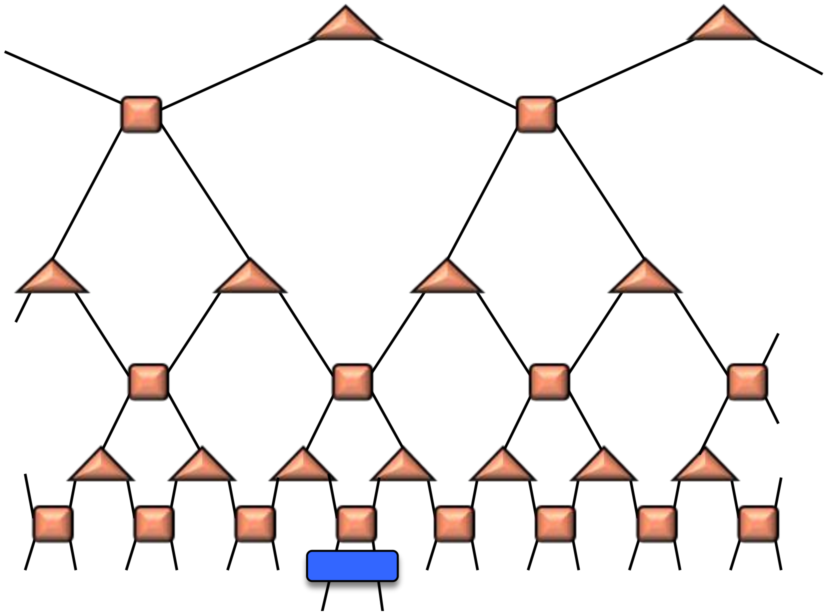


Expectation values





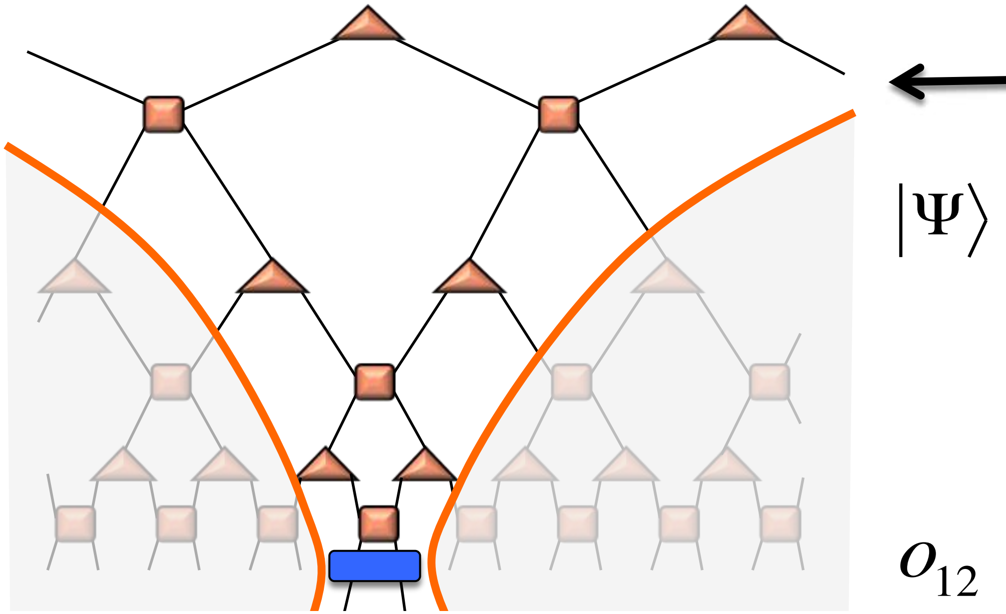
Expectation values



$$|\Psi\rangle$$

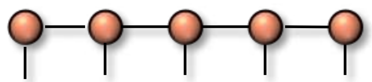
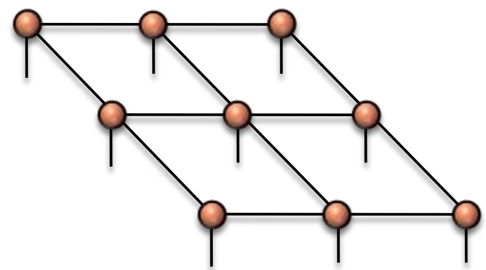
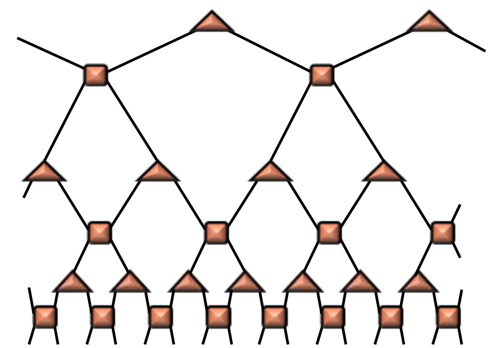
$$o_{12}$$

Expectation values



Only tensors inside of the causal cone contribute to the expectation value

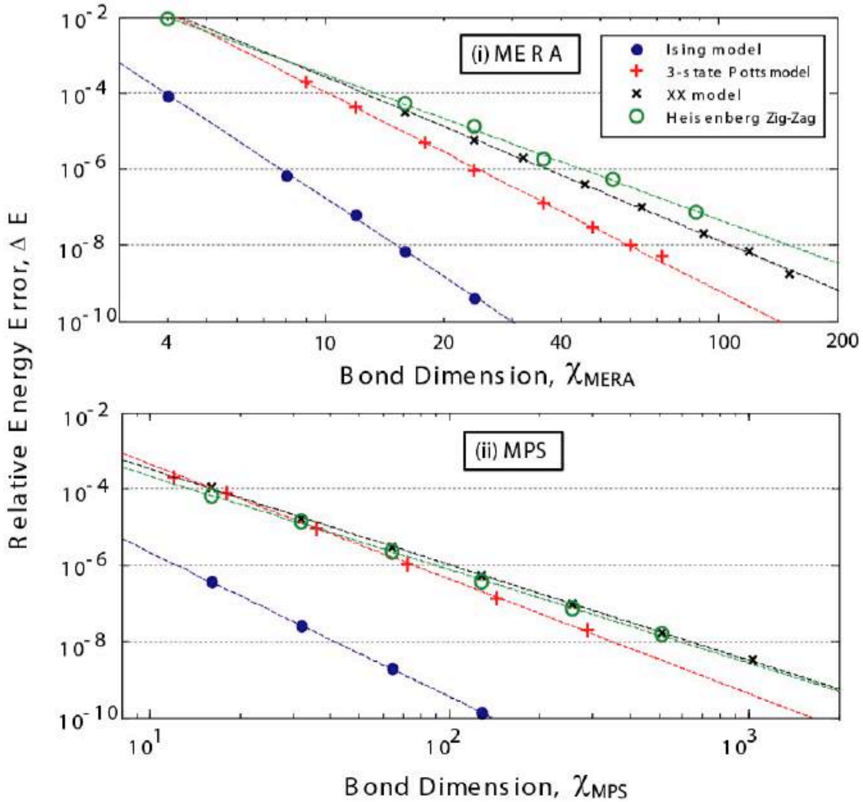
Comparison

	MPS in 1d 	*PEPS in 2d* 	*MERA in 1d* 
Ent. entropy	$S(L) = O(1)$	$S(L) = O(L)$	$S(L) = O(\log L)$
Exact contraction	efficient	inefficient	efficient
Corr. length	finite	finite & infinite	finite & infinite
To/from	1d Ham.	2d Ham.	1d Ham.
Tensors	arbitrary	arbitrary	constrained

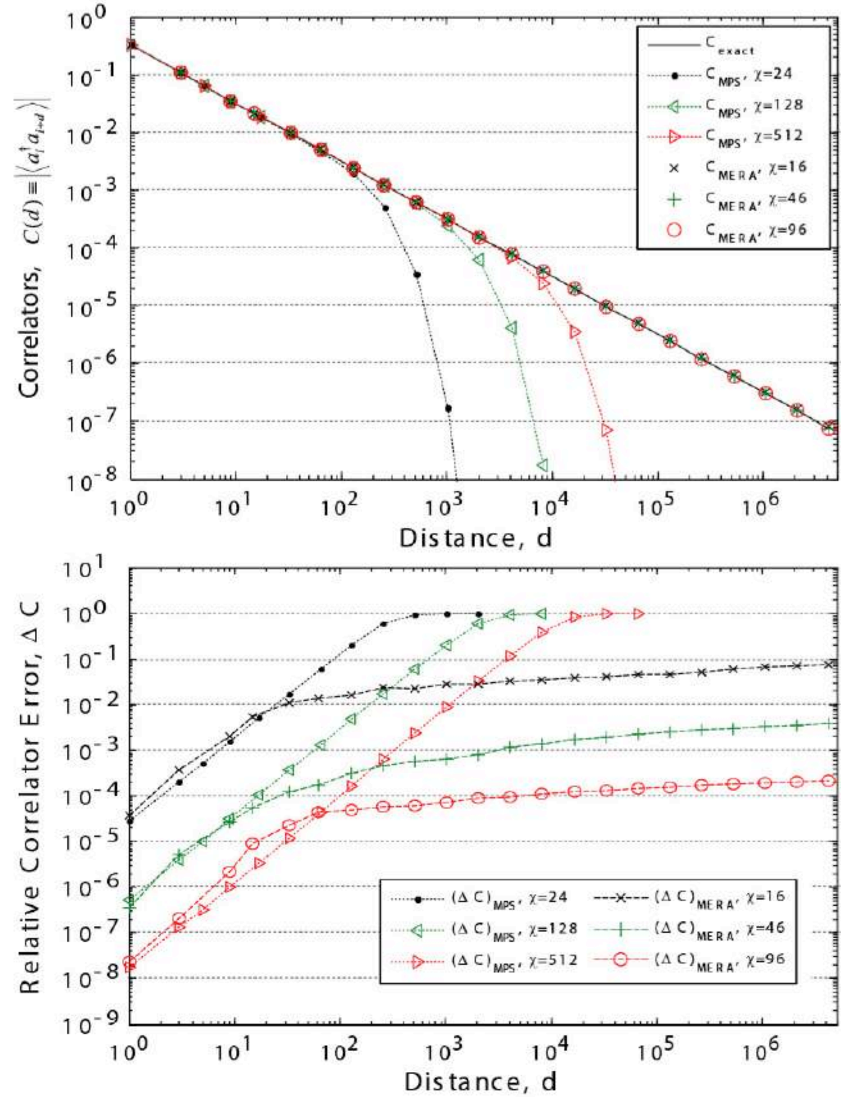
A numerical example:
1d critical systems

G. Evenbly, G. Vidal, in "Strongly Correlated Systems. Numerical Methods", Springer, Vol. 176 (2013)

Critical Energies



Critical XX Correlators



Outline

1) Basics

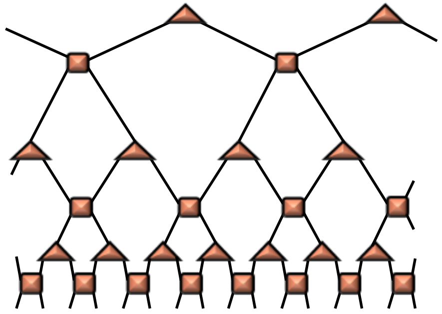
2) 1d MPS

3) 2d PEPS

4) Numerical algorithms

5) MERA

6) Extras



Outline

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6) Extras

Machine Learning

E.g., E. M. Stoudenmire, D. J. Schwab, Advances in Neural Information Processing Systems 29, 4799 (2016)

Supervised Kernel Learning

Supervised Learning With Quantum-Inspired Tensor Networks

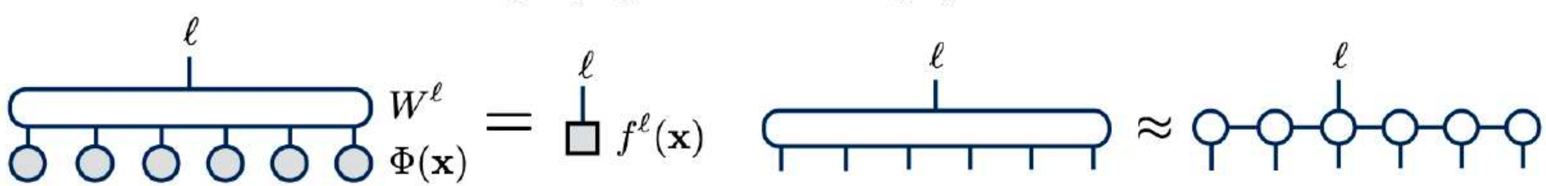
E. Miles Stoudenmire[1,2] and David J. Schwab[3]

[1]*Perimeter Institute for Theoretical Physics, Waterloo, Ontario, N2L 2Y5, Canada*
[2]*Department of Physics and Astronomy, University of California, Irvine, CA 92697-4575 USA*
[3]*Dept. of Physics, Northwestern University, Evanston, IL*

(Dated: May 22, 2017)

Tensor networks are efficient representations of high-dimensional tensors which have been very successful for physics and mathematics applications. We demonstrate how algorithms for optimizing such networks can be adapted to supervised learning tasks by using matrix product states (tensor trains) to parameterize models for classifying images. For the MNIST data set we obtain less than 1% test set classification error. We discuss how the tensor network form imparts additional structure to the learned model and suggest a possible generative interpretation.

$$f^{\ell}(\mathbf{x}) = W^{\ell} \cdot \Phi(\mathbf{x})$$



$$C = \frac{1}{2} \sum_{n=1}^{N_T} \sum_{\ell} (f^{\ell}(\mathbf{x}_n) - \delta^{\ell}_{L_n})^2$$

Minimize using TN methods

Anomaly detection

Anomaly Detection with Tensor Networks

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Abstract

Originating from condensed matter physics, tensor networks are compact representations of high-dimensional tensors. In this paper, the prowess of tensor networks is demonstrated on the particular task of one-class anomaly detection. We exploit the memory and computational efficiency of tensor networks to learn a linear transformation over a space with dimension exponential in the number of original features. The linearity of our model enables us to ensure a tight fit around training instances by penalizing the model's global tendency to a predict normality via its Frobenius norm—a task that is infeasible for most deep learning models. Our method outperforms deep and classical algorithms on tabular datasets and produces competitive results on image datasets, despite not exploiting the locality of images.

Anomaly detection

Anomaly Detection with Tensor Networks

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Beyond MPS

Machine Learning by Unitary Tensor Network of Hierarchical Tree Structure

Ding Liu,[1,2] Shi-Ju Ran,[3,2,*] Peter Wittek,[4,5,6,7,†] Cheng Peng,[8]
Raul Blázquez García,[2] Gang Su,[8,9] and Maciej Lewenstein[2,10]

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[2] *ICFO-Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain*
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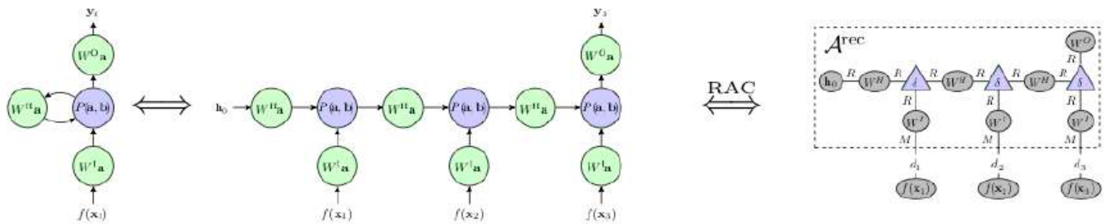
[10] *ICREA, Passeig Lluis Companys 23, 08010 Barcelona, Spain*

The resemblance between the methods used in quantum-many body physics and in machine learning has drawn considerable attention. In particular, tensor networks (TNs) and deep learning architectures bear striking similarities to the extent that TNs can be used for machine learning. Previous results used one-dimensional TNs in image recognition, showing limited scalability and flexibilities. In this work, we train two-dimensional hierarchical TNs to solve image recognition problems, using a training algorithm derived from the multi-scale entanglement renormalization ansatz. This approach introduces mathematical connections among quantum many-body physics, quantum information theory, and machine learning. While keeping the TN unitary in the training phase, TN states are defined, which encode classes of images into quantum many-body states. We study the quantum features of the TN states, including quantum entanglement and fidelity. We find these quantities could be properties that characterize the image classes, as well as the machine learning tasks.

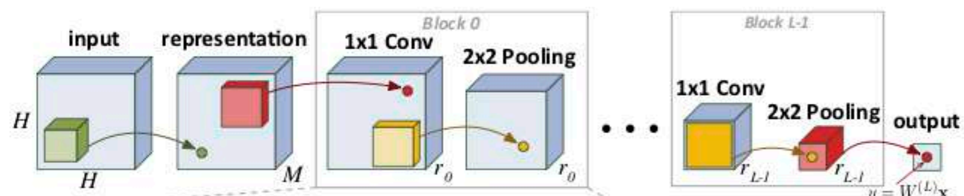
TN structure of some neural networks

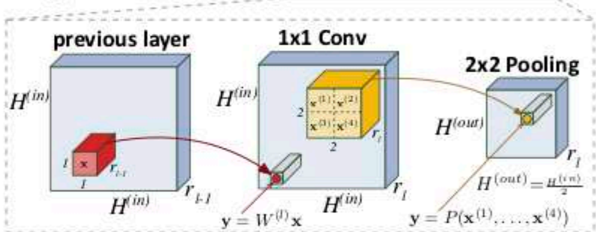
Y. Levine et al, Phys. Rev. Lett. 122, 065301 (2019)

Recurrent neural networks are MPS (hidden Markov models)

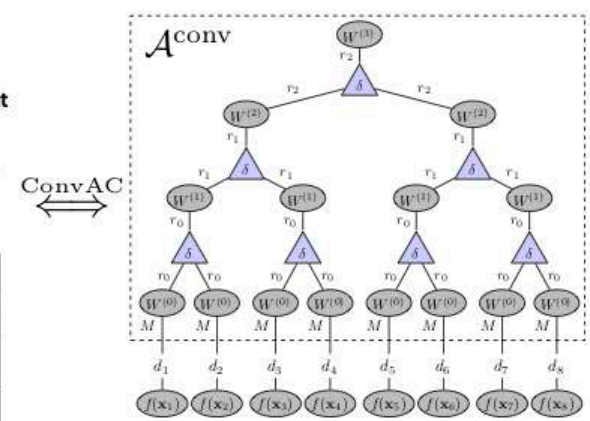


Deep convolutional networks are TTNs





	$P:\mathbb{R}^{r_1}\times\ldots\times\mathbb{R}^{r_1}\to\mathbb{R}^{r_1}$
ConvNet	$y_i \equiv \max_j \max\left\{x_i^{(j)}, 0\right\}$
RNN	$y_i \equiv \tanh\left(\sum_j x_i^{(j)}\right)$
ConvAC & RAC	$y_i \equiv \prod_j x_i^{(j)}$

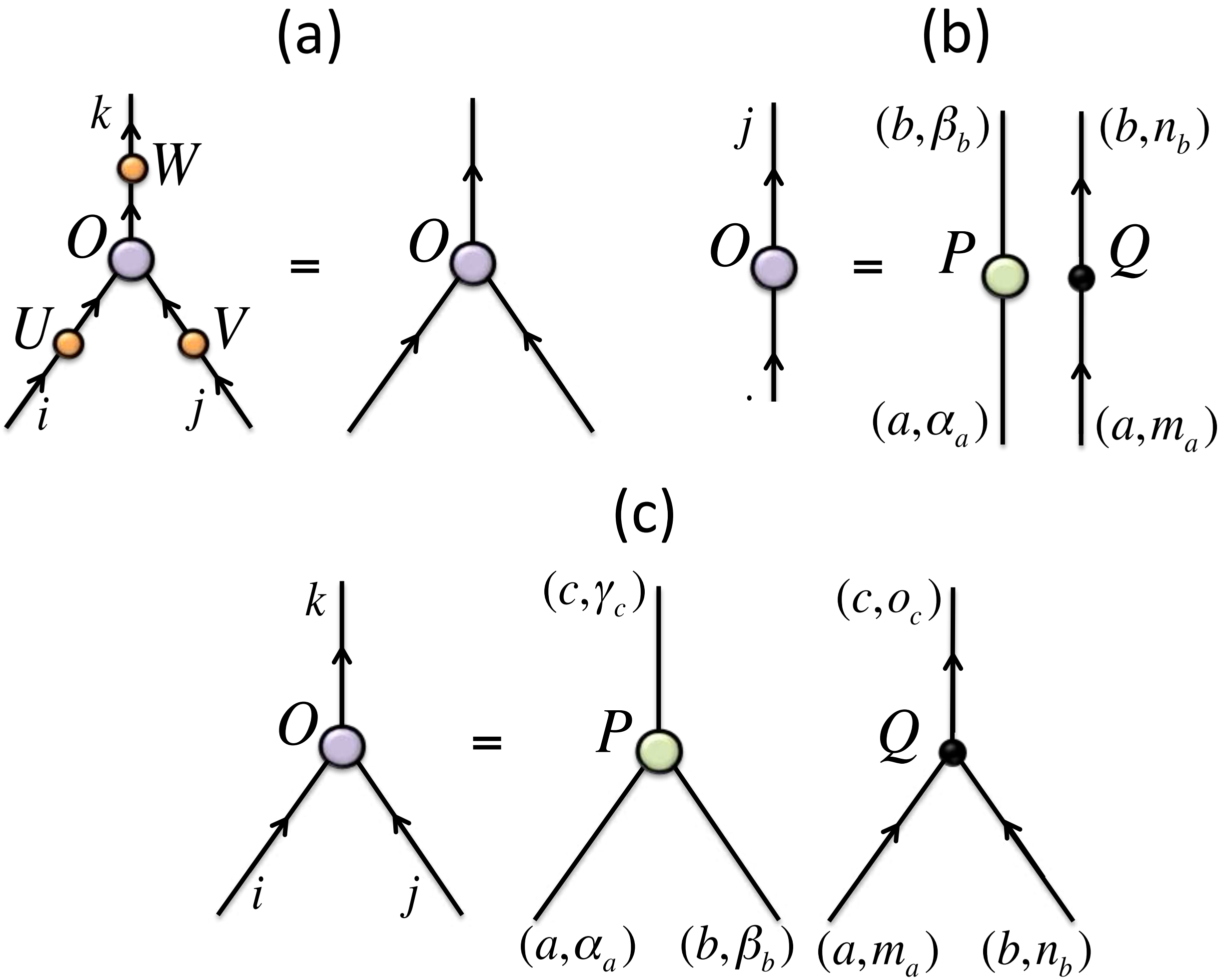


Symmetries

e.g., P. Schmoll et al, Annals of Physics 419 (2020) 168232

Symmetric tensors and Schur's lemma

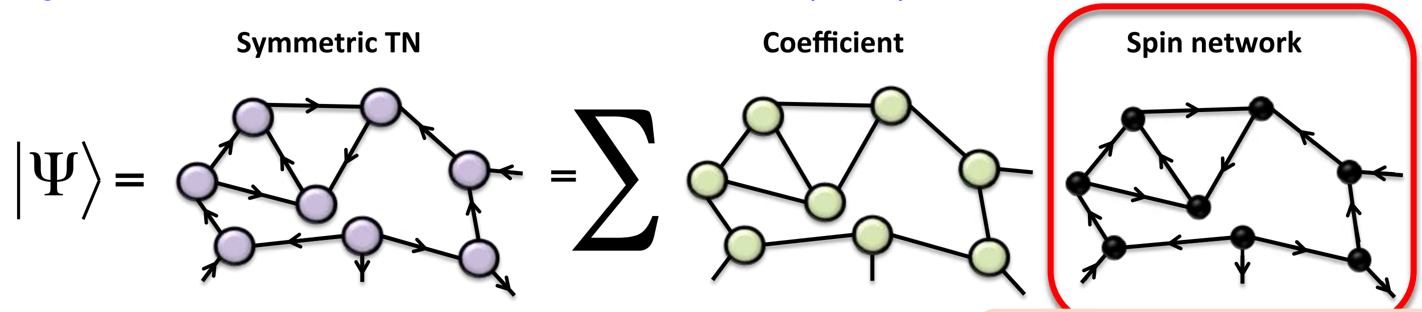
e.g., S. Singh, R. N. C. Pfeifer, G. Vidal, PRA 82, 050301 (2010)



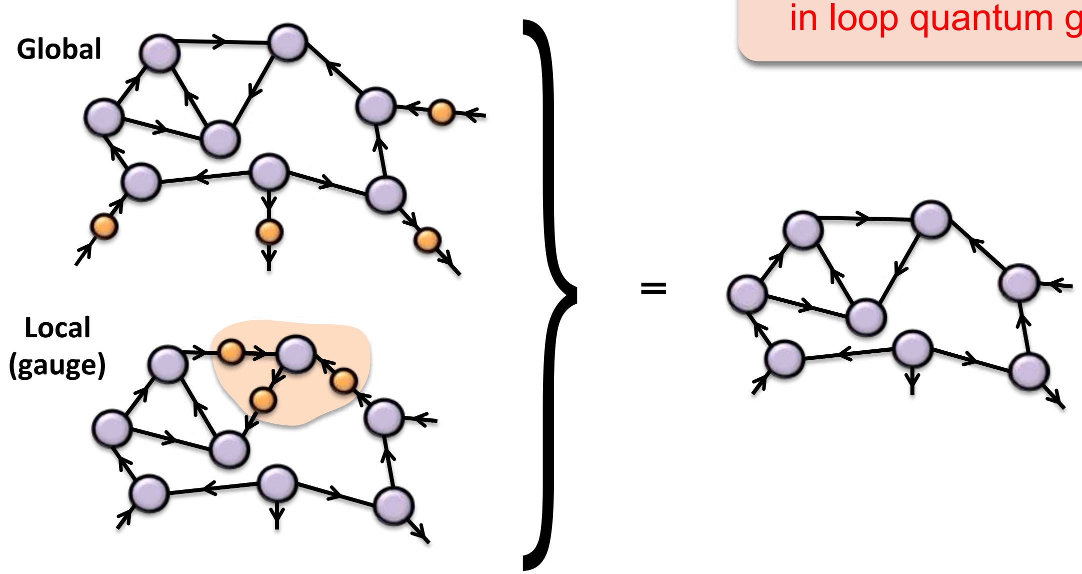
Structural part depends only on the group properties (intertwiners)

Emergent spin networks

e.g., S. Singh, R. N. C. Pfeifer, G. Vidal, PRA 82, 050301 (2010)



States of quantum geometry in loop quantum gravity...

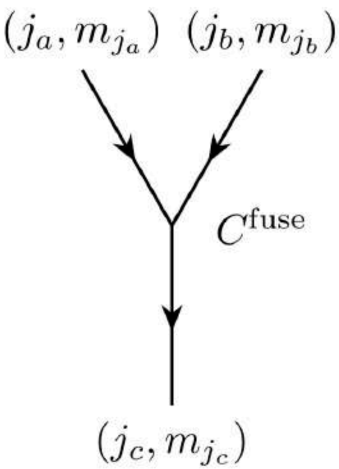


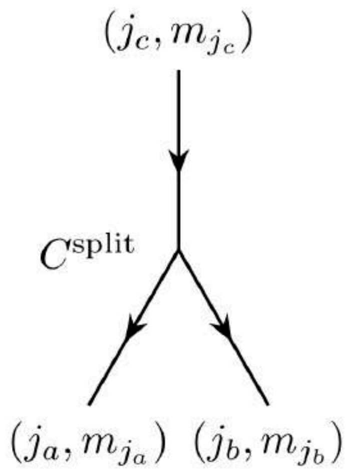
Global and gauge symmetries are handled naturally

Fusion tree formalism

e.g., P. Schmoll, S. Singh, M. Rizzi, RO, arXiv:1809.08180

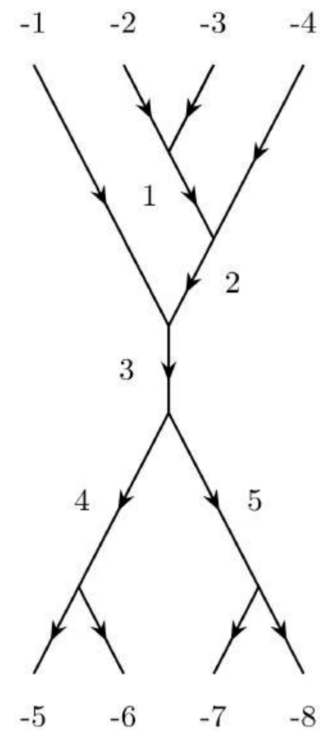
Fuse and Split tensors





e.g., SU(2) Clebsh-Gordan

Structural tensors are fusion trees



One stores the *structure* of the tree, and not the tensor itself

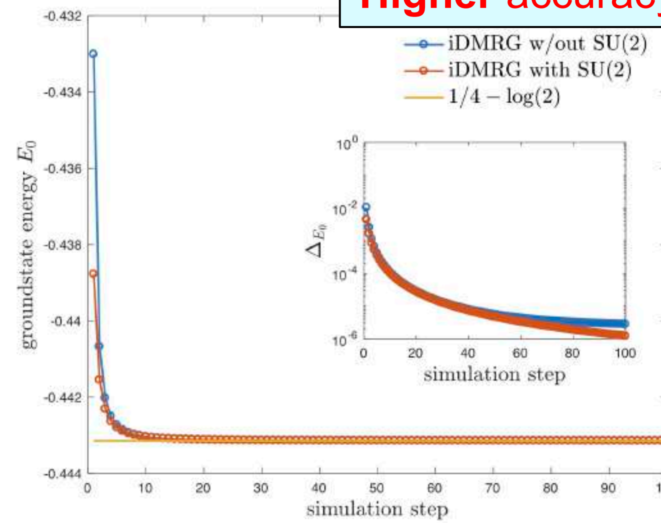
Symmetries are useful for numerics!

e.g., P. Schmoll, S. Singh, M. Rizzi, RO, arXiv:1809.08180

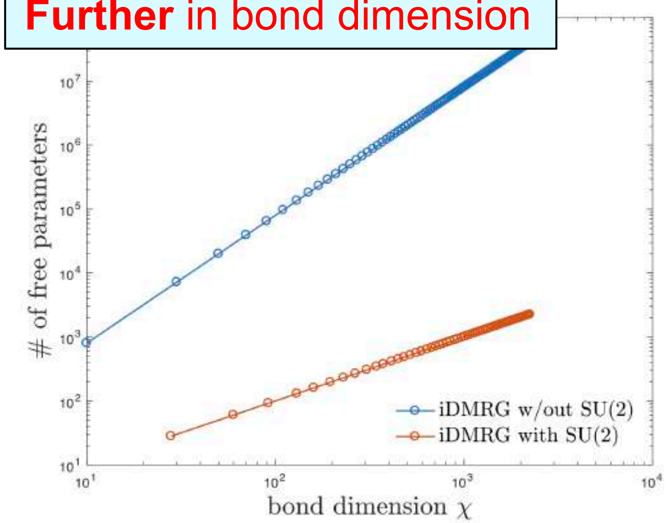
Canonical example: spin-1/2 Heisenberg quantum spin chain with SU(2)-iDMRG

$$H = \frac{1}{2}\sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \sigma_i^z \sigma_{i+1}^z$$

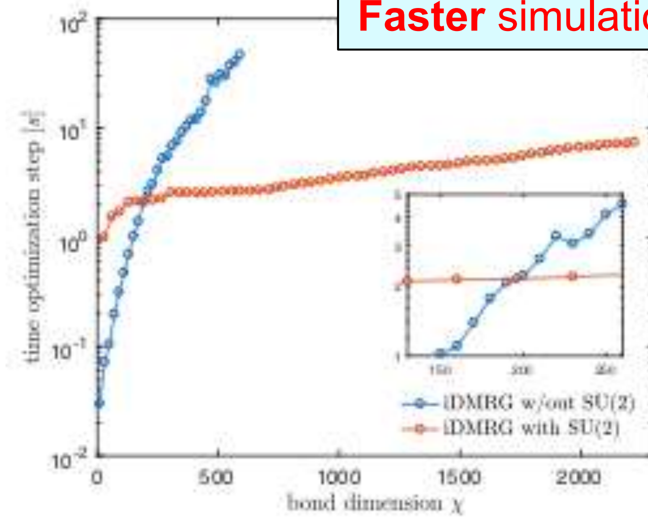
Higher accuracy



Further in bond dimension



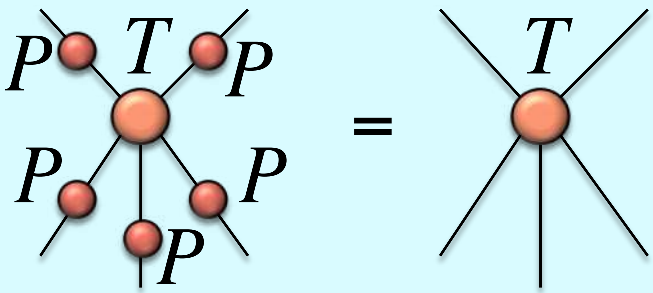
Faster simulation



Fermionic systems

e.g. P. Corboz, R. Orús, B. Bauer, G. Vidal, PRB 81, 165104 (2010)

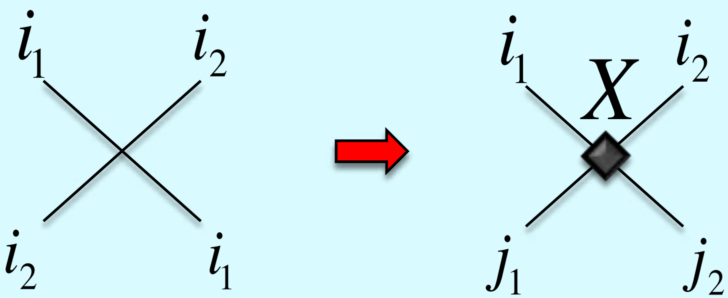
Rules (ask me later if interested in technical details or derivations)



Use parity-preserving tensors

$$T_{i_1 i_2 \ldots i_M} = 0 \ \text{ if } \ P(i_1)P(i_2)\ldots P(i_M) \neq 1$$

Symmetry of the Hamiltonian



Replace crossings by fermionic swap gates

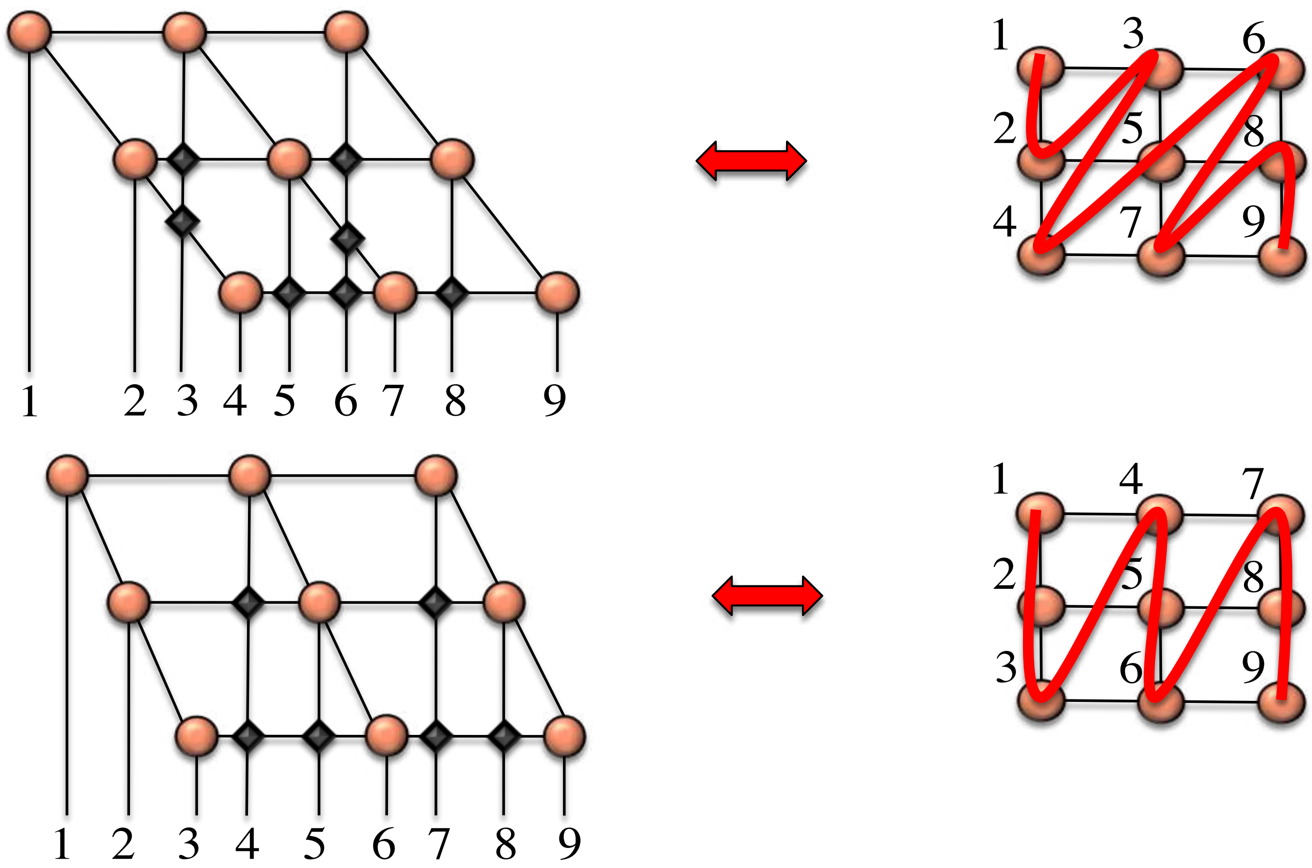
$$X_{i_2 i_1 j_1 j_2} = \delta_{i_1 j_1}\delta_{i_2 j_2} S(P(i_1),P(i_2))$$

$$S(P(i_1),P(i_2)) = \begin{cases} -1 & \text{if } \ P(i_1) = P(i_2) = -1 \\ +1 & \text{otherwise} \end{cases}$$

Fermionic operators **anti**commute

The leading order of the computational cost is the same as in the bosonic case

fermionic order ~ graphical projection

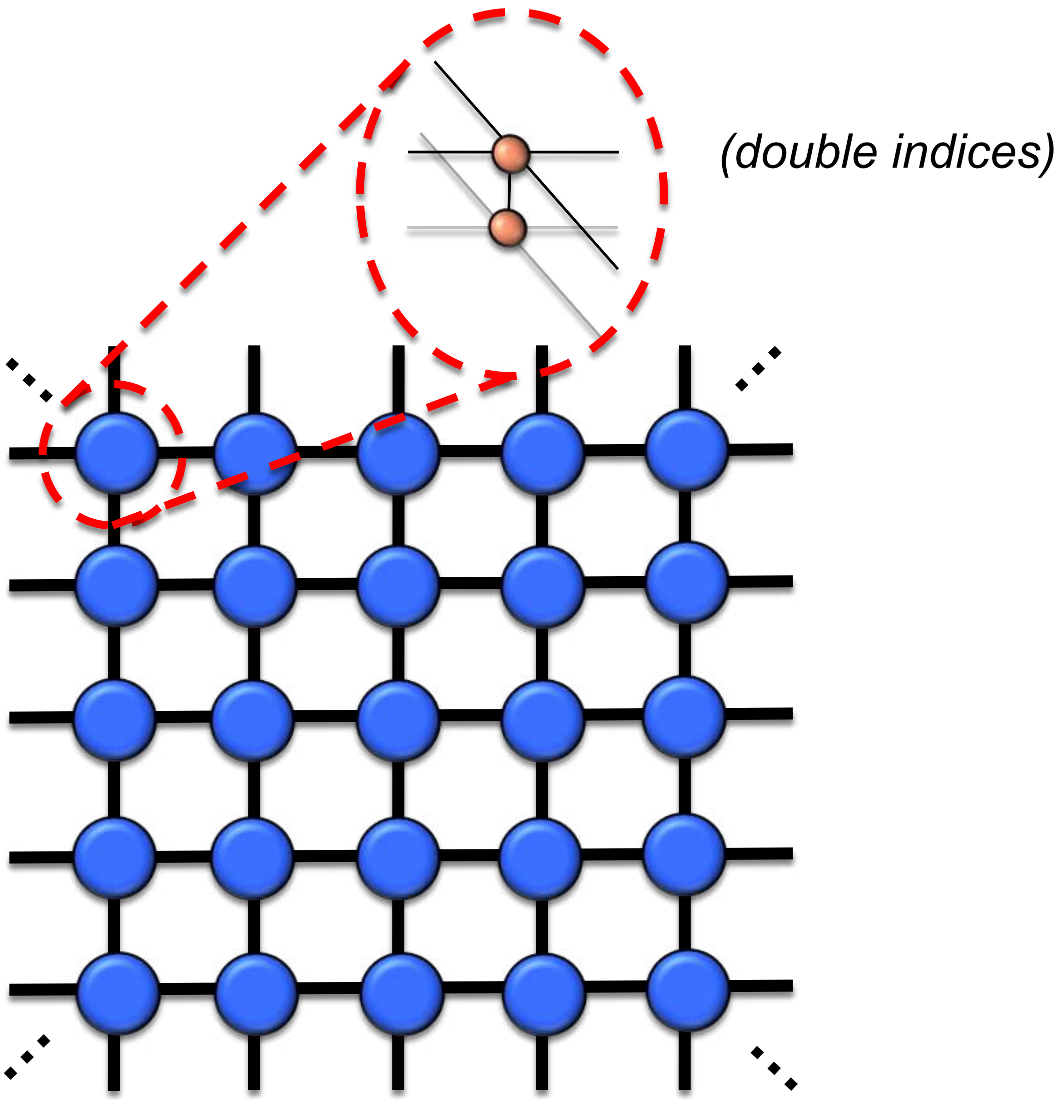


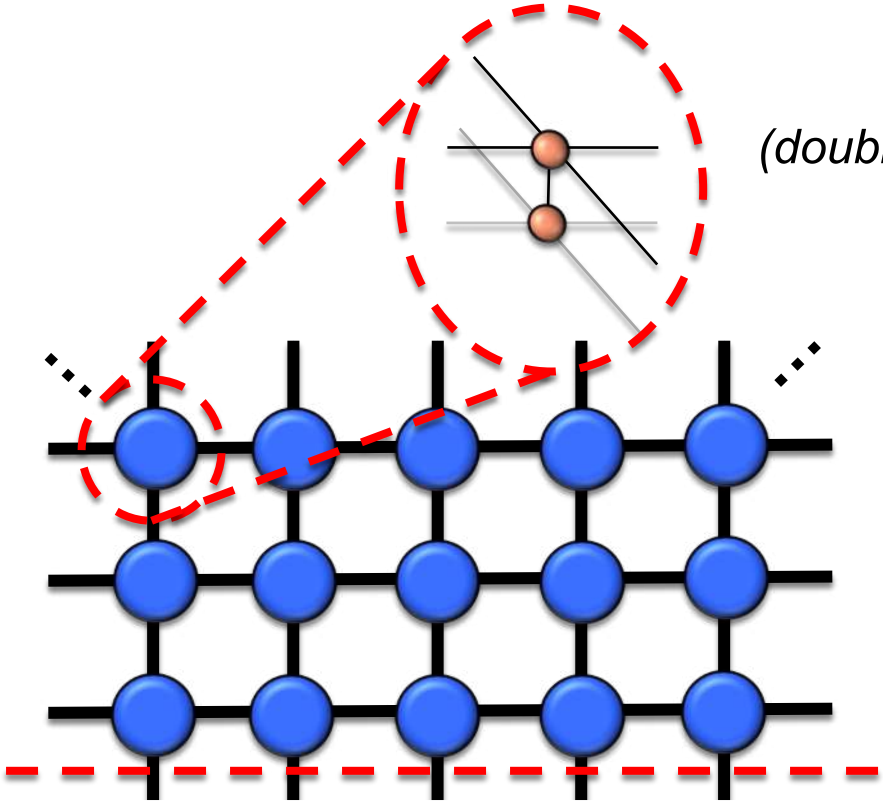
physics is independent of the order
↕
physics is independent of graphical projection

(different choices of Jordan-Wigner transformation, if mapping to a spin system)

PEPS & Entanglement Hamiltonians

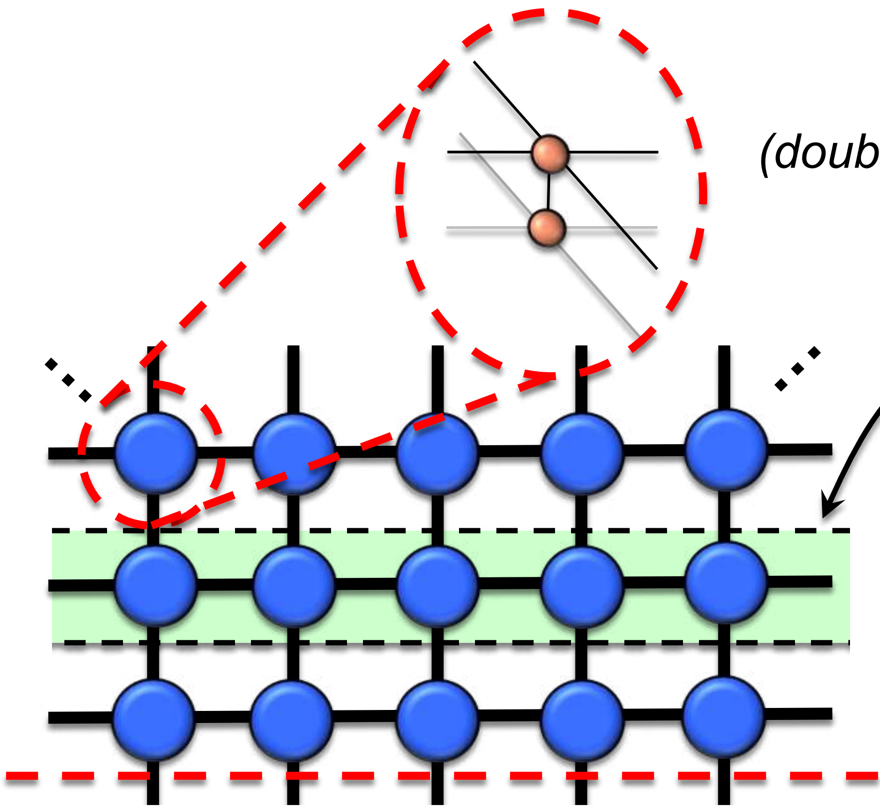
e.g. I. Cirac et al, PRB 83, 245134 (2011), N. Schuch et al, PRL 111, 090501 (2013)


(double indices)



Boundary

How is physics described here?



(double indices)

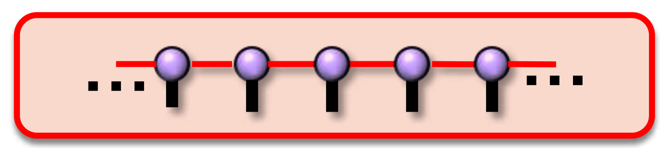
1-dim transfer matrix:

dominant eigenvector?

Boundary

How is physics described here?

Can be approximated using infinite MPS



iTEBD, iDMRG, PWFRG, etc

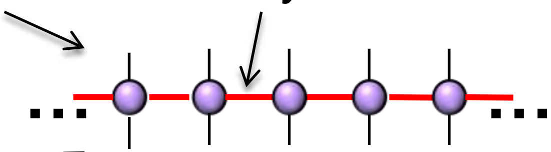
Emergent Hamiltonians

Remember it has double indices…

Emergent Hamiltonians

Virtual indices of bra 1…D

Boundary virtual index 1…χ



Virtual indices of ket 1…D

It is also hermitian and positive by construction (up to finite-χ effects)

||

1d Entanglement Hamiltonian

$$\rho = \exp(-H_E)$$

Who is H_E ???

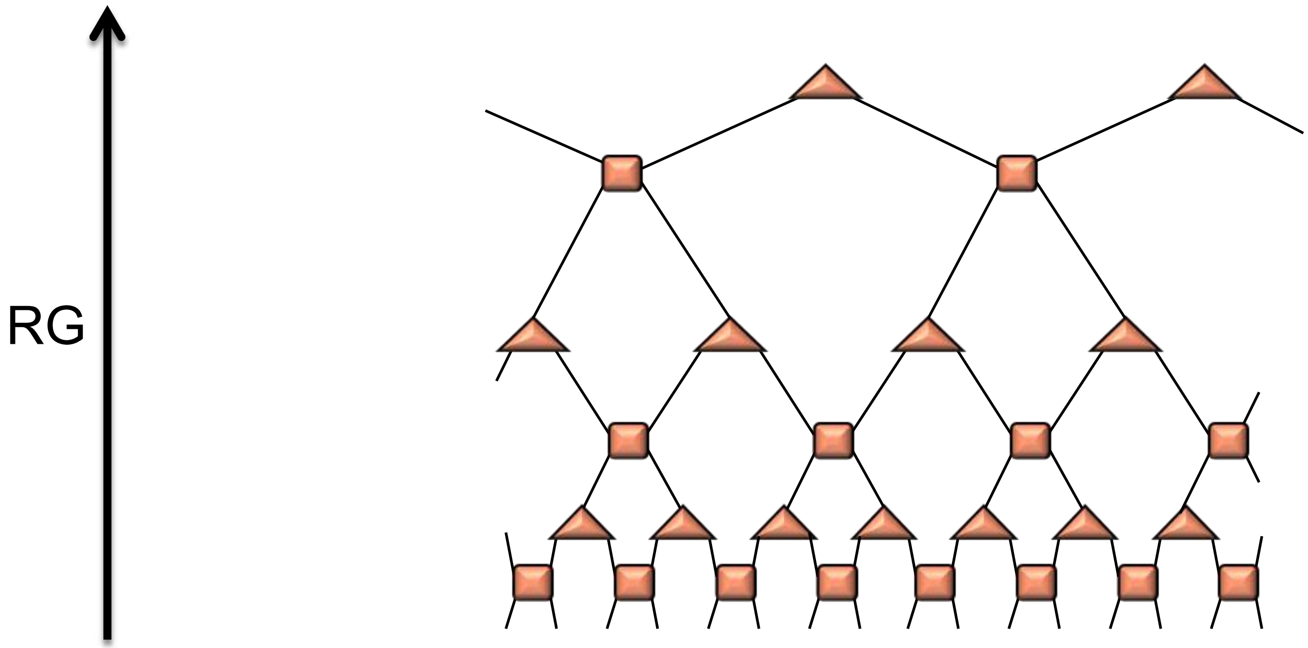
Bulk	Correspondence	Boundary
Gapped 2d systems, trivial phase		1d Hamiltonian, short-range
Critical 2d systems		1d Hamiltonian, long-range
Gapped 2d systems, topological order		Completely non-local (projector)
Chiral topological order, gapless		(1+1)d Conformal field theory

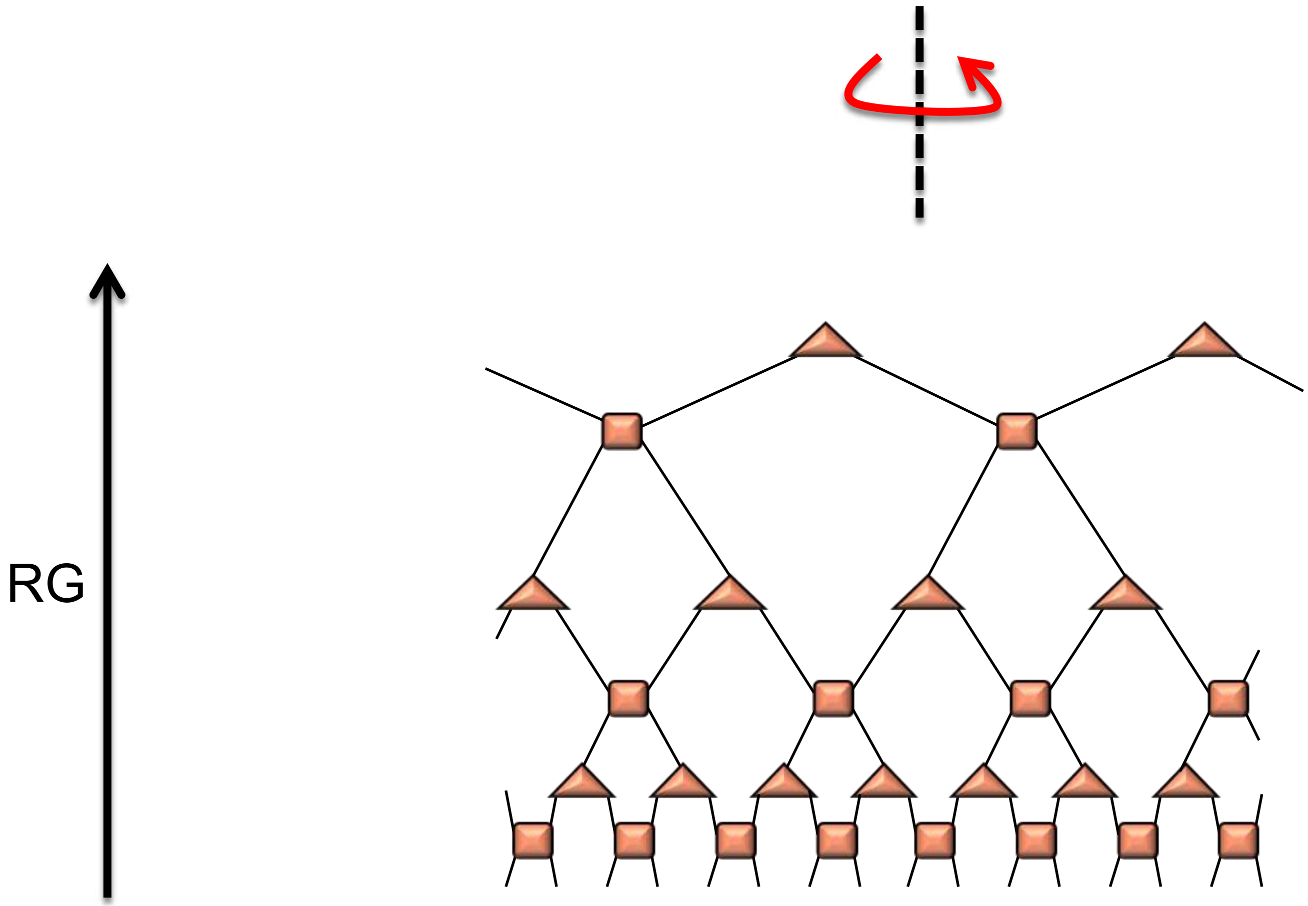
RO, M. Mambrini, D. Poilblanc, work in progress

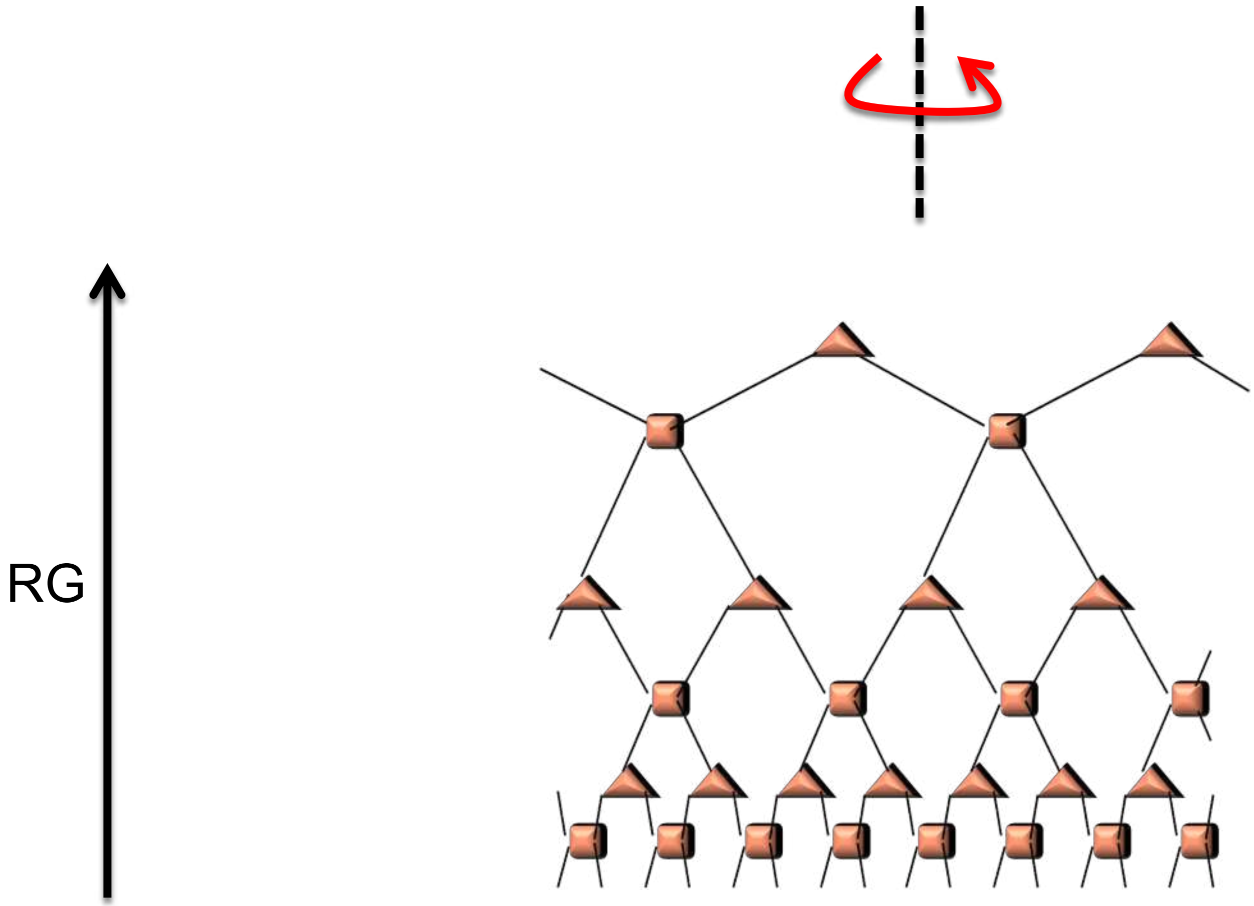
Particles and energies from Hamiltonians, and Hamiltonians from networks of entanglement + bulk-boundary correspondence

“branching” MERA

G. Evenbly, G. Vidal, PRL 112, 240502 (2014)


RG


RG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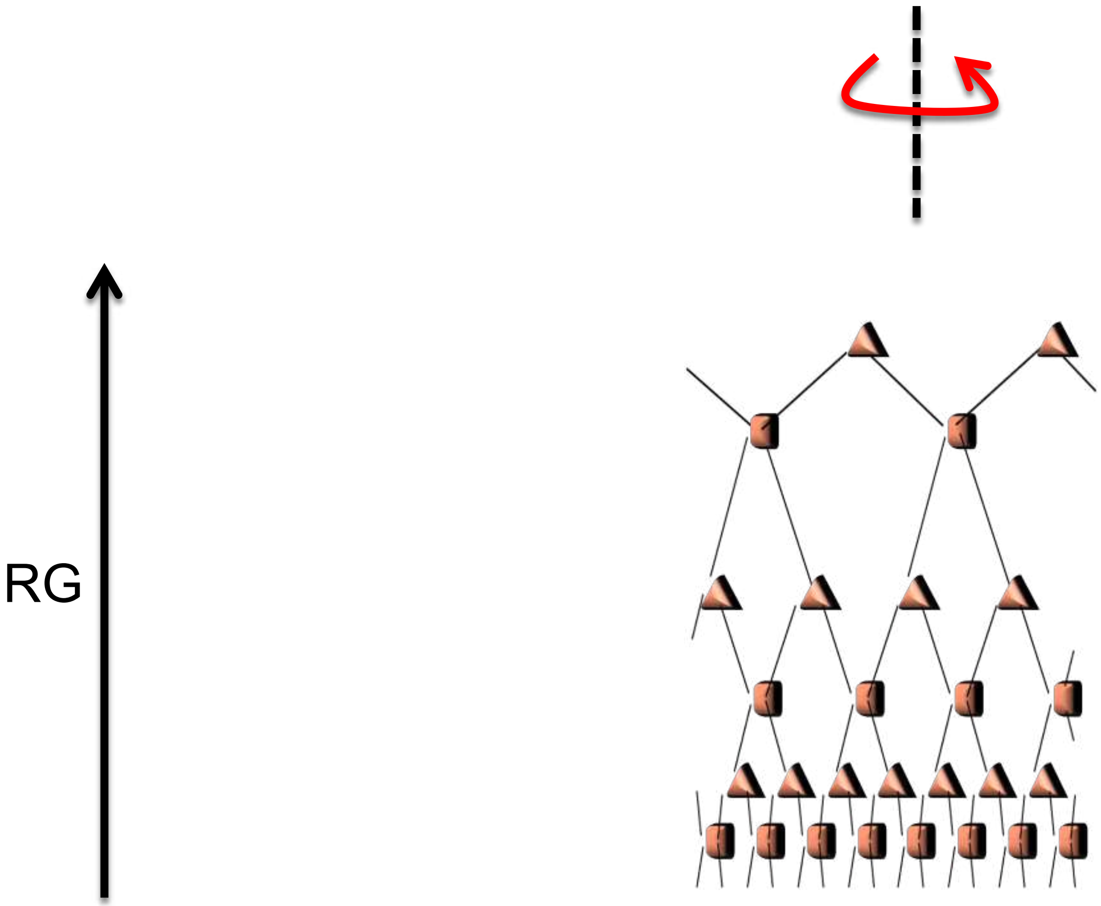
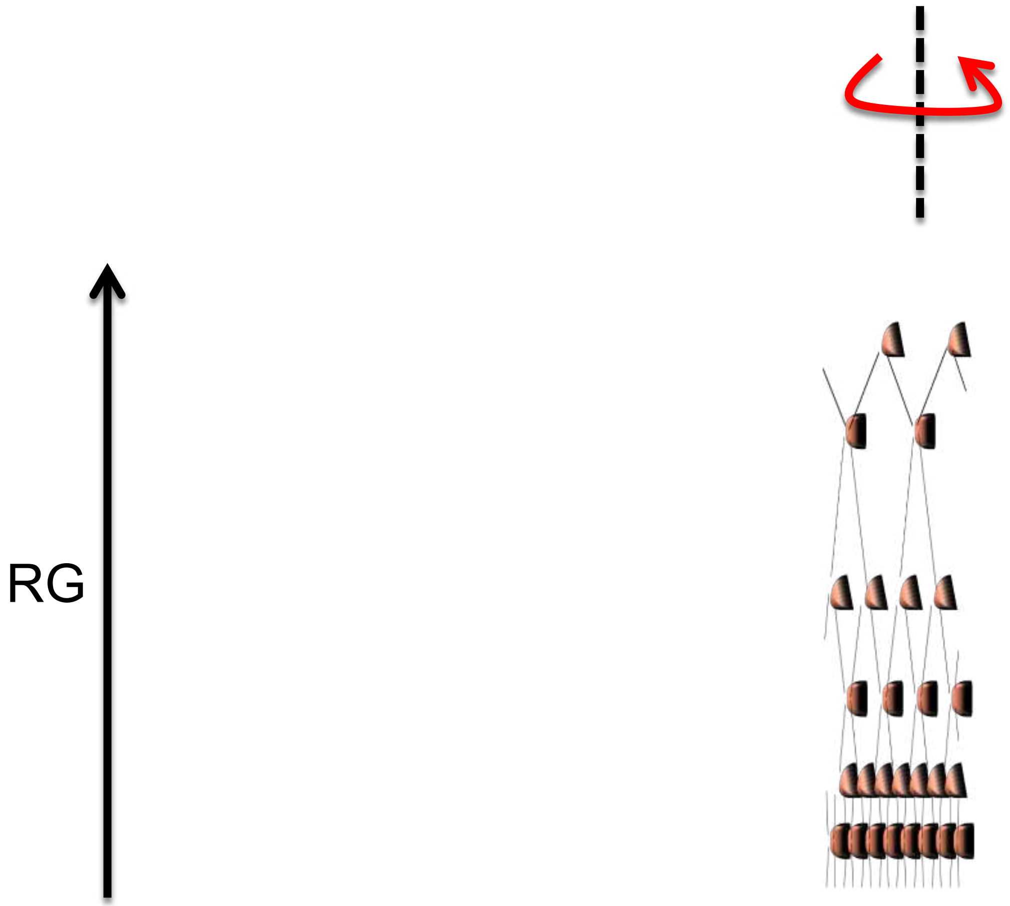
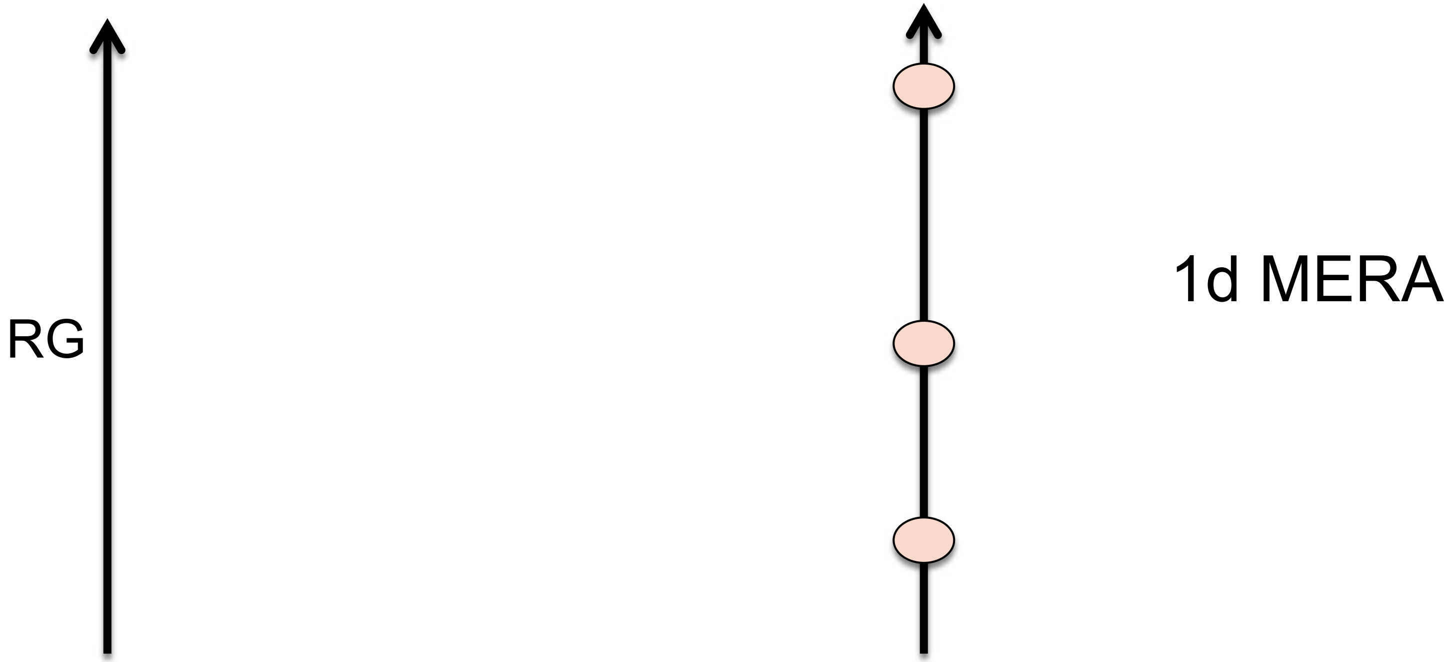
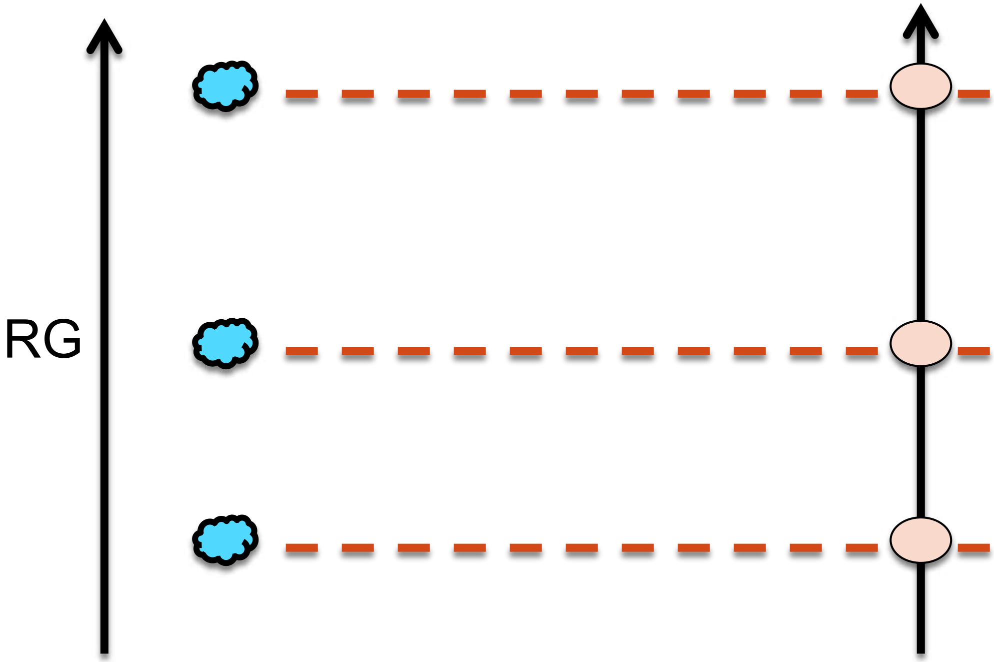
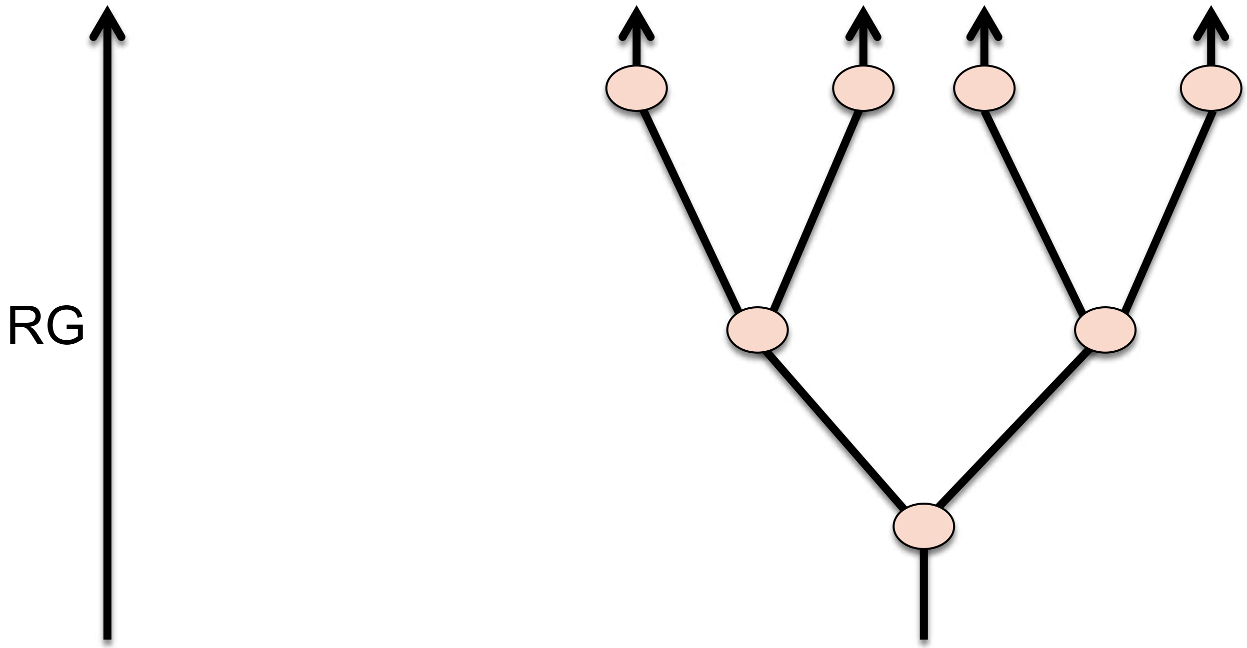

RG


RG

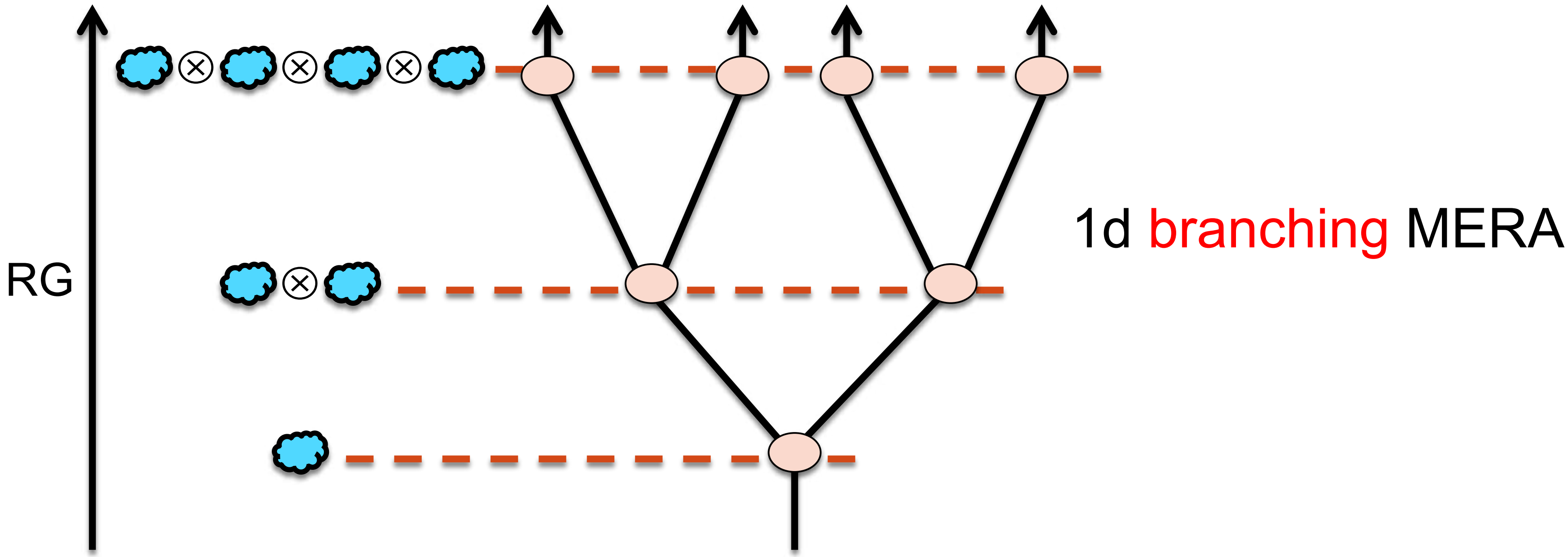

RG
1d MERA



1d MERA



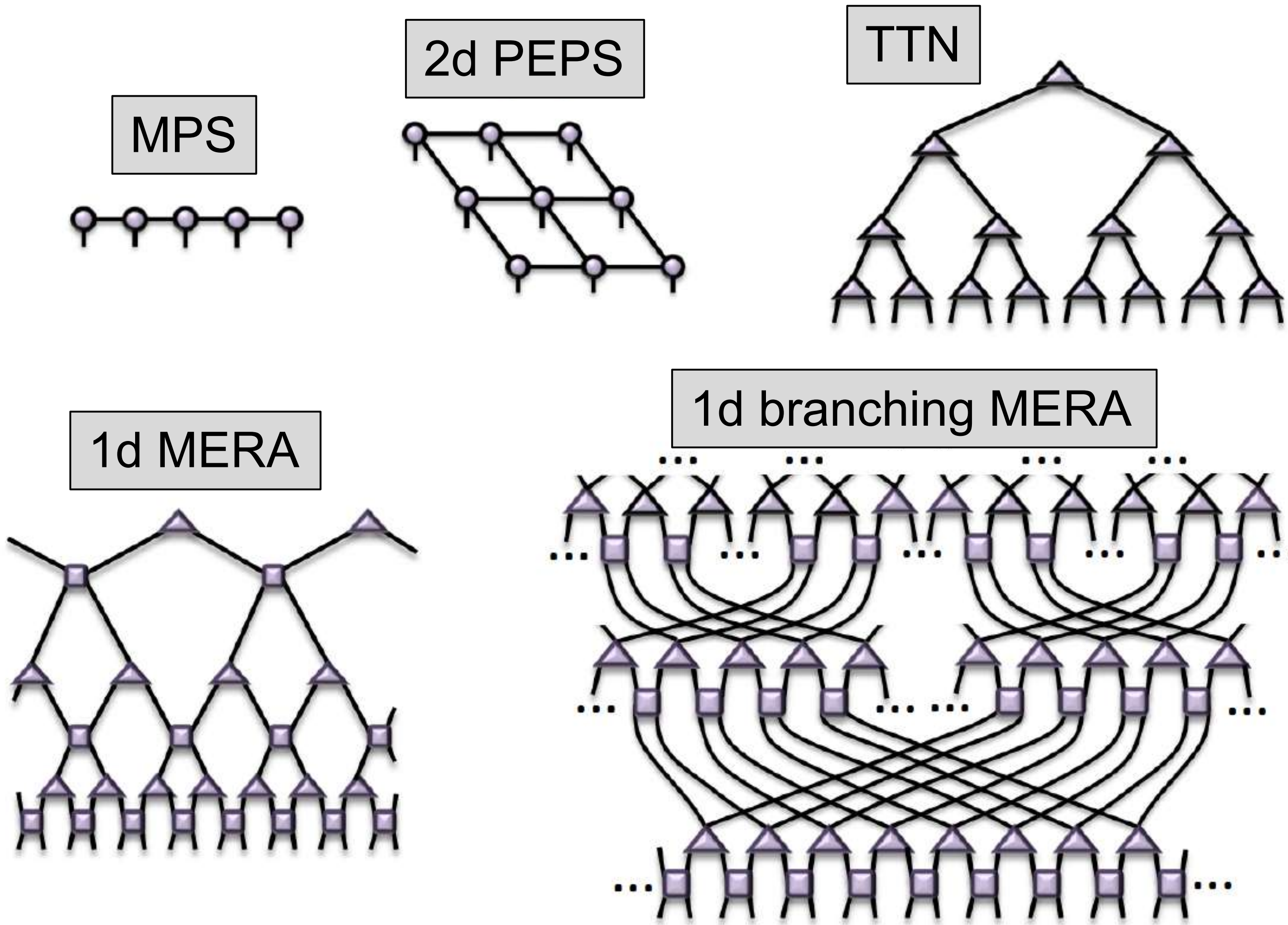
1d branching MERA



Decoupling of degrees of freedom along RG (e.g. spin-charge separation), and allows arbitrary scalings of the entanglement entropy

In 2d, ansatz for e.g., Fermi & Bose liquids, $S(L) \approx L \log L$

Increasing complexity...

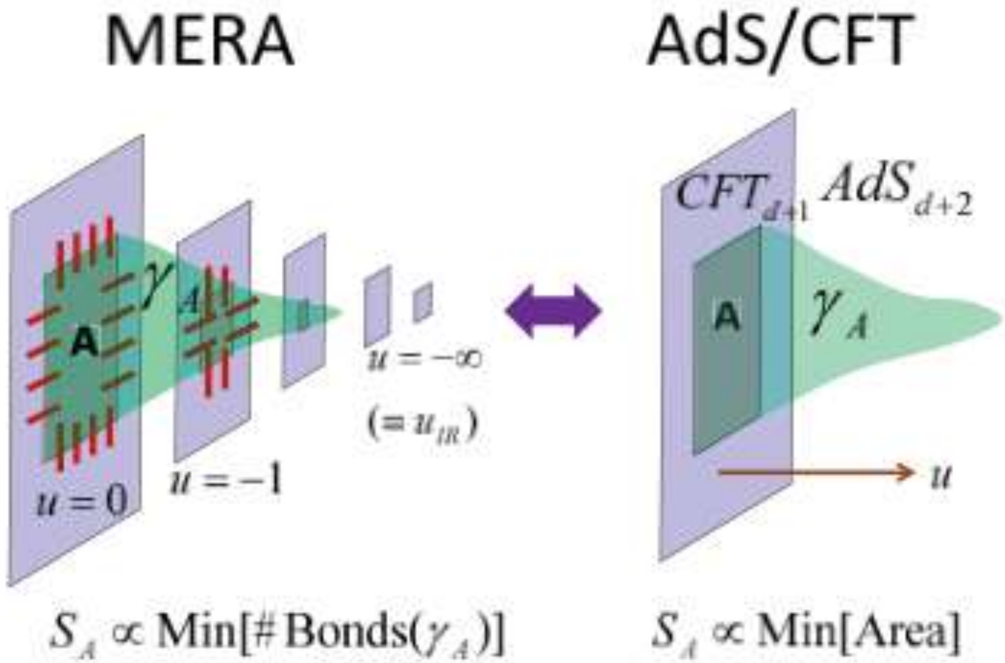


Exact in many cases
Variational ansatz for numerical simulations (e.g. DMRG)

MERA & AdS/CFT

e.g. B. Swingle, PRD 86, 065007 (2012), G. Evenbly,G. Vidal, JSTAT 145:891-918 (2011)

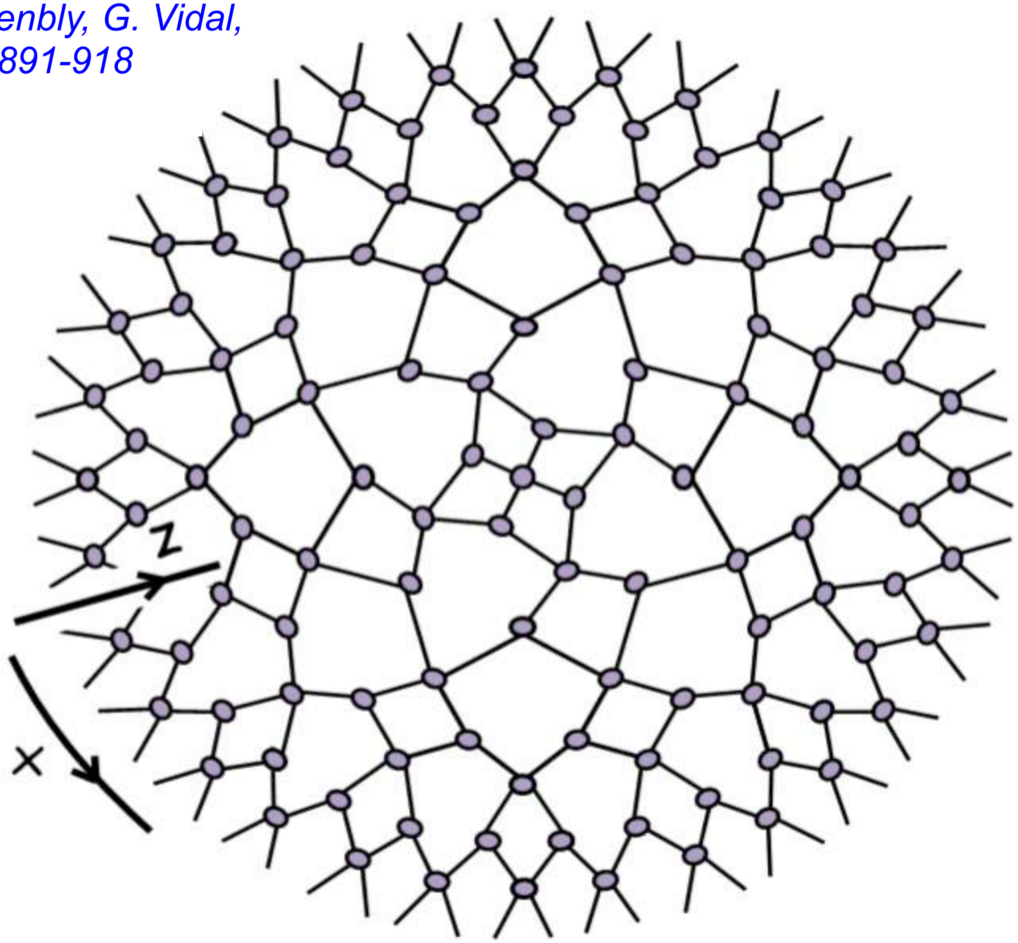
Emergent space-time



Picture from M. Nozaki, S. Ryu, T. Takayanagi, JHEP10(2012)193

MERA entropy ~ Ryu-Takayanagi prescription

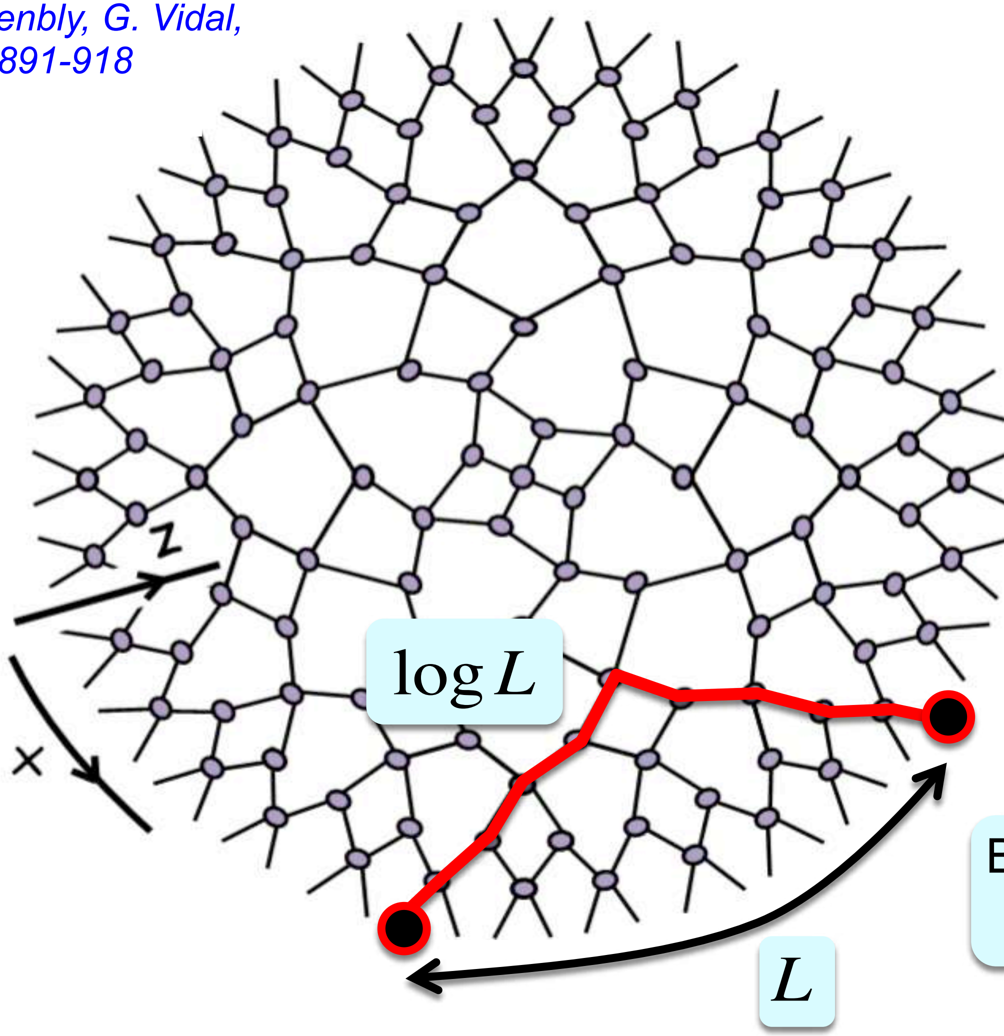
Picture from G. Evenbly, G. Vidal, (2011) JSTAT 145:891-918



(time slice)

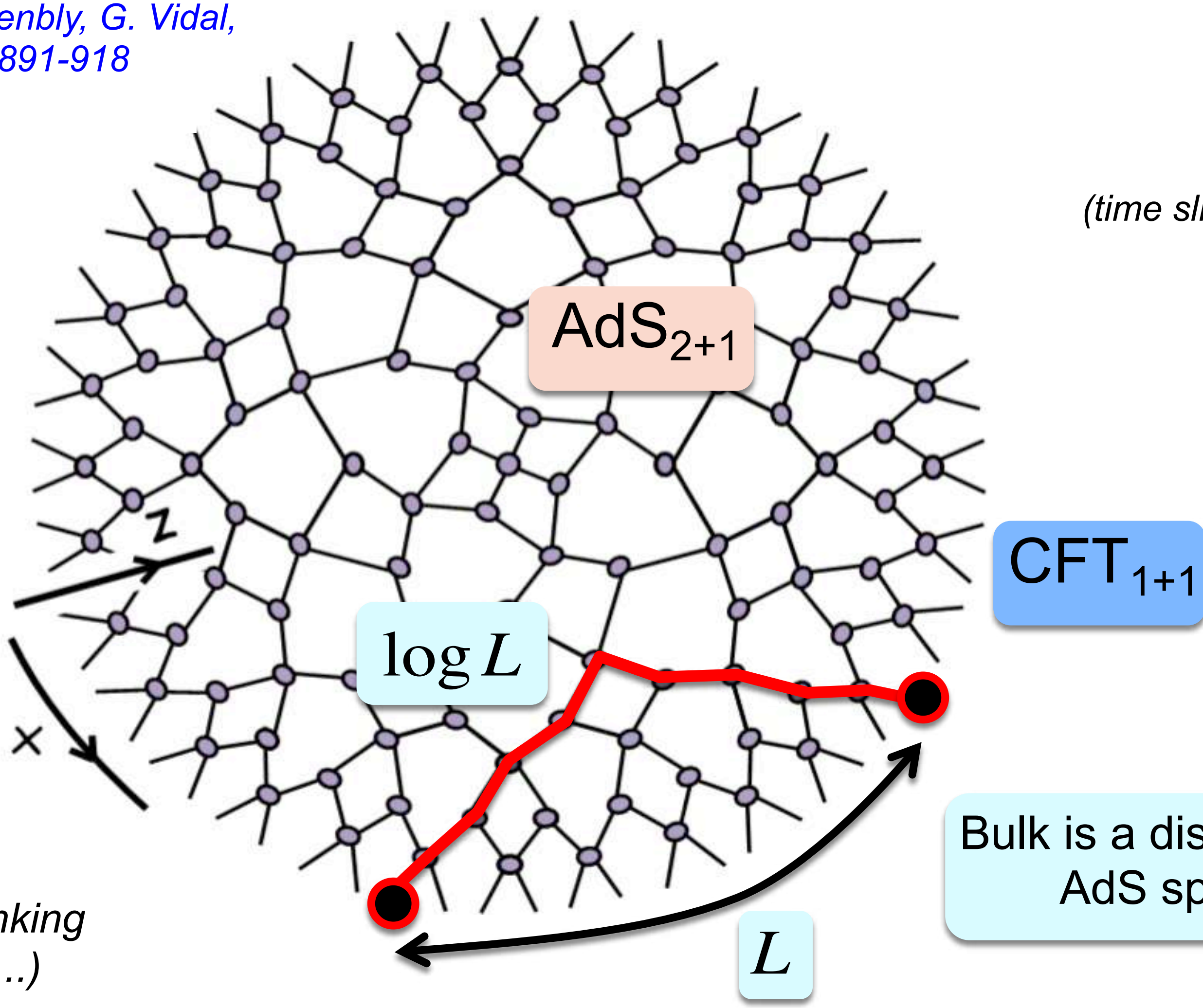
Picture from G. Evenbly, G. Vidal, (2011) JSTAT 145:891-918

(time slice)



Bulk is a discretized AdS space

Picture from G. Evenbly, G. Vidal, (2011) JSTAT 145:891-918



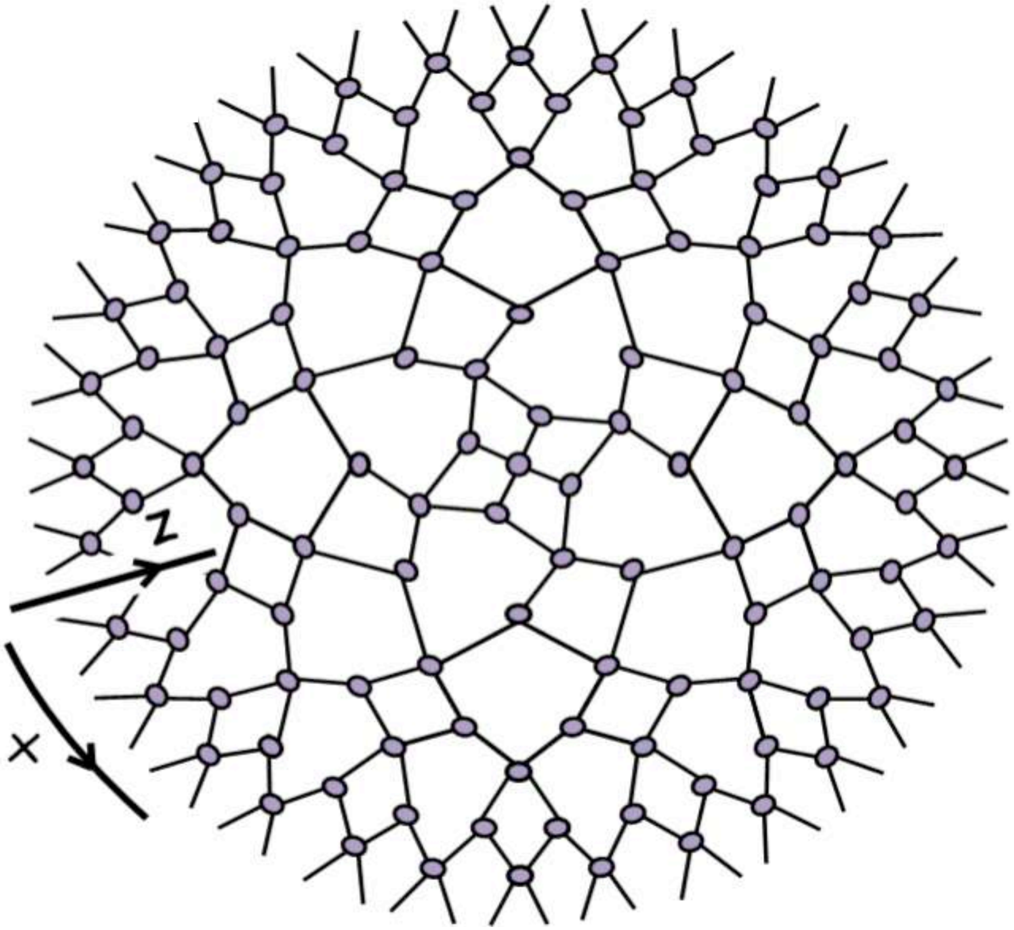
(time slice)

Bulk is a discretized AdS space

(and we were not thinking about gravity at all...)

For a scale-invariant MERA, the tensors of a critical model with a CFT limit correspond to a „gravitational“ description in a discretized AdS space: „lattice“ realization of AdS/CFT correspondence

Let's now play
some jazz...



(time slice)

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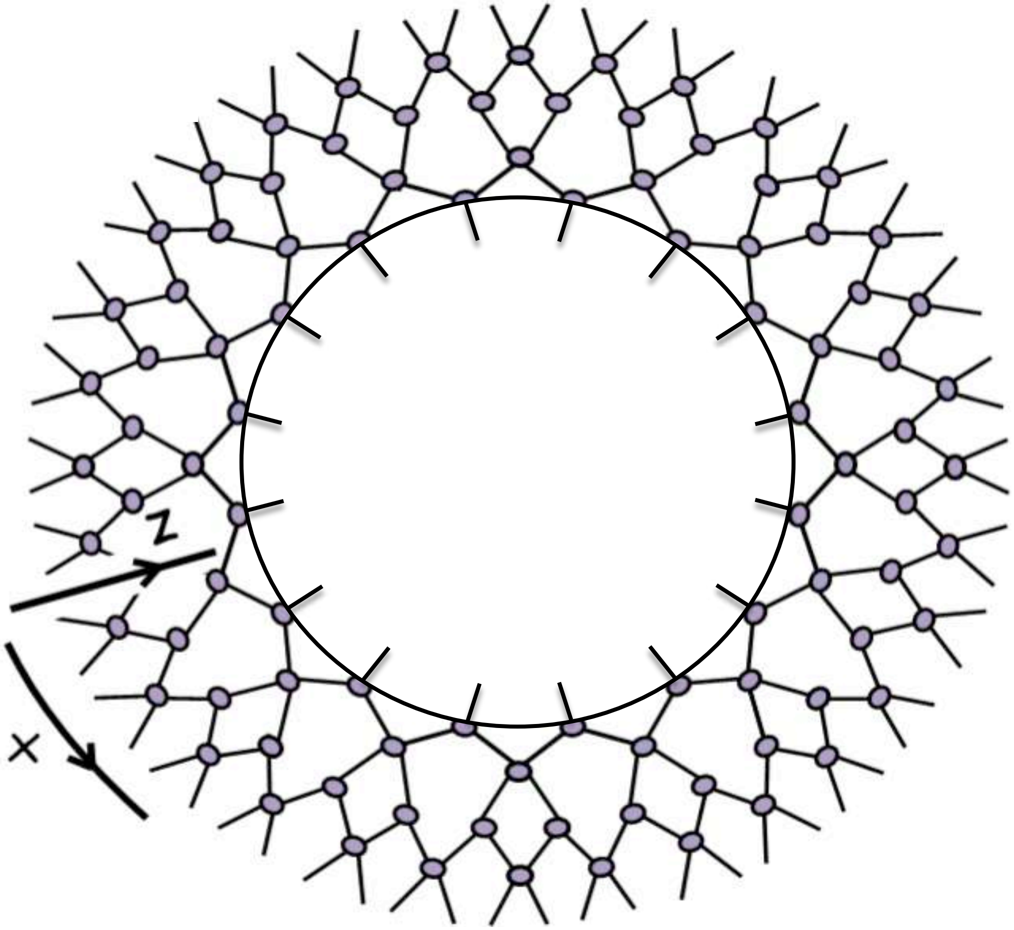
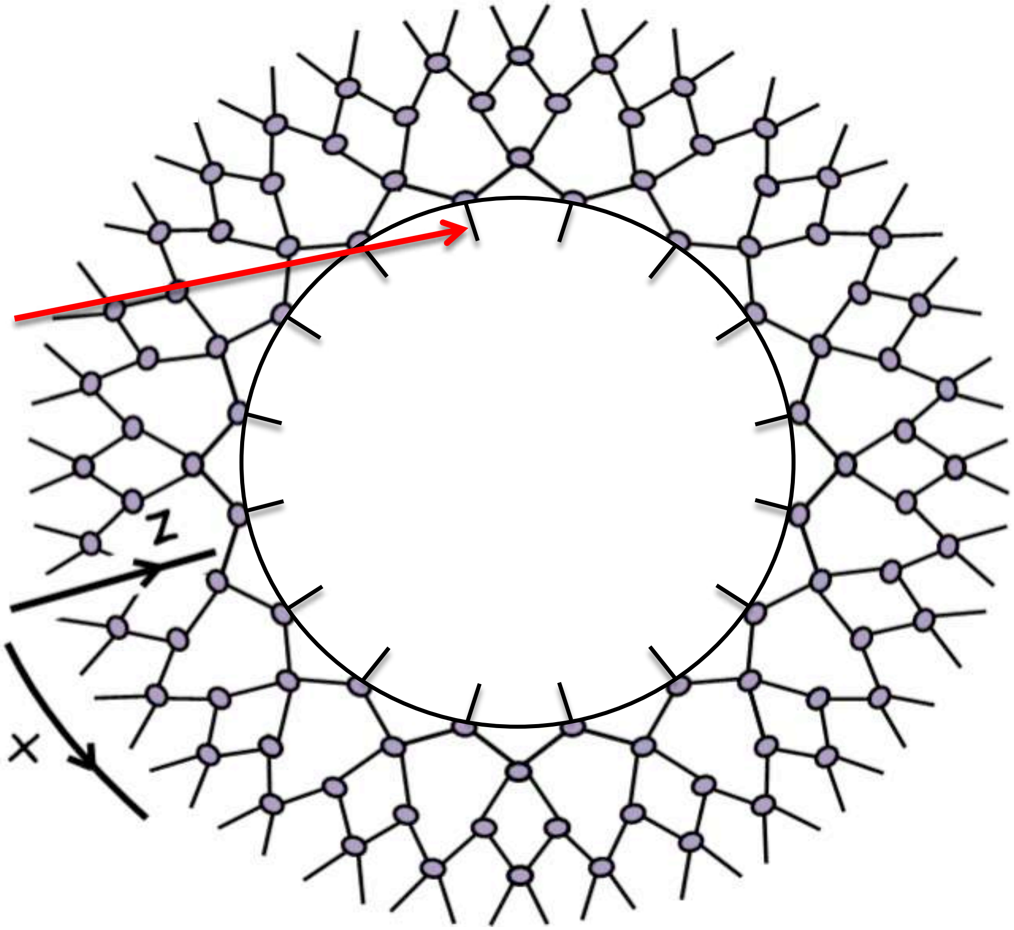


(time slice)

Finite correlation length (gapped systems) = finite number of layers

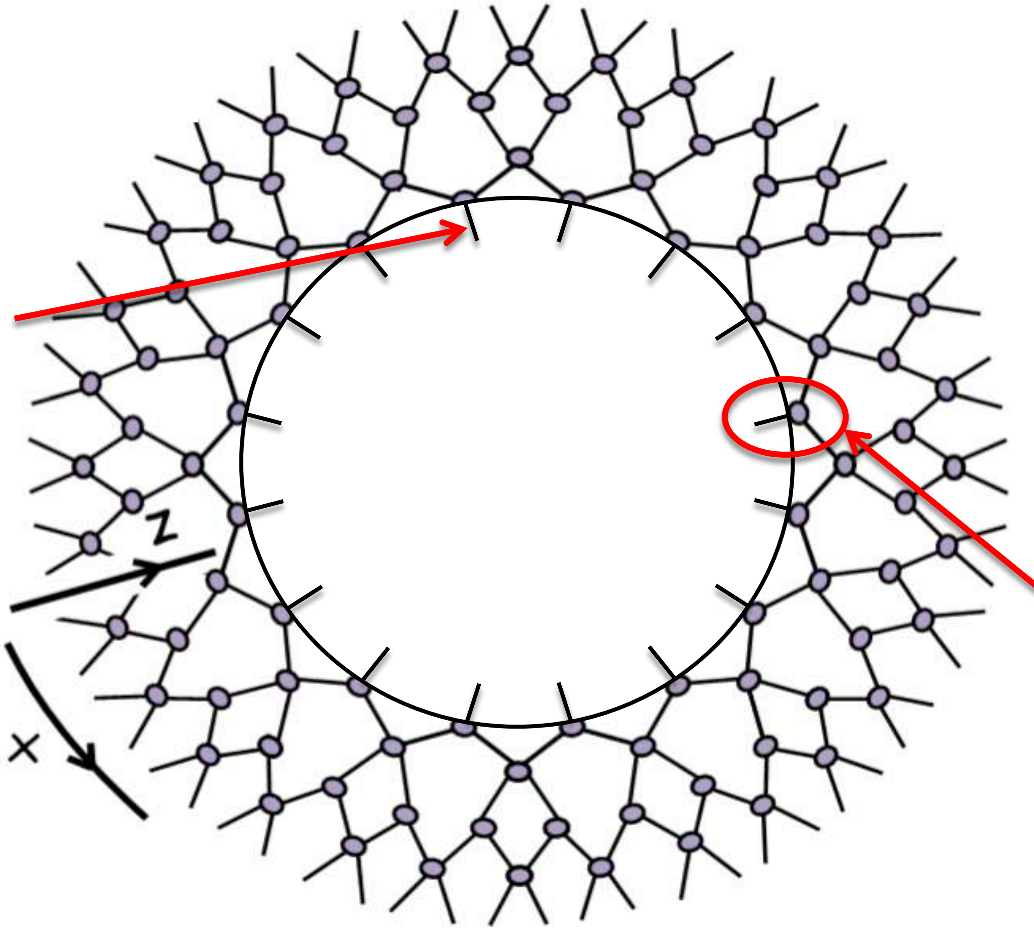
Let‘s now play some jazz...



Finite correlation length (gapped systems) = finite number of layers

Let's now play some jazz...

Product state = trivial fixed point



(time slice)

If arbitrary, then we can have non-trivial thermal states.

If isommetry, then all information is encoded in the network of correlations and

$$\rho_{in} = I$$

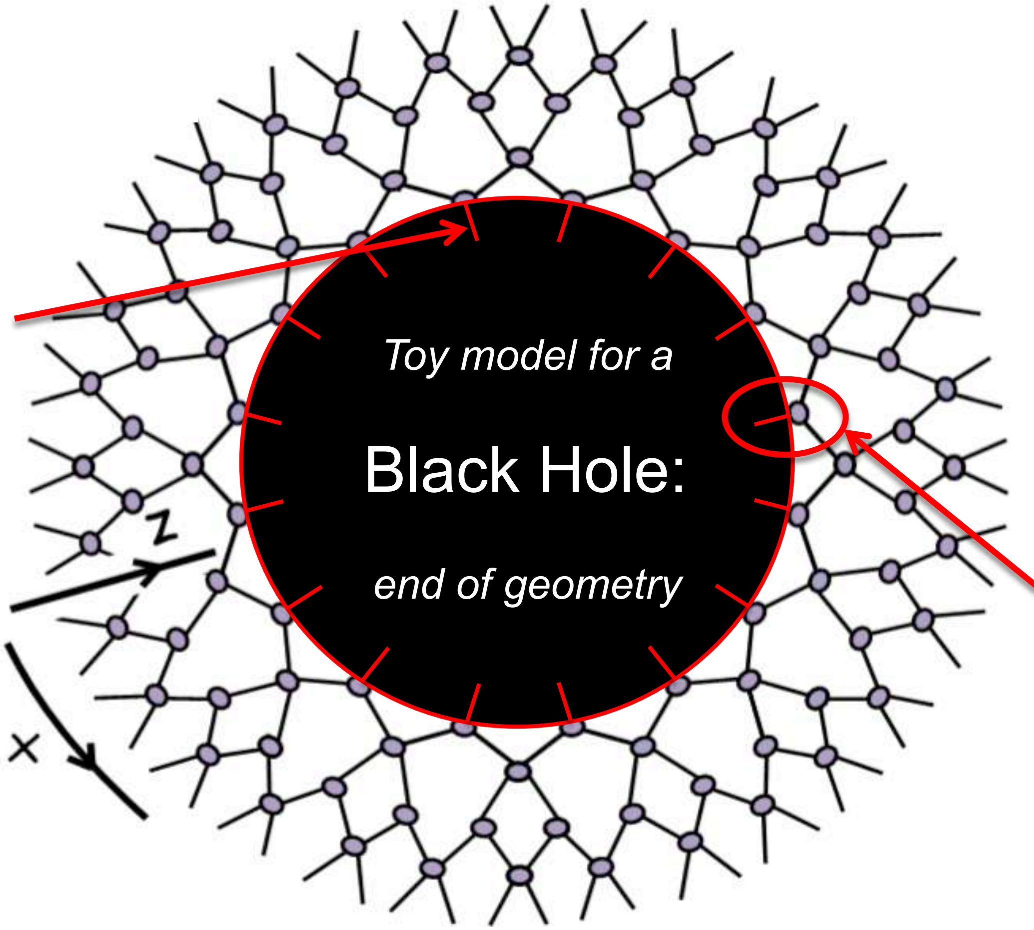
Finite correlation length (gapped systems) = finite number of layers

$$\left.\begin{aligned}\rho_{in} &= tr_{out}\left(|\Psi\rangle\langle\Psi|\right)\\ \rho_{out} &= tr_{in}\left(|\Psi\rangle\langle\Psi|\right)\end{aligned}\right\}$$

*Same **thermal** spectrum (entanglement Hamiltonian)*
finite temperature, scale invariance broken

Let‘s now play some jazz...

Product state = trivial fixed point



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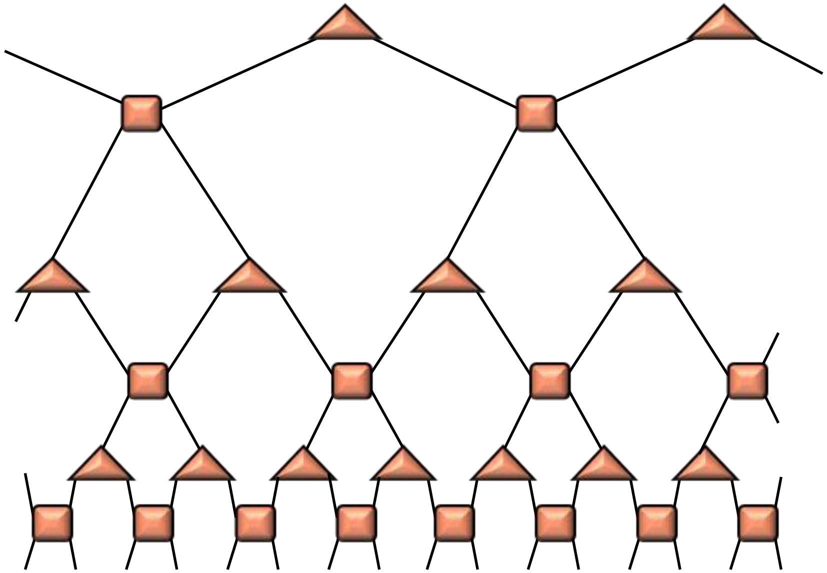
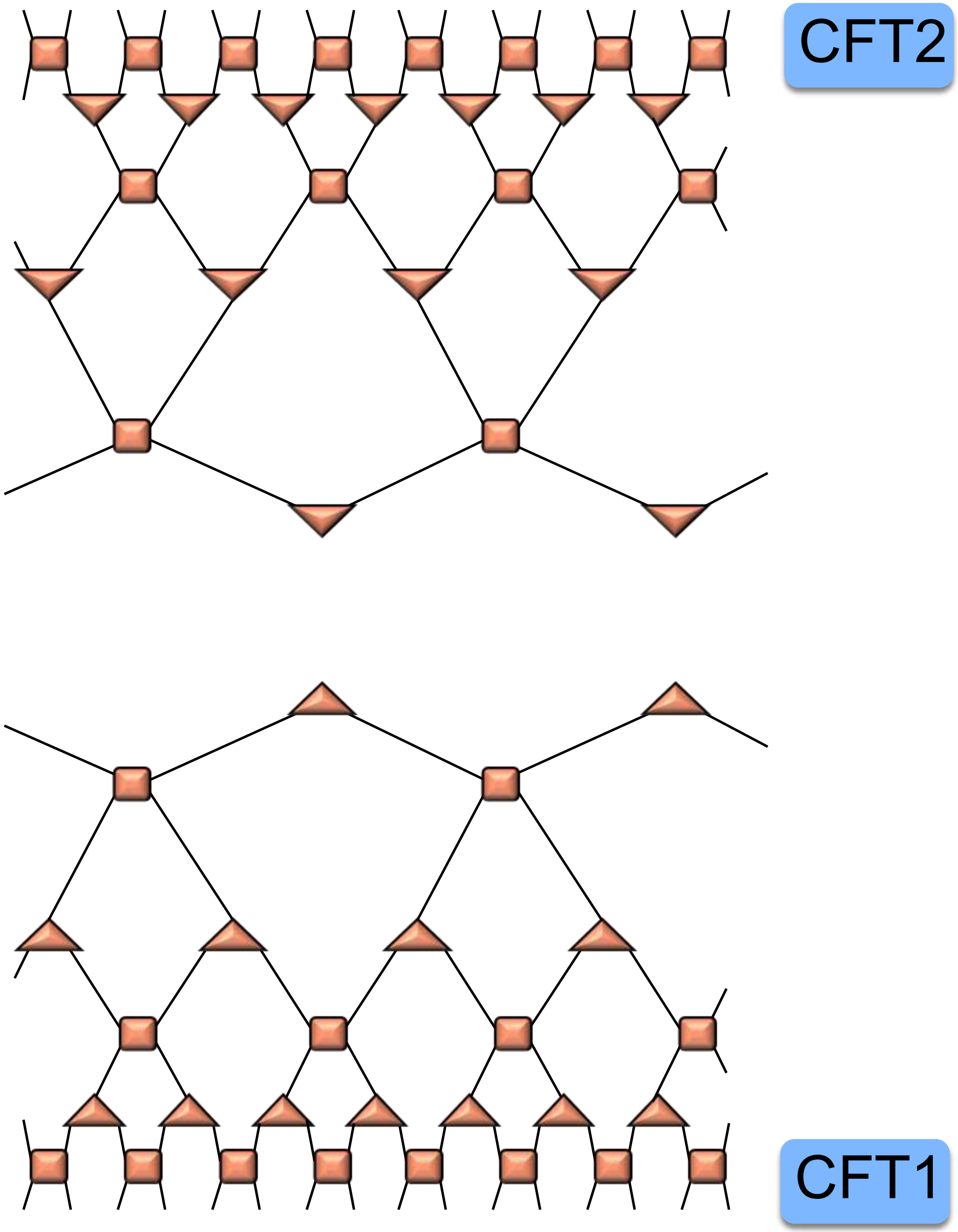
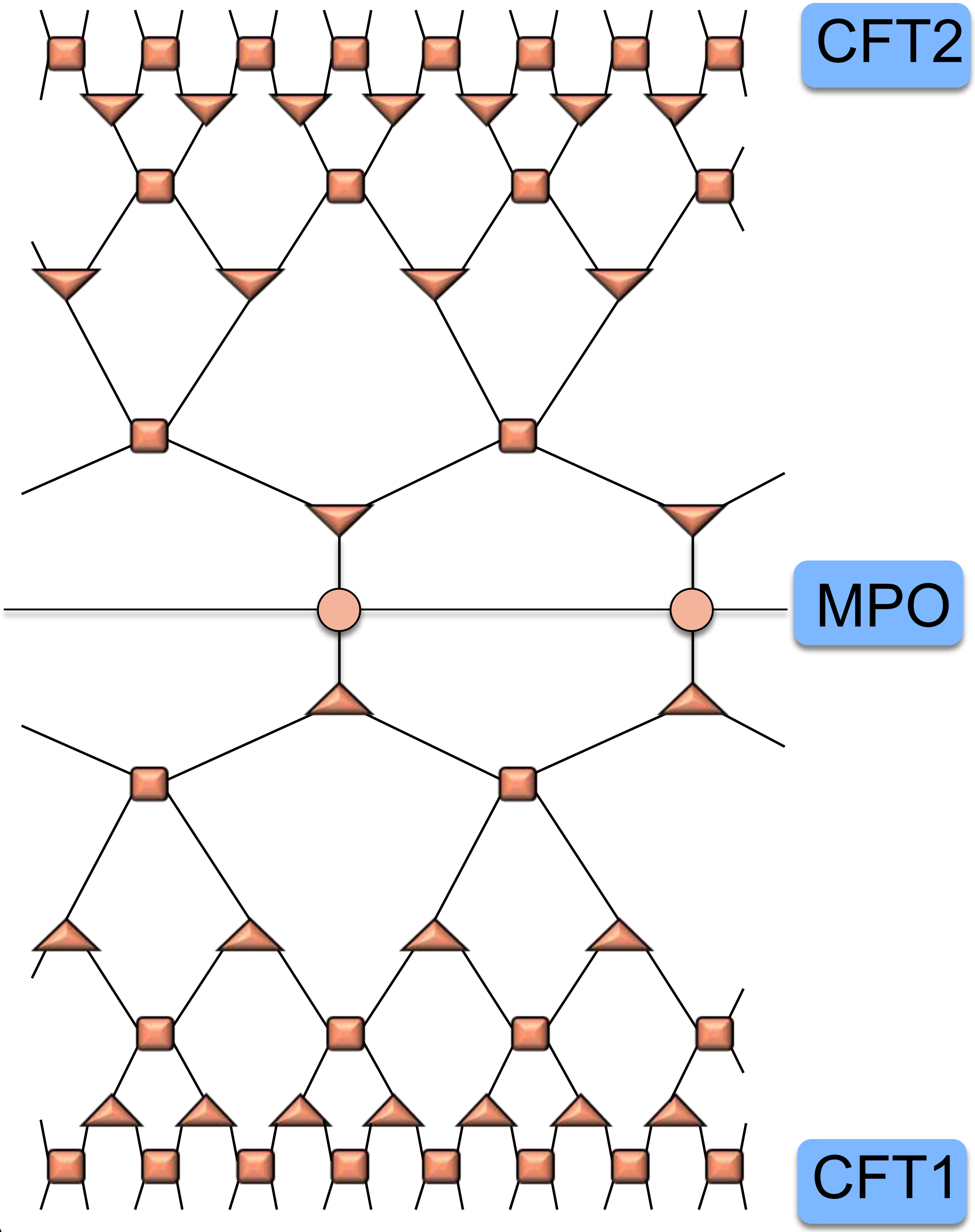
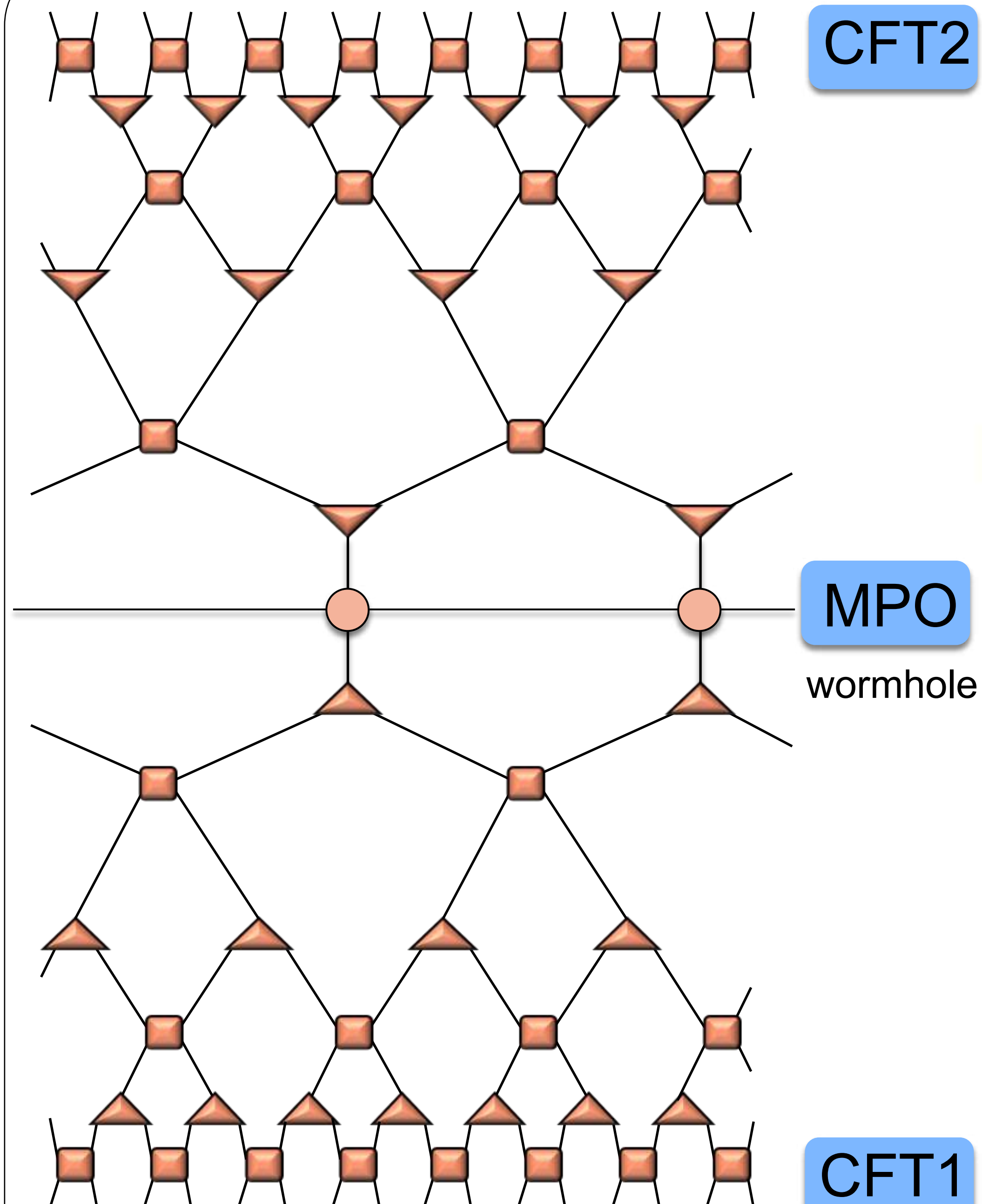


CFT1


CFT2
CFT1


CFT2
MPO
CFT1

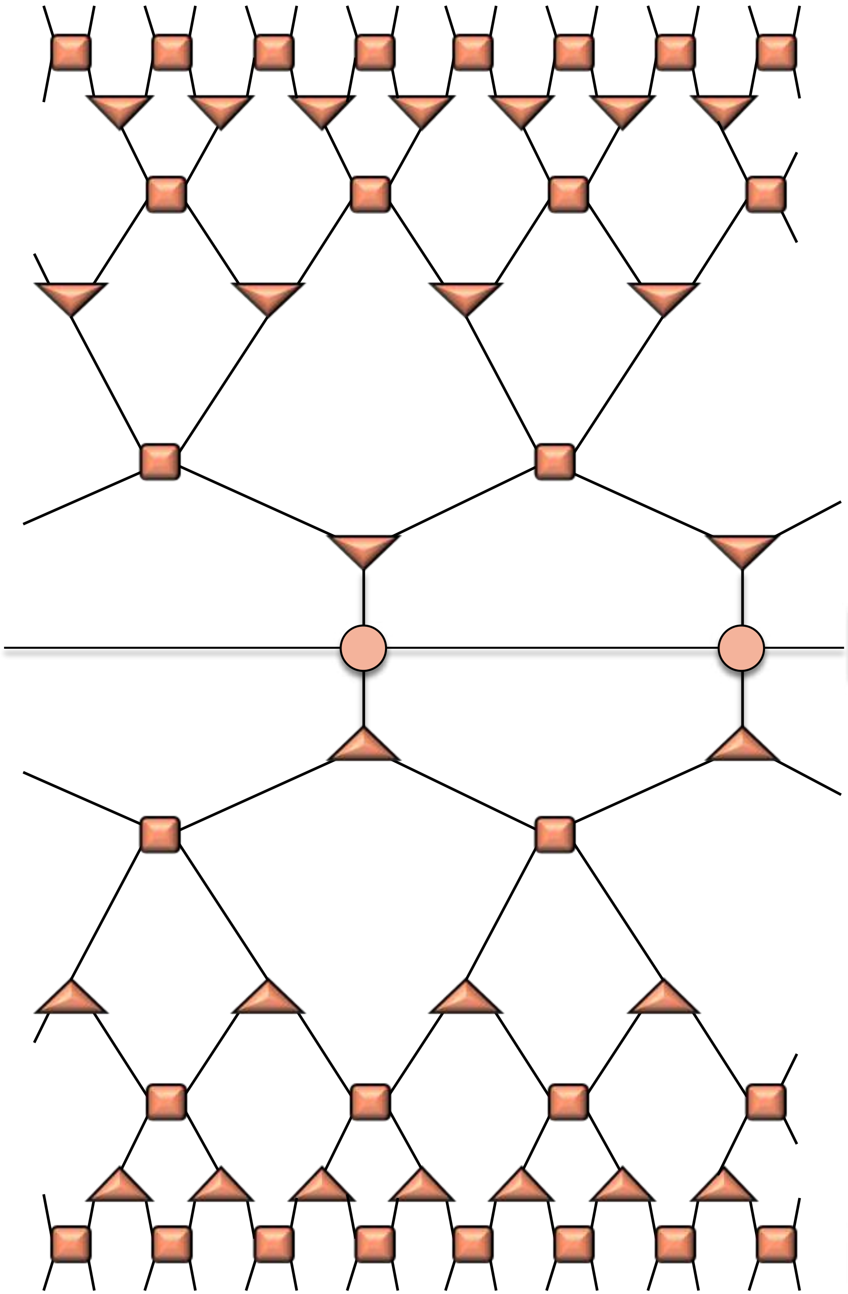


e.g., T. Hartman, J. Maldacena, JHEP05(2013)014

Thermofield double state

Eternal AdS black-hole

$$|TFD\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |n\rangle_1 |n\rangle_2$$



e.g., T. Hartman, J. Maldacena, JHEP05(2013)014

Thermofield double state

Eternal AdS black-hole

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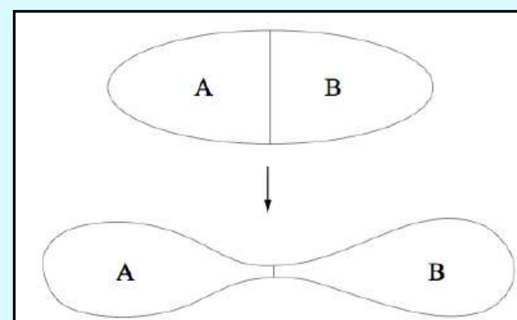
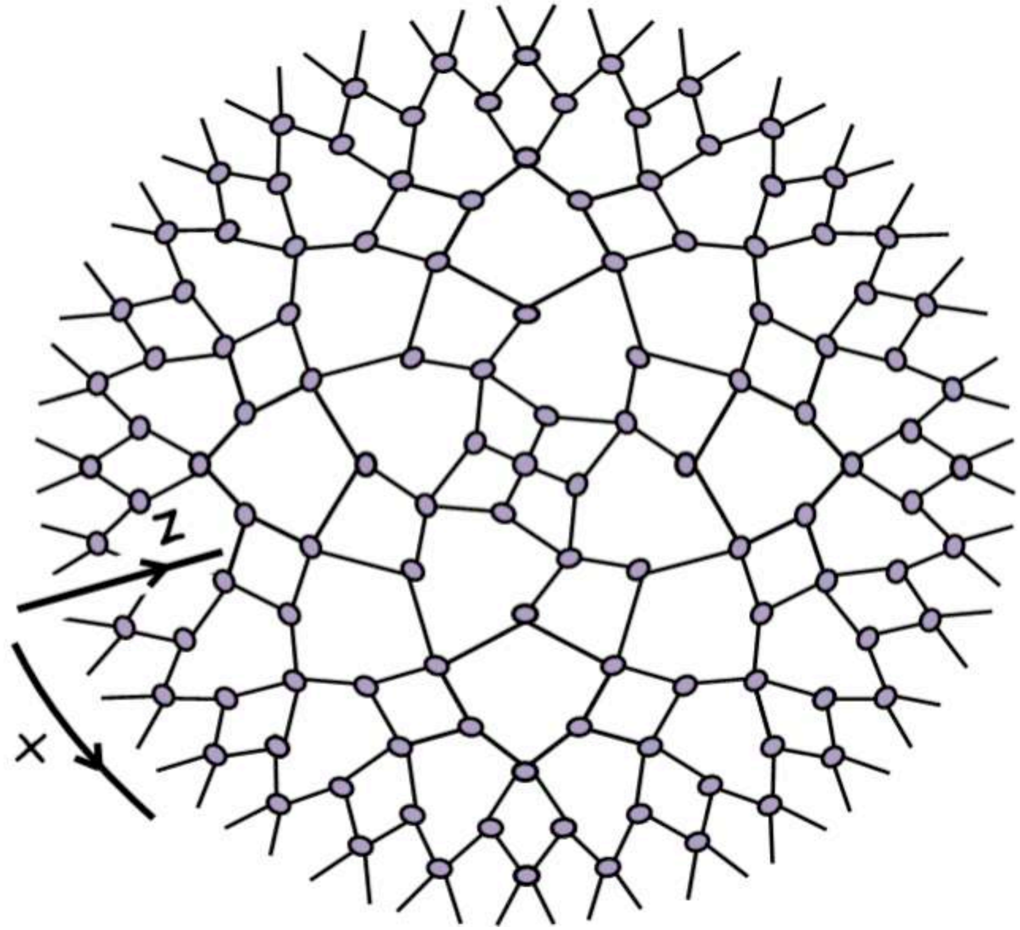
Entanglement connects upper and lower spacetimes



M. Van Raamsdonk, arXiv:0907.2939

ER=EPR, Maldacena & Susskind

MERA



cMERA

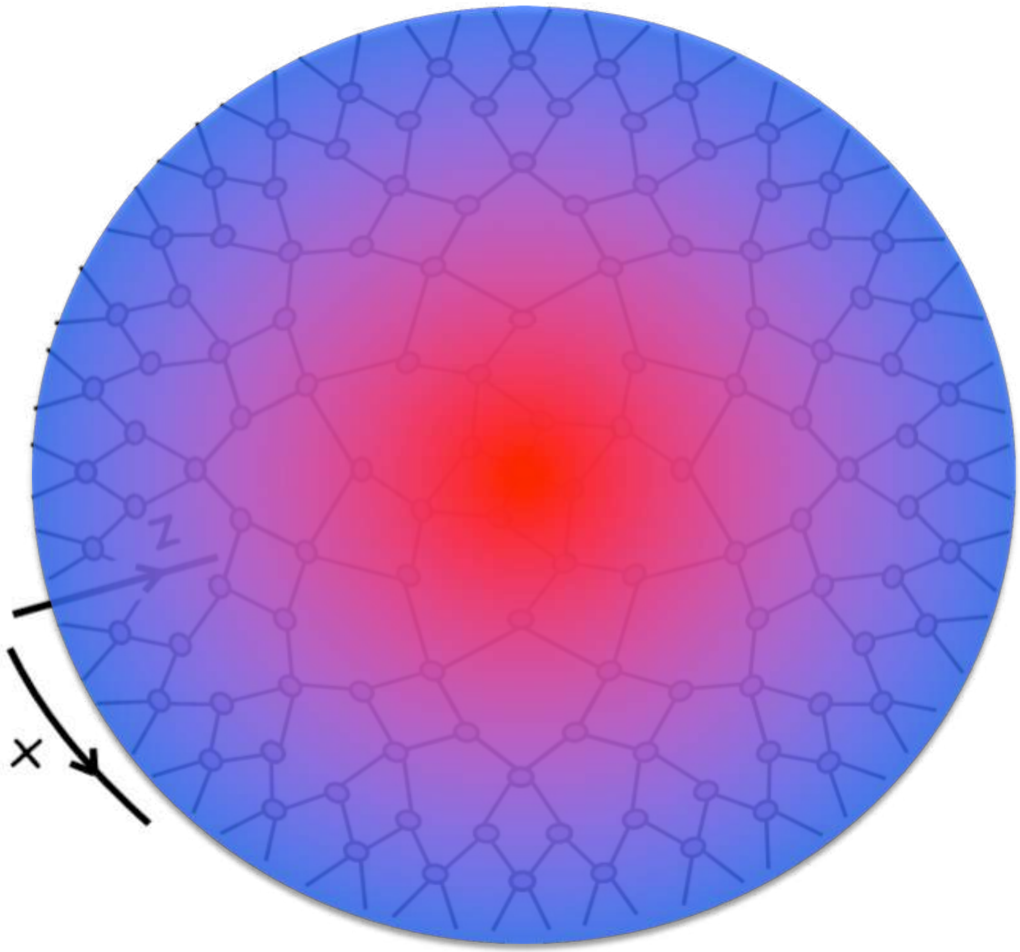
(continuum)

$$\left|\psi\right\rangle = P e^{-i\int_{u2}^{u1}\left(K(u)+L\right)du}\left|\Omega\right\rangle$$

J. Haegeman et al,
Phys. Rev. Lett. 110, 100402 (2013)

$K(u)$ Disentangler generator

L Isommetry generator



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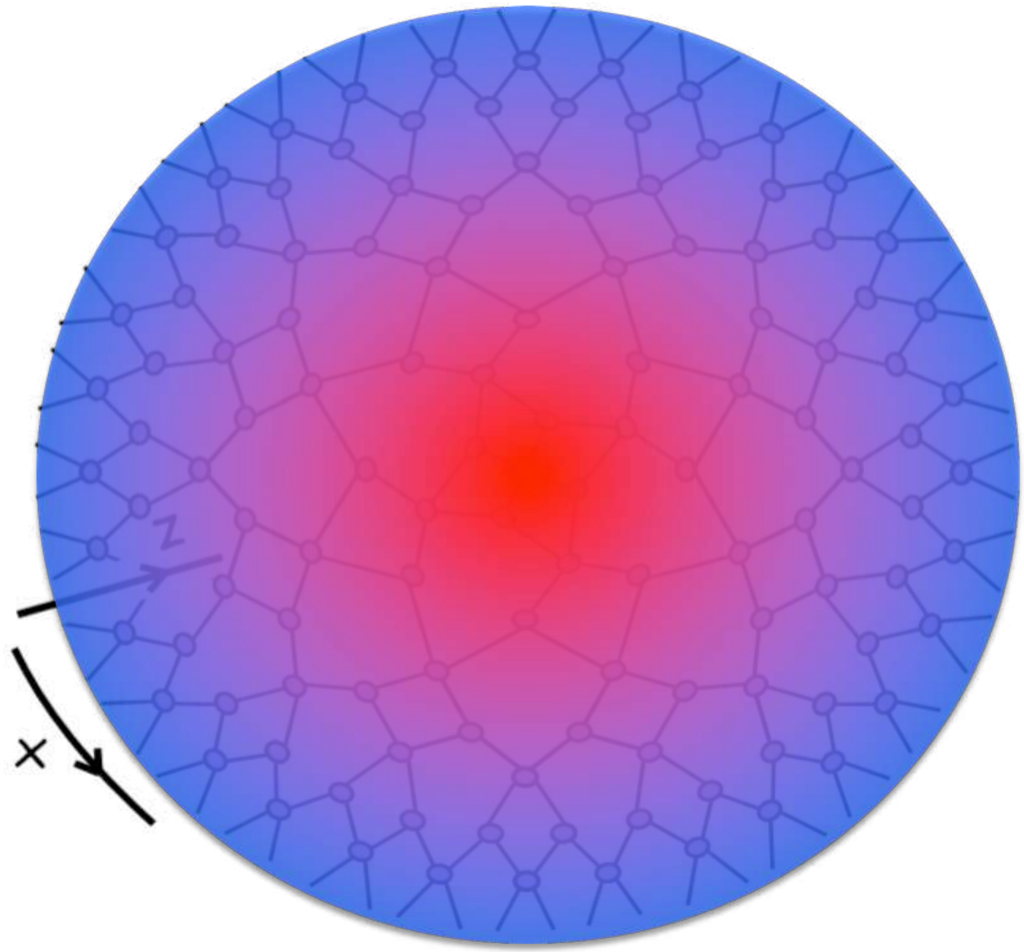


$$g_{uu}(u)du^2 = \mathcal{N}^{-1}\left(1 - \left|\langle\Psi(u)|e^{iL\cdot du}|\Psi(u+du)\rangle\right|^2\right)$$

Measures the density of strength of disentanglers.
Compatible with AdS metric

M. Nozaki, S. Ryu, T. Takayanagi, JHEP10(2012)193

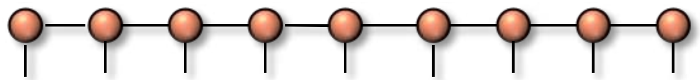
curvature ~ change of entanglement at every length scale

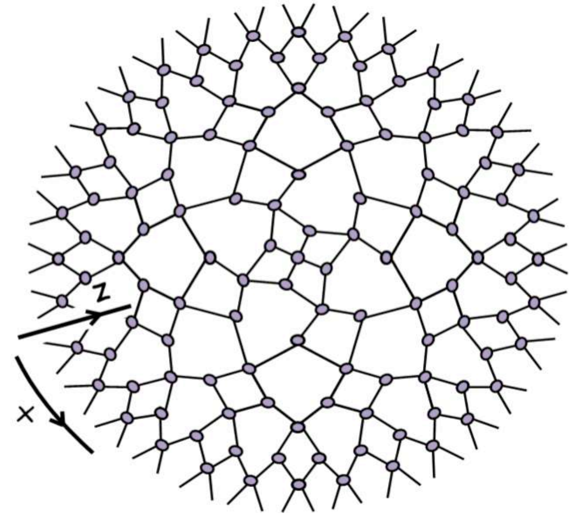
Lots of other things:

Lots of other things:

Chiral PEPS
Entanglement measures
Time evolution, TDVP
Infinite systems
Classical systems
Corner Transfer Matrices
Tensor Renormalization Group
Variational updates
PESS, gPEPS, TgPEPS
Phases of matter
3d PEPS
Topological systems
Excited states
Thermal states & dissipation
Continuous tensor networks
TNs & MonteCarlo, DFT, FRG...
Practical implementations
Entanglement Hamiltonians
Symmetries

Blablablablablablabla...





Thank you!