Dynamical hysteresis properties of the driven-dissipative Bose-Hubbard model with a Gutzwiller Monte Carlo approach

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1. Introduction

The dynamics of out of equilibrium systems are generally obtained through computational simulation. As the size of the Hilbert-space of these systems usually scales exponentially with system size, a clear need for efficient simulation algorithms arises. We study the driven-dissipative Bose-Hubbard model by using the quantum trajectory method. As a wave function ansatz we utilise the (Cluster) Gutzwiller ansatz. We investigate the dynamical hysteresis properties of this model and compare with mean-field results.

2. The model and method

The model Hamiltonian is given by

$$\hat{H} = \sum_i \left(-\Delta \hat{a}_i^{+} \hat{a}_i + \frac{U}{2} \hat{a}_i^{+} \hat{a}_i^{+} \hat{a}_i \hat{a}_i + F(\hat{a}_i + \hat{a}_i^{+}) - \frac{J}{z} \sum_{<i,j>} (\hat{a}_i^{+} \hat{a}_j + \hat{a}_j^{+} \hat{a}_i) \right),$$

with Δ the laser detuning, U the Kerr non-linearity, F the pumping strength, J the hopping amplitude and z the number of nearest neighbours.

Dissipation is introduced as the emission of a particle with a dissipation rate γ, the open system dynamics are governed by

$$\partial_t \hat{\rho} = -i[\hat{H}, \hat{\rho}] + \frac{\gamma}{2} \sum_j (2\hat{a}_i \hat{\rho} \hat{a}_i^{+} - \{\hat{a}_i^{+} \hat{a}_i, \hat{\rho}\}).$$

Equivalent approach: the quantum trajectory method. We evolve the wave function with a non-unitary Hamiltonian H

$$\psi(t) = \frac{\exp(-iHt)\tilde{\psi}}{\|\exp(-iHt)\tilde{\psi}\|} \qquad and \qquad H = \hat{H} - i\frac{\gamma}{2}\sum_i \hat{a}_i^{+} \hat{a}_i ,$$

and sample quantum jumps, i.e. the spin flips using a Monte Carlo scheme

$$\psi \to \frac{\hat{a}_i \psi}{\|\hat{a}_i \psi\|}.$$

Computationally more efficient

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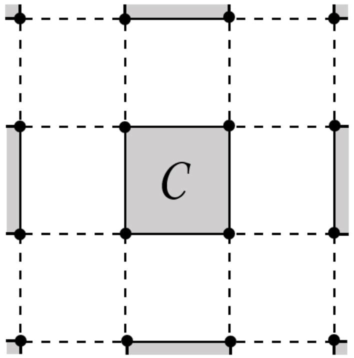
3. The wave function Ansatz

Cluster Gutzwiller wave function ansatz [1, 2]

$$\psi_{CGW} = \prod_{C} \psi_C,$$



Where ψ_C lies in the Hilbert-space H_C of the cluster C.

- Inclusion of short-range quantum correlations
- Inclusion of classical correlations
- Linear scaling of Hilbert space with cluster Hilbert size.

4. The parameter regime

We perform a linear sweep in pumping strength, that is

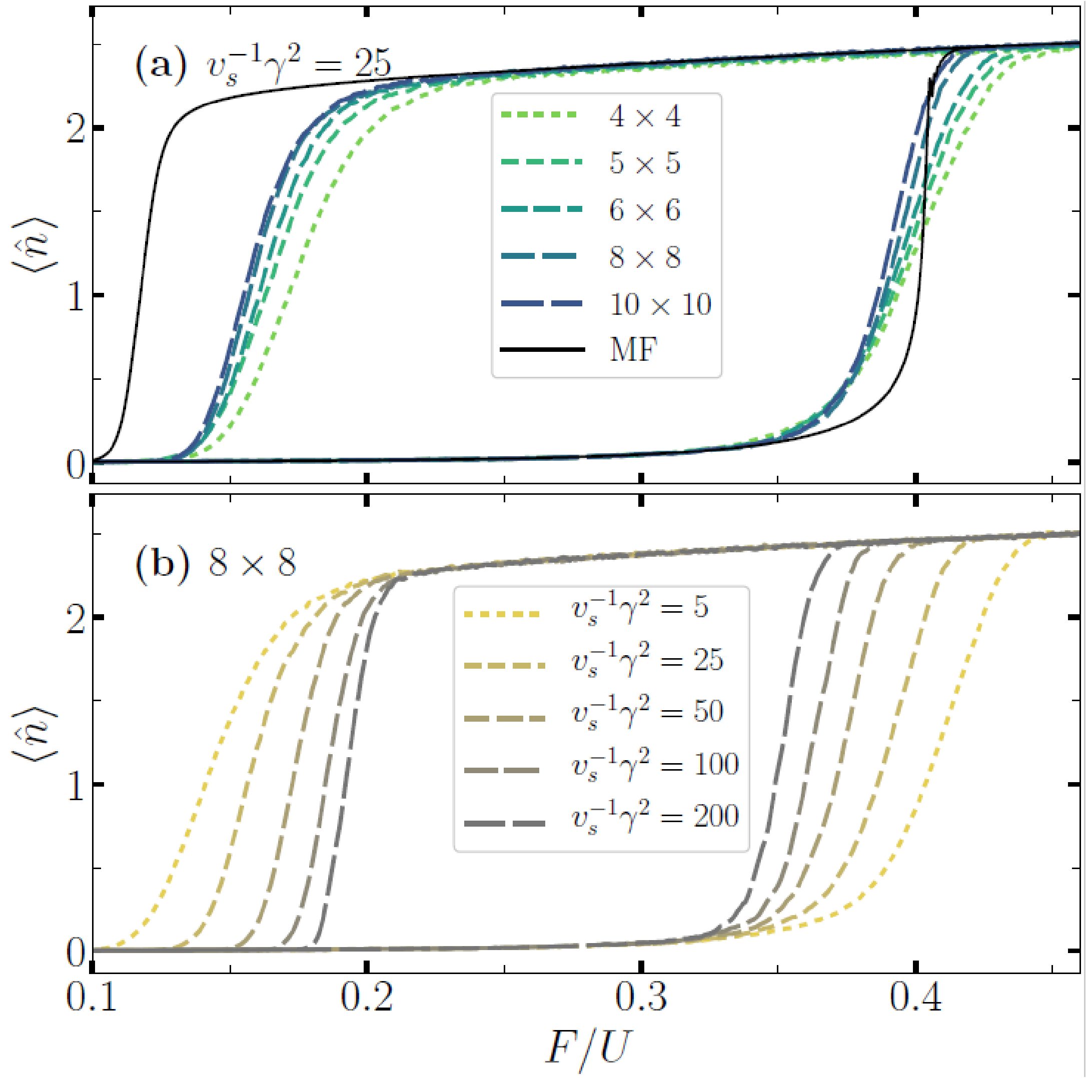
$$F(t) = (F_{start} + v_s t)\theta\left(t < \frac{t_s}{2}\right) + \left(F_{end} - v_s\left(t - \frac{t_s}{2}\right)\right)\theta\left(t \geq \frac{t_s}{2}\right),$$

with t_s the total sweep time and $v_s = \frac{2(F_{end} - F_{start})}{t_s}$.

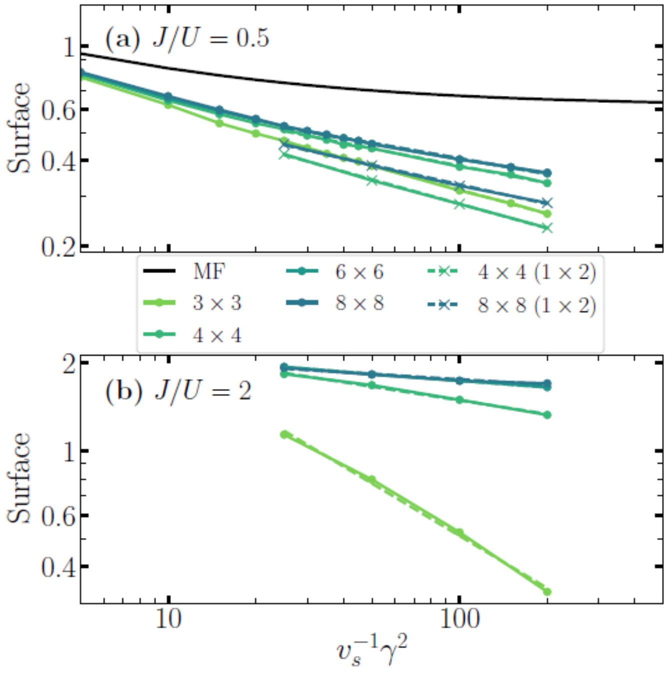
[1] W. Casteels, R. M. Wilson, and M. Wouters, Phys. Rev. A **97**, 062107 (2018)
[2] D. Huybrechts and M. Wouters, Phys. Rev. A **99**, 043841 (2019)

5. Results: dynamical hysteresis of $\langle \hat{n} \rangle = \frac{1}{N}\langle \sum_i \hat{a}_i^+ \hat{a}_i \rangle$

If we choose clusters of size one, i.e. the single cavities, we include classical fluctuations and on-site quantum fluctuations.



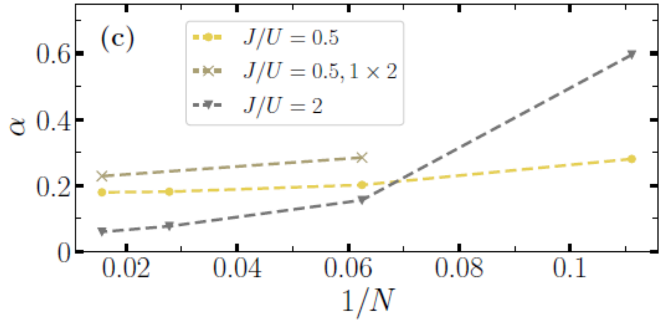
6. Results: the hysteretic surface



We study the surface between the hysteresis curves, defined as

$$S_h = \int_{F_{start}}^{F_{end}} \big(n_\uparrow(F) - n_\downarrow(F)\big) dF,$$

with $n_\uparrow$ the top branch and $n_\downarrow$ the bottom branch.



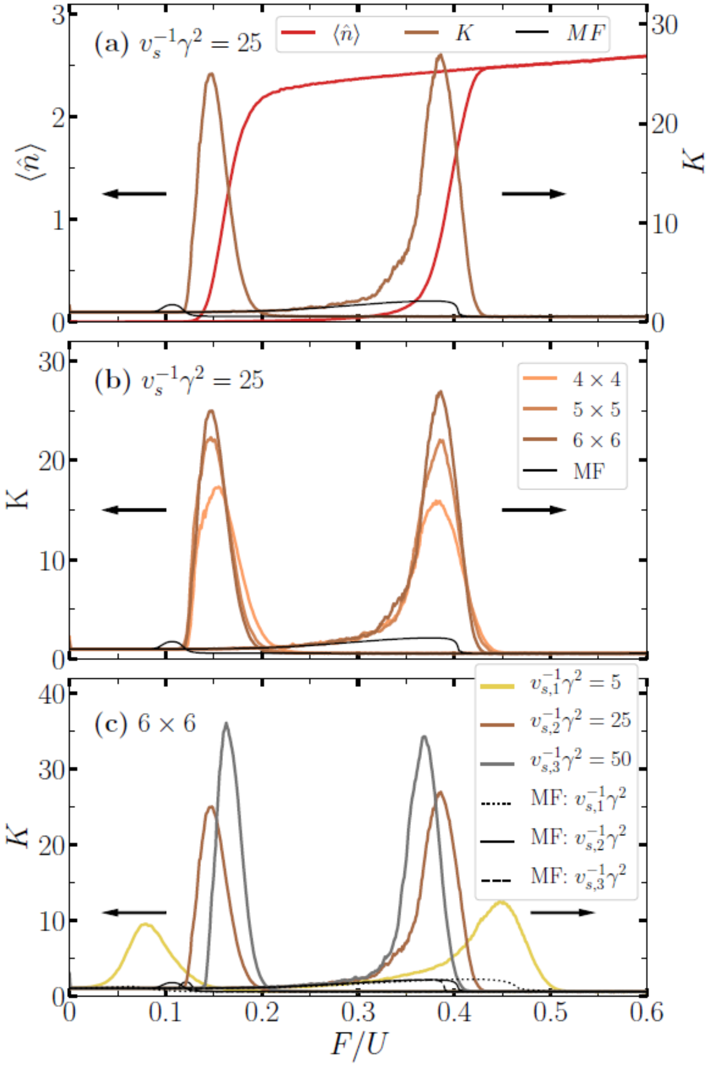
We show the exponent of the power law fit

$$S_h \propto (v_s^{-1})^{-\alpha},$$

in the long-time limit.

Note: 1×2 stands for clusters of this size.

7. Results: the compressibility



We show the compressibility, defined as

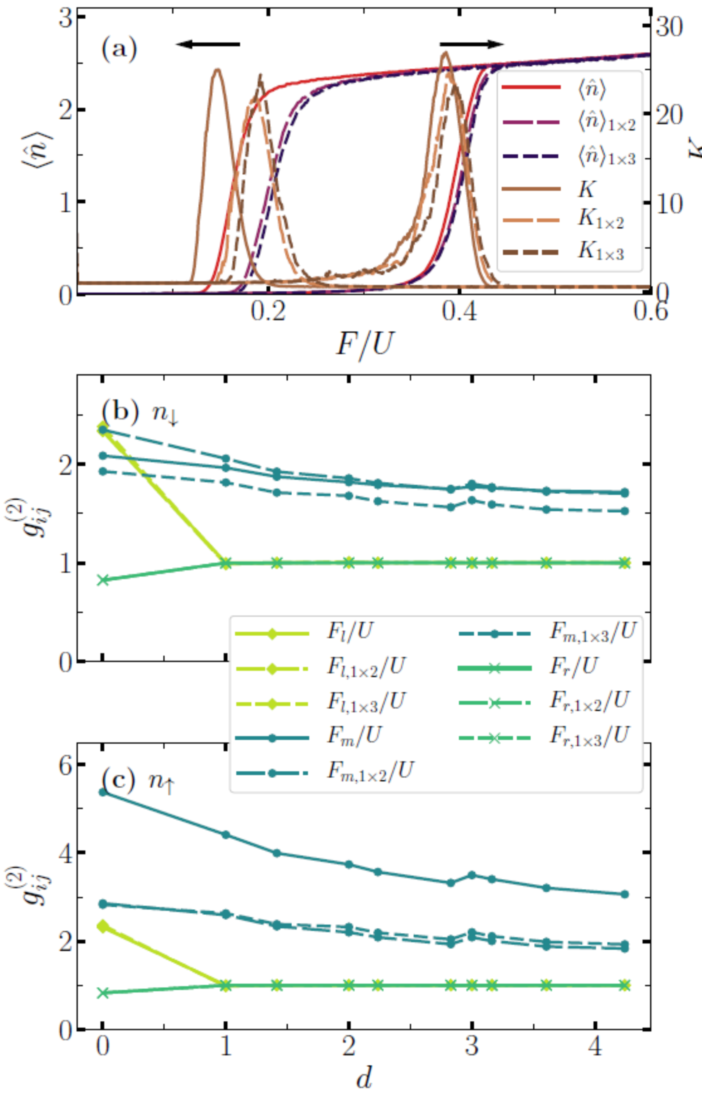
$$K = \frac{\langle \widehat{N^2} \rangle - \langle \widehat{N} \rangle^2}{\langle \widehat{N} \rangle},$$

With the number operator $\langle \widehat{N} \rangle = \langle \sum_i \hat{a}_i^+ \hat{a}_i \rangle$.

→The mean-field method drastically underestimates the number of fluctuations.

→ Subextensive scaling of the maxima of the peaks, indicating the formation of domains.

8. Results: the correlation function



→ Short-range quantum correlations cause an additional shift of the left transition.

The correlation function is given by

$$g_{ij}^{(2)} = \frac{\langle \hat{a}_i^{+} \hat{a}_j^{+} \hat{a}_i \hat{a}_j \rangle}{\langle \hat{N} \rangle^2}.$$

On the figure $d = |i - j|$.

[3] W. Casteels, R. Fazio, and C. Ciuti, Phys. Rev. A **95**, 012128 (2017)

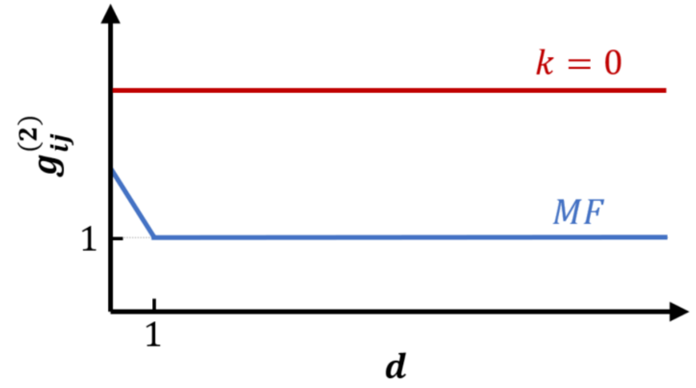
9. Results: mapping to single Kerr cavity

There exists a mapping [3] of the model Hamiltonian onto a single Kerr cavity. It is based on the Fourier transform of the Hamiltonian and the assumption that all $\boldsymbol{k} \neq 0$ modes can be neglected.

$$\hat{H}_{BH} \xrightarrow{maps\ onto} \hat{H} = \omega_0 \hat{a}_0^{+} \hat{a}_0 + F_{eff}(\hat{a}_0^{+} + \hat{a}_0) + \frac{U_{eff}}{2} \hat{a}_0^{+} \hat{a}_0^{+} \hat{a}_0 \hat{a}_0$$

The validity of neglecting this term is however not obvious.

For the $\boldsymbol{k} = 0$ mode and the mean-field method one expects results like



→ mapping not valid!

Manuscript in preparation.