Probing many-body entanglement via inverse statistical methods

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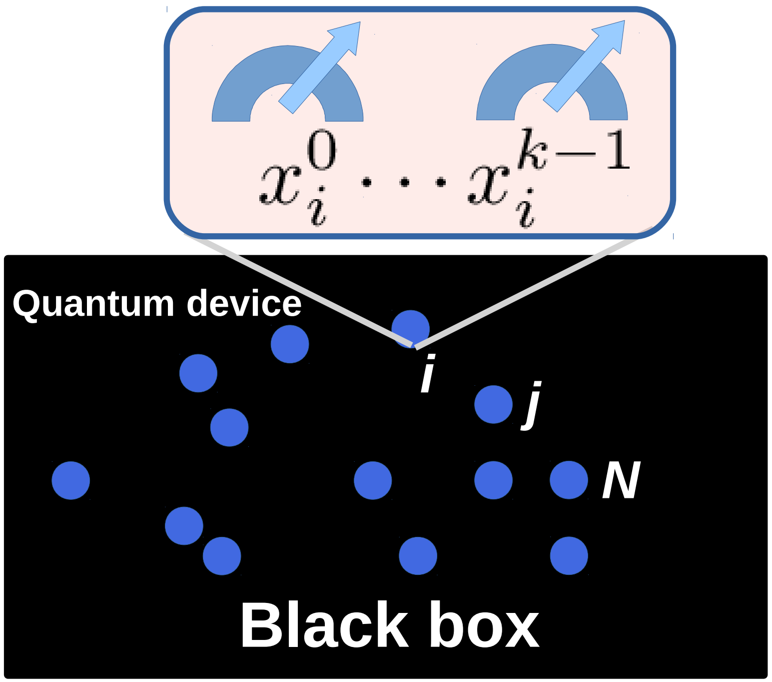
Certifying entanglement via Bell's inequalities



Quantum data

$$\langle f(\sigma)\rangle_{\mathrm{Q}} \quad (\sigma = \{x_i^a\})$$
$$(\text{e.g.} \langle x_i^a\rangle_{\mathrm{Q}}, \langle x_i^a x_j^b\rangle_{\mathrm{Q}}, \cdots)$$

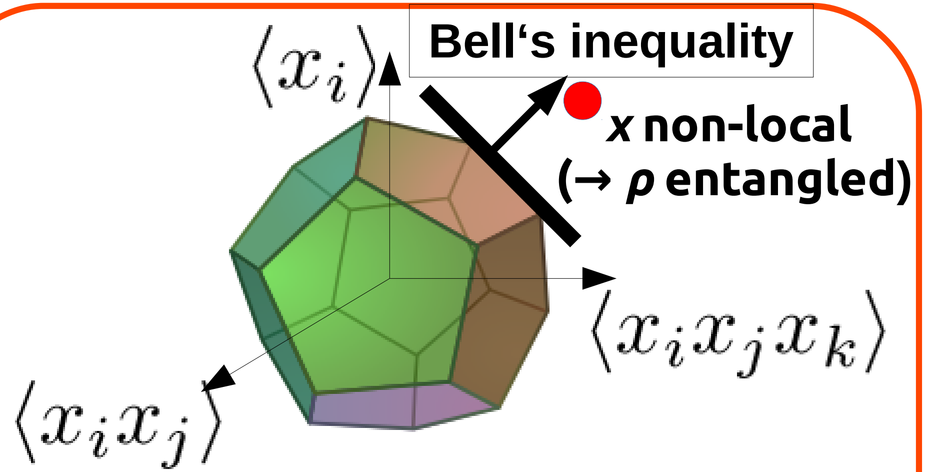
Bell's **local-variable (LV) model (classical data):**

Global probability distribution $P_{\mathrm{LV}}(\sigma)$ s.t.

$$\langle f_r(\sigma)\rangle_{\mathrm{LV}} = \langle f_r(\sigma)\rangle_{\mathrm{Q}} \quad (r = 1, \cdots R)$$

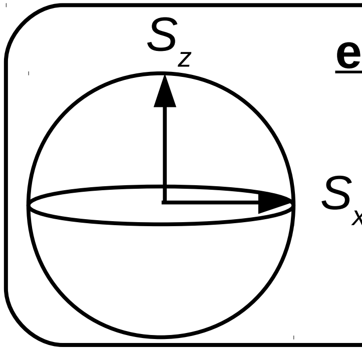
Always exists if the state is *not* entangled

J. S. Bell, Physics 1964
A. Fine, Phys. Rev. Lett. 1982



Polytope of classical data
\# vertices: ~exp(*N*)
\# facets: ~exp(exp(*N*))

e.g. quantum spin 1/2



$$[S_x, S_z] \neq 0$$
$$P(S_x, S_z) = ?$$

Membership problem:
NP hard
→ No scalable approach solving every instance

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N. Brunner *et al*, Rev. Mod. Phys. 2014

Statistical-inference approach to Bell's inequalities

IF & T. Roscilde, arxiv: 2004.07796 (2020)

<u>State-of-the-art approaches:</u>

- Brute force (linear program, solve convex hull problem → limited to small scenarios)
- Try some existing Bell inequalities (not generic)
- Remove structure from the data (symmetrize them; misses some data points)

Tura *et al* (Science 2014), Wang *et al* (PRL 2017), Fadel & Tura (PRL 2017)

- Approximate the polytope from the outside (hierarchy of semi-definite programs)

Baccari *et al* (PRX 2017)

<u>Inverse problem: max entropy solution</u>

$$P_{\rm LV}(\sigma) = Z^{-1} \exp[\sum_{r=1}^{R} K_r f_r(\sigma)]$$

Boltzmann distribution over a classical model

E.T. Jaynes, Phys. Rev. 1957

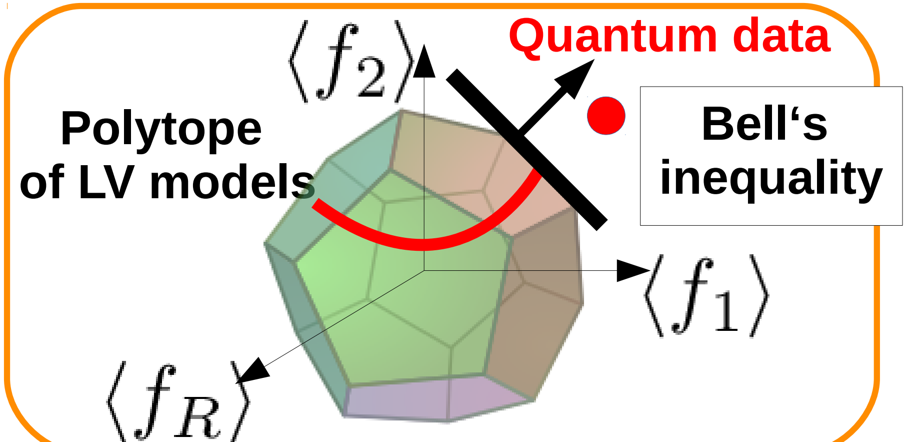
<u>Update rule:</u> iterate

$$K'_r = K_r - \epsilon(\langle f_r \rangle_{\rm LV} - \langle f_r \rangle_{\rm Q})$$

gradient-descent of the log-likelihood **(convex)**

Nguyen *et al.*, Adv. Phys. 2017

Trajectory of classical data converging to the quantum data

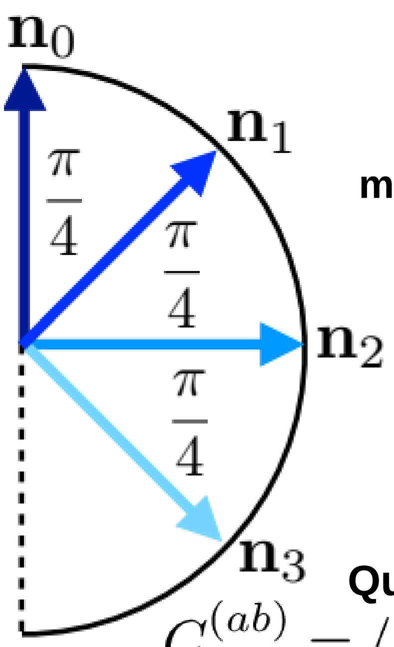


<u>Generic</u>: applies to any quantum data
<u>Scalable</u>: Monte-Carlo sampling of classical statistical models (unless e.g. spin glass)

Application: Heisenberg antiferromagnets

$$\mathcal{H} = J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j$$

S=1/2 quantum spins on a square lattice
Ground state: many-body singlet $S_{tot}=0$

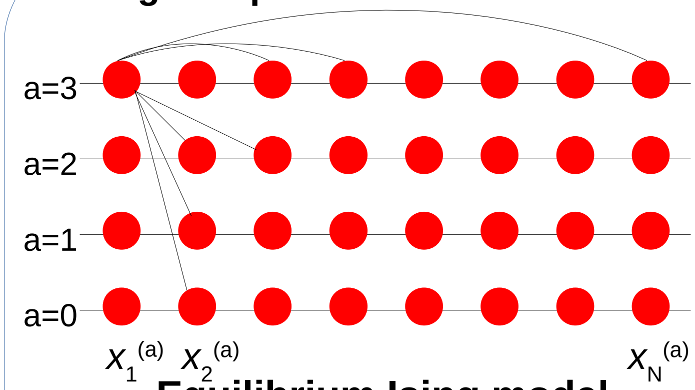


Uniform measurement strategy (k=4)

Quantum data:

$$C_{ij}^{(ab)} = \langle \hat{\sigma}_i^{\mathbf{n_a}} \hat{\sigma}_j^{\mathbf{n_b}} \rangle_Q$$

Fitting the quantum data with LV models



Equilibrium Ising model

$$H = -\sum_{i<j} \sum_{a,b} K_{ij}^{(ab)} x_i^{(a)} x_j^{(b)}$$

Novel Bell inequality (permutationally invariant)

$$\langle B \rangle_{\text{LV}} = S_{11} + S_{12} + S_{22} + S_{23} + \dots + S_{kk} - S_{k1} \geq -2N(k-1)$$

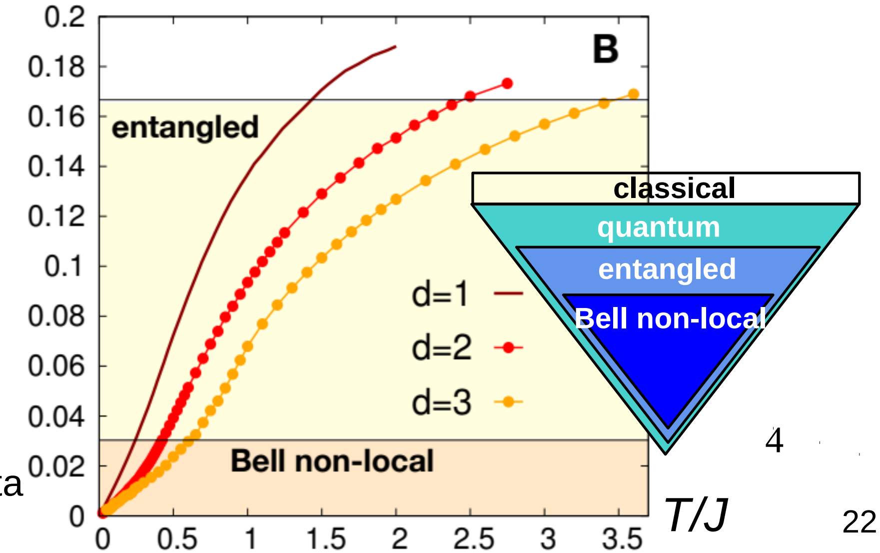
$$S_{ab} = \sum_{i \neq j} C_{ij}^{(ab)}$$

Many-body singlets: $\langle B \rangle_Q = -Nk[1 + \cos(\pi/k)]$
(maximal quantum violation)

Assuming SU(2) invariance of the quantum state:

$$\langle \hat{J}_z^2 \rangle / N < \frac{1}{4} - \frac{k-1}{2k[1+\cos(\pi/k)]}$$ (Bell non-locality)

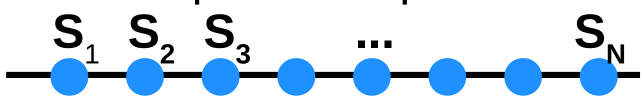
Compared to: $$\langle \hat{J}_z^2 \rangle / N < \frac{1}{6}$$ (entanglement, Tóth *et al.* PRA 2004)



Similar structure to Schmied *et al* (Science 2016); emerges from the data
IF & T. Roscilde, arxiv: 2004.07796 (2020)

Extension: solving to the separability problem

N quantum spin-1/2

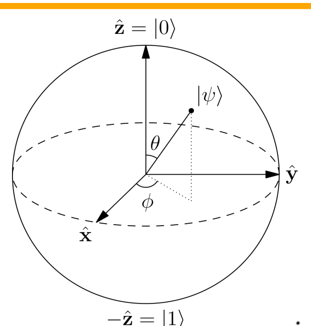


$$\rho_{\rm sep} = \prod_{i=1}^{N} \int_{|\mathbf{S}_i|=1} d\mathbf{S}_i \; P(\mathbf{S}_1, \cdots \mathbf{S}_N) \; \rho_1(\mathbf{S}_1) \otimes \cdots \otimes \rho_N(\mathbf{S}_N)$$

with $\rho(\mathbf{S}_i) := |\psi(\mathbf{S}_i)\rangle\langle\psi(\mathbf{S}_i)|$

***P* : statistical distribution over classical rotators**

Bloch sphere



$$|\mathbf{S}(\theta,\phi)|^2 = 1$$

$$|\psi(\mathbf{S})\rangle = \cos(\theta/2)|\uparrow\rangle + \sin(\theta/2)e^{i\phi}|\downarrow\rangle$$

Quantum data = 2-body correlation matrix:

$$C_{ij}^{ab} := \langle \hat{\sigma}_i^a \hat{\sigma}_j^b \rangle \equiv_{(\rho_{\rm sep})} \langle S_i^a S_j^b \rangle_P$$

$$P(\{\mathbf{S}_i\}) \propto \exp[\sum_{i,j=1}^{N} \sum_{a,b\in\{x,y,z\}} K_{ij}^{ab} \; S_i^a S_j^b]$$

Solution: **Generalized XYZ model for classical rotators**

Tomographically complete

(no other possible entanglement witness based on 2-body correlators)

General case: local Hilbert space dim = d:

- 2d – 2 classical variables
- d^2 - 1 measurements (e.g. generators of SU(d))

(unpublished)