Particle Density Mobility Edge

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Single Particle Mobility Edge

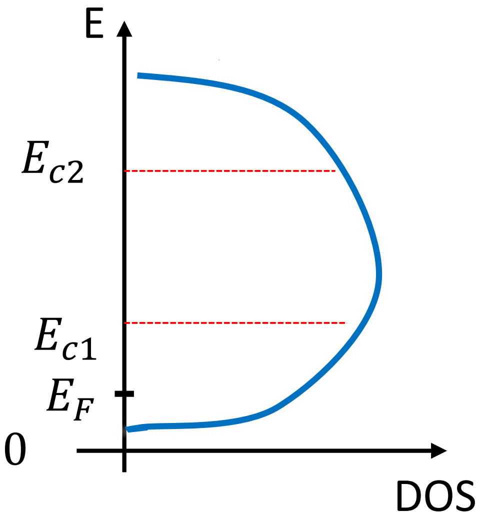
- Non-interacting fermions in a random potential (Anderson, 1958):

$$\hat{H} = t \sum_{\langle i,j \rangle} (c_i^\dagger c_j + \text{h.c.}) + \sum_i \epsilon_i \hat{n}_i \,, \quad \epsilon_i \in [-W, W]$$

- Resulting eigenstates are either localized or extended

$$|\psi_l(i)|^2 \sim \begin{cases} \frac{1}{\xi^d} \exp\left(-\frac{|i-l|}{\xi}\right) & \textit{localized} \\ \frac{1}{V} & \textit{extended} \end{cases}$$

- In d=1,2 all eigenstates are localized $\forall\, W > 0$ (Abrahams et al.,1979)
- In d>2 localized and extended states occupy different bands, separated by mobility edges E_c (Mott, 1967).



$$\sigma(T) \propto \exp(-\frac{E_c - E_F}{T})$$

Metal-insulator transition at $T = 0$

Many-Body Mobility Edge

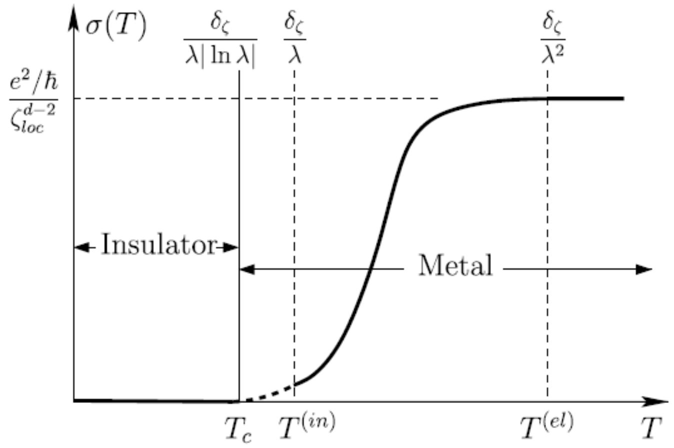
- Weakly interacting fermions in a random potential (Basko et al., 2006; Gornyi et al., 2005):

$$\hat{H} = t \sum_{\langle i,j \rangle} (c_i^\dagger c_j + \text{h.c.}) + \sum_i \epsilon_i \hat{n}_i + V \sum_{\langle i,j \rangle} \hat{n}_i \hat{n}_j$$

- Localization is stable under determinate conditions
- Basko et al. predict extensive many-body mobility edge (MBME) $\mathcal{E}_c$:

$$\sigma(T) = \sigma[E(t)], \quad E(T) > \mathcal{E}_c$$

$$\sigma(T) \propto \exp\left(-\frac{\mathcal{E}_c - E(T)}{T}\right), \quad E(T) < \mathcal{E}_c$$

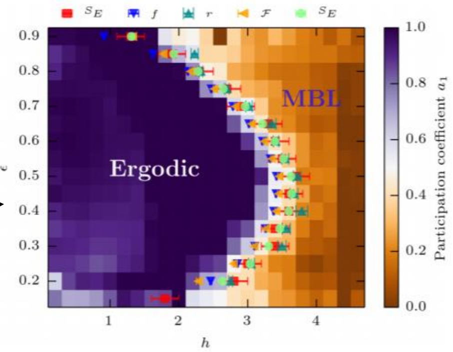


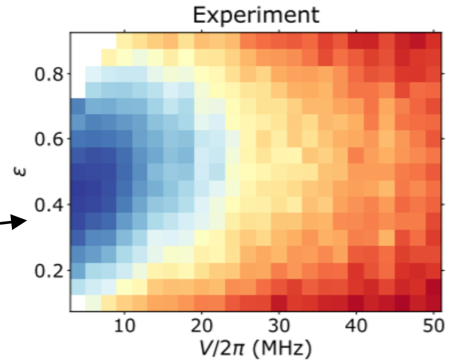
- Metal-insulator transition at $T_c > 0$

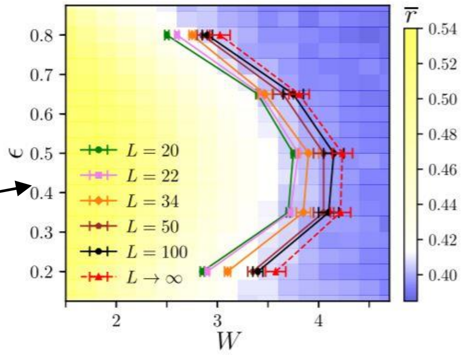
Stability of Many-Body Mobility Edge

Numerical Studies

- Evidence of MBME in small systems through exact diagonalization (ED) - Luitz et al. PRB **91**, 2015; Serbyn et al. PRX **5**, 2015; Geraedts et al. PRB **95**, 2017
- Recent experiments also reported mobility edge at least on intermediate timescales - Guo et al. arXiv:1912.02818, 2019
- Novel tensor network approaches show marks of MBME in large systems - Brighi et al. arXiv:2005.02999, 2020; Chanda et al. arXiv:2006.02860, 2020



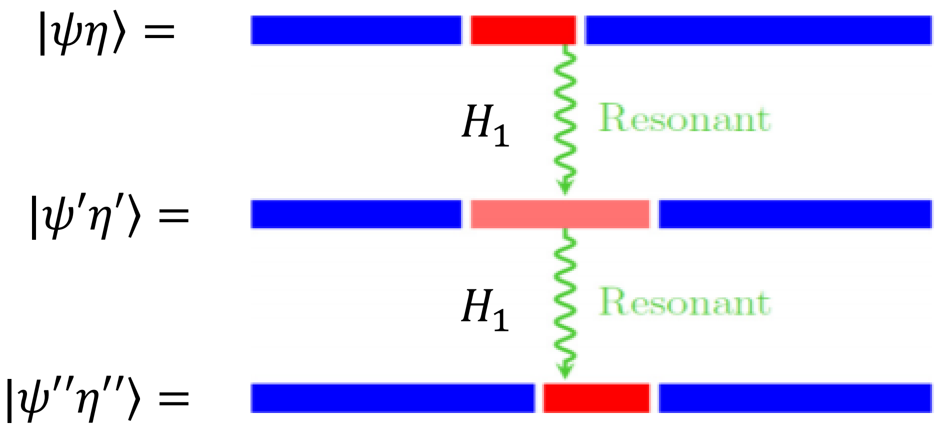




Arguments Supporting Instability

- De Roeck et al. (PRB **93**, 2016) recently questioned stability of localization in systems with MBME
- Locally ergodic regions (bubbles), due to flucutations, move through the lattice and resonantly hybridize many-body states

$$\frac{|\langle \Psi\eta|H_1|\Psi'\eta'\rangle|}{|E(\eta)-E(\eta')+E(\Psi)-E(\Psi')|} \gg 1$$



Mobility Edge in Particle Density

Constrained Hopping Model

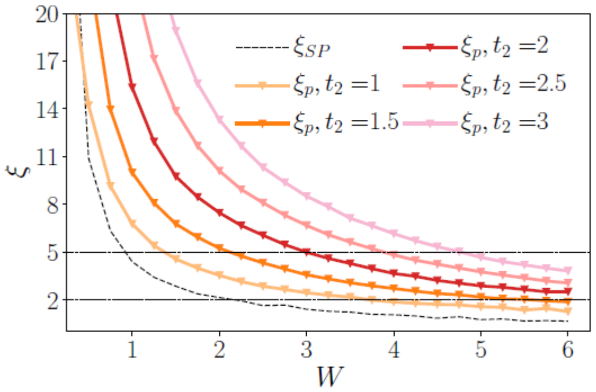
- Using a kinetically constrained model, we obtain MBME in particle density ν

$$\hat{H} = t_1 \sum_{i=1}^{L-1} (c_i^\dagger c_{i+1} + \text{h.c.}) + \sum_{i=1}^{L} \epsilon_i \hat{n}_i + t_2 \sum_{i=2}^{L-1} (c_{i-1}^\dagger \hat{n}_i c_{i+1} + \text{h.c.})$$

- Hopping parameter t_2 implements pair hopping:

t_2: ●●○ ↔ ○●●

- Tuning t_2/t_1 we can achieve density dependent localization



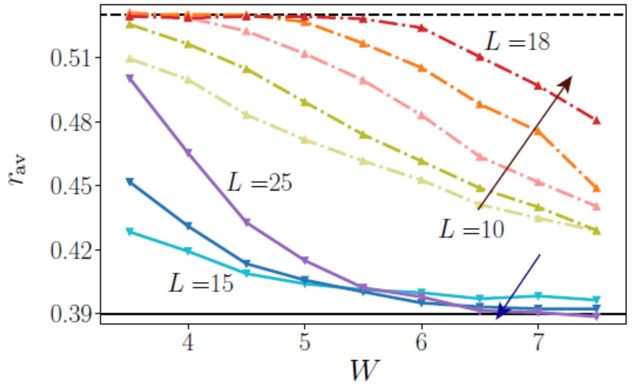
- Novel approach, albeit qualitatively equivalent, to the mobility edge

- Density dependent localization enables simple initialization of bubbles in matrix product states (MPS)

ED Evidence of Particle Density Mobility Edge

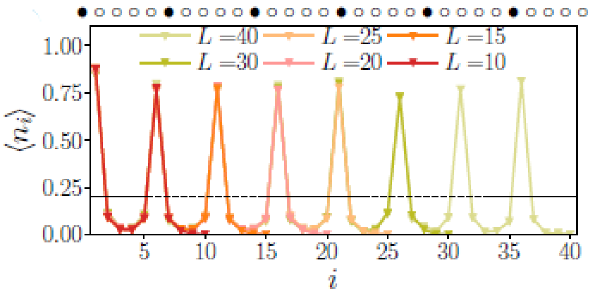
- Study of typical eigenstate measures as level spacing ratio r through exact diagonalization and shift-invert

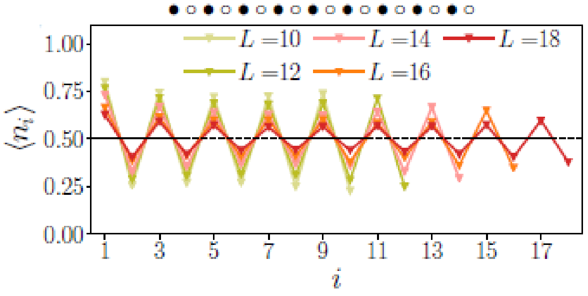
$$r = \left\langle \frac{\min(\delta_n, \delta_{n+1})}{max(\delta_n, \delta_{n+1})} \right\rangle$$



- Comparison of $\nu = 1/5$ (blue curves) and $\nu = 1/2$ (red curves) shows different behavior after a critical disorder

- TEBD quench dynamics of density waves (DW) shows similar differences at large system size



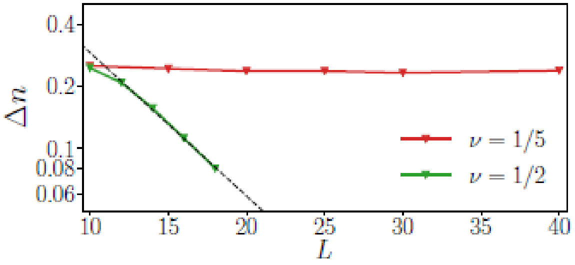


Mobility Edge in Particle Density

Evidence of MBME from Quench Dynamics

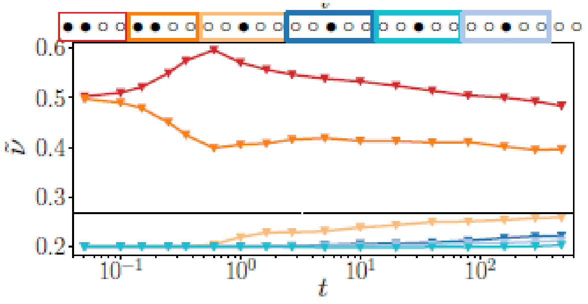
- Scaling of the deviation from the thermal density Δn reveals exponential decay for $\nu = 1/2$. Lower density has instead constant Δn

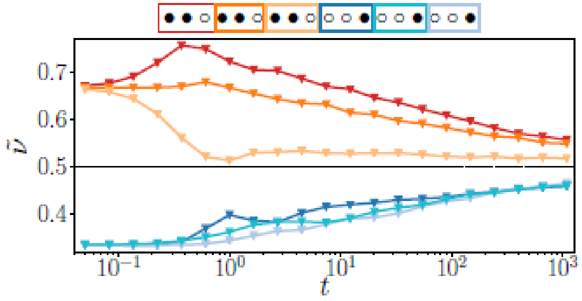
$$\Delta n = \frac{1}{L}\sum_{i=1}^{L} |\langle \hat{n}_i(T_{\max})\rangle - \nu|$$



- Dynamics of the coarse grained density $\tilde{\nu}$ from initial states including thermal bubbles highlights the absence of relaxation at low density even at large system size

$$\tilde{\nu} = \sum_{j=i}^{i+k} \langle \hat{n}_j \rangle$$





Discussion and Outlook

- Our model shows strong evidence of many-body mobility edge in particle density both in eigenstates statistics and quench dynamics for large systems
- The mobility edge in particle density enables a simple initialization of states in a certain density sector allowing the study of MBME in large systems through MPS
- Furthermore, it allows the study of states including thermal regions, thus enabling direct investigation of the arguments of De Roeck et al.
- With respect to that, our simulations seem to rule out the mobility of the bubble in the localized regime
- Interestingly, a mechanism similar to the one described by our model could be realized in state of the art experiments with the Aubry-Andre' bosonic Hamiltonian
- Finally, more efficient algorithms could lead to the study of even larger systems

Thank you for your attention!

(For major details please refer to: arXiv:2005.02999)