Loschmidt echo singularities as dynamical signatures of strongly localized phases

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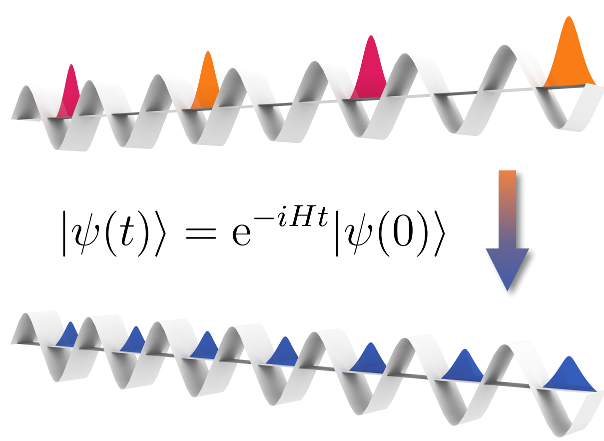
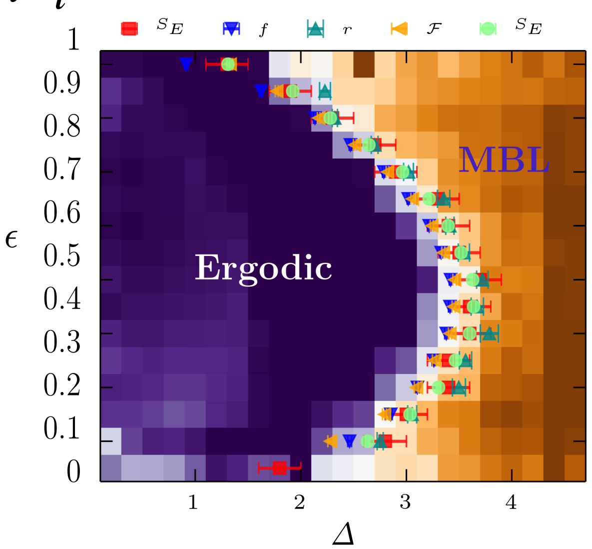
Model and protocol

$$H = J\sum_{i=1}^{N-1}\left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + J_z S_i^z S_{i+1}^z\right) + \sum_{i=1}^{N}\phi_i S_i^z$$

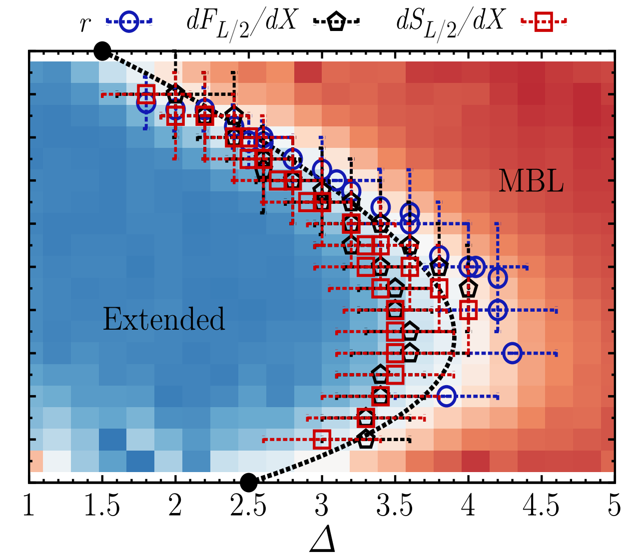
$\phi_i \in [-\Delta, \Delta]$ random

$\phi_i = \Delta\cos(2\pi\kappa i + \phi_0)$

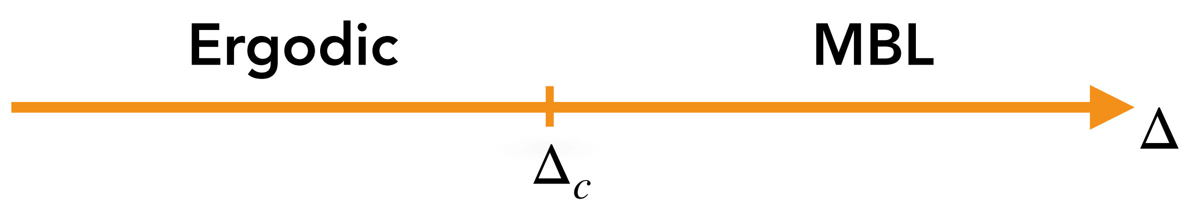




D. Luitz et al., Phys. Rev. B 91, 081103 (2015)



P. Naldesi et al., SciPost Phys. 1, 010 (2016)



- *Quench protocol*

  We prepare the system in the initial product state $|\psi(0)\rangle = |\uparrow,\downarrow,\cdots,\uparrow,\downarrow\rangle$ and we study the non-equilibrium dynamics driven by the MBL Hamiltonian

- *Main observable*

  Quantum overlap between state at time *t* and initial state

$$\mathcal{G}(t) = \langle\psi(0)|\psi(t)\rangle = \langle\psi(0)|e^{-iHt}|\psi(0)\rangle$$

Associated probability: **Loschmidt Echo**

$$\mathcal{L}(t) = |\mathcal{G}(t)|^2 = |\langle\psi(0)|e^{-iHt}|\psi(0)\rangle|^2$$

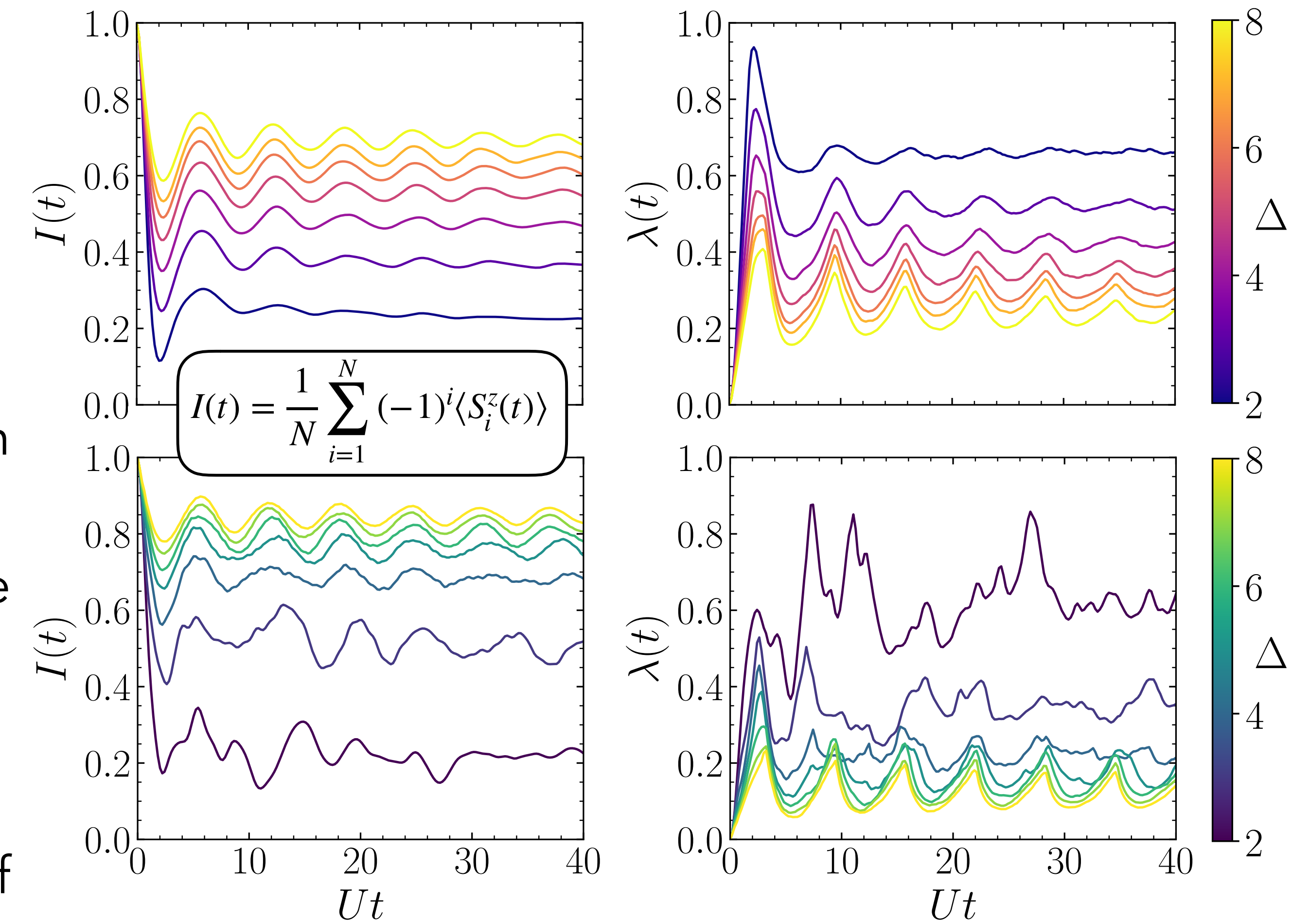
- ✦ Quantum chaos
- ✦ Quantum computation and information
- ✦ Linear waves (elastic waves, microwaves, …) and non-linear waves (BEC, …)
- ✦ Quantum criticality and non-equilibrium dynamics of many-body systems (DQPTs, …)

Quench dynamics

Loschmidt "return rate"

$$\lambda(t) = -\lim_{N\to\infty}\frac{1}{N}\log L(t)$$

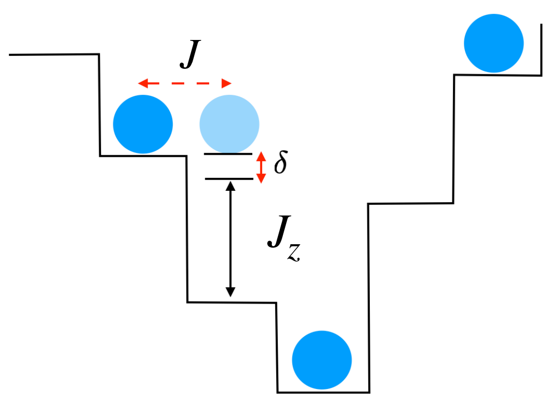
- ED simulations for N up to 22 sites
- Disorder averages over *O(10³)* realisations
- Emergence of a periodic structure of singularities in the ***short-time dynamics*** at times $t_n^* = \pi + 2n\pi$
- Same periodicity of imbalance oscillations in the same disorder regime

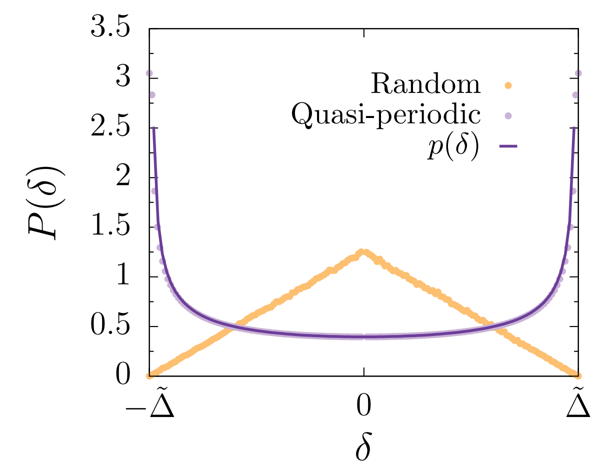


Where do these singularities come from?

- Simple model of Rabi oscillations of an ensemble of independent two-level systems detuned by $\delta + J_z$
- For a collection of N two-level systems with local detunings δ_j , the Loschmidt rate function is

$$\lambda^{2S}(t;\{\delta_j\}) = -\log\prod_{j=1}^{N}L(t;\delta_j) = -\sum_{j=1}^{N}\log\left(1-\frac{J^2}{J^2+\delta_j^2}\sin^2(\Omega t)\right)$$





- A disorder average is then an average over all possible detunings, and sampling the probability distribution $P(\delta)$ leads to

$$\lambda^{2S}(t) = -\int d\delta P(\delta)\log\left(1-\frac{J^2}{J^2+\delta^2}\sin^2(\Omega t)\right)$$

Rabi frequency

$$\Omega = \frac{\sqrt{J^2+\delta_j^2}}{2}$$

Exact form for QP potentials:

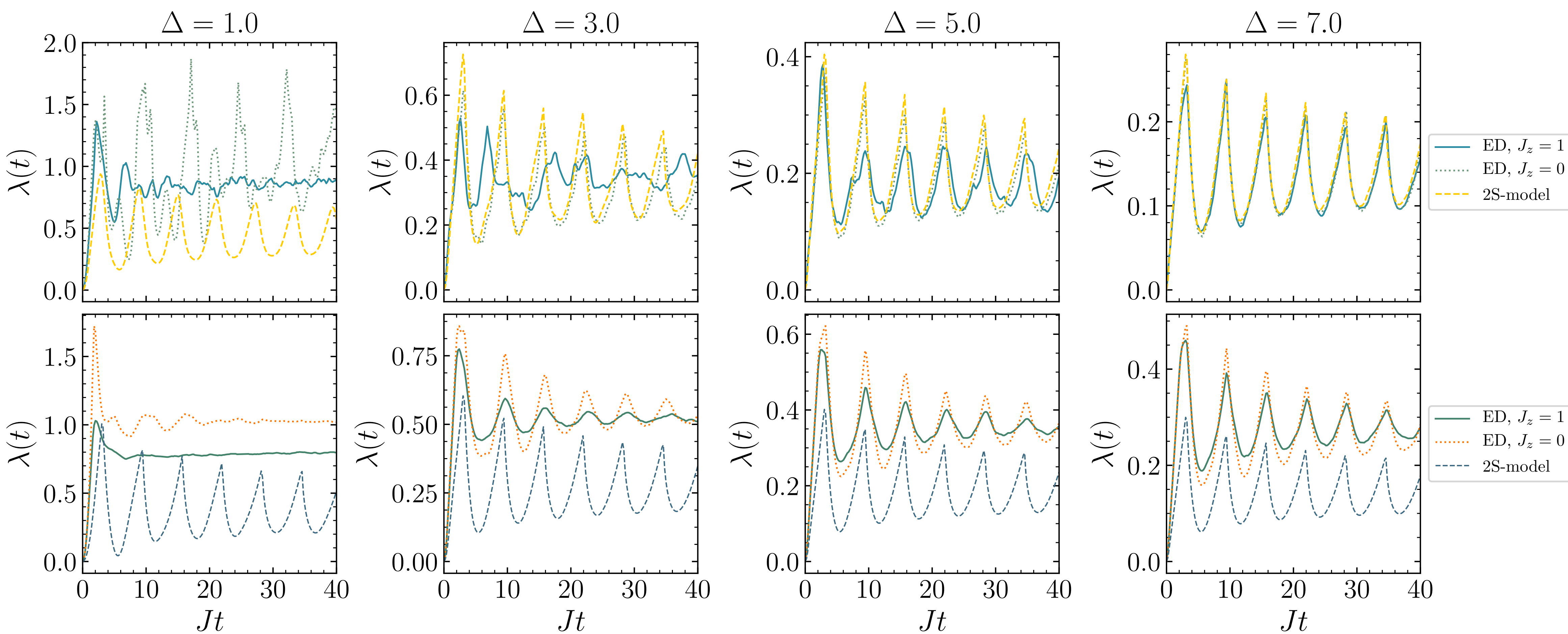
$$p(\delta)\mathrm{d}\delta = \frac{1}{\pi\Delta}\left(1-\frac{\delta^2}{\Delta^2}\right)^{-1/2}$$

V. Guarrera et al., NJP. 9, 107 (2007)

2S-model predictions

QP

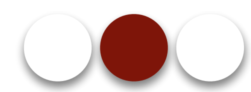
Random



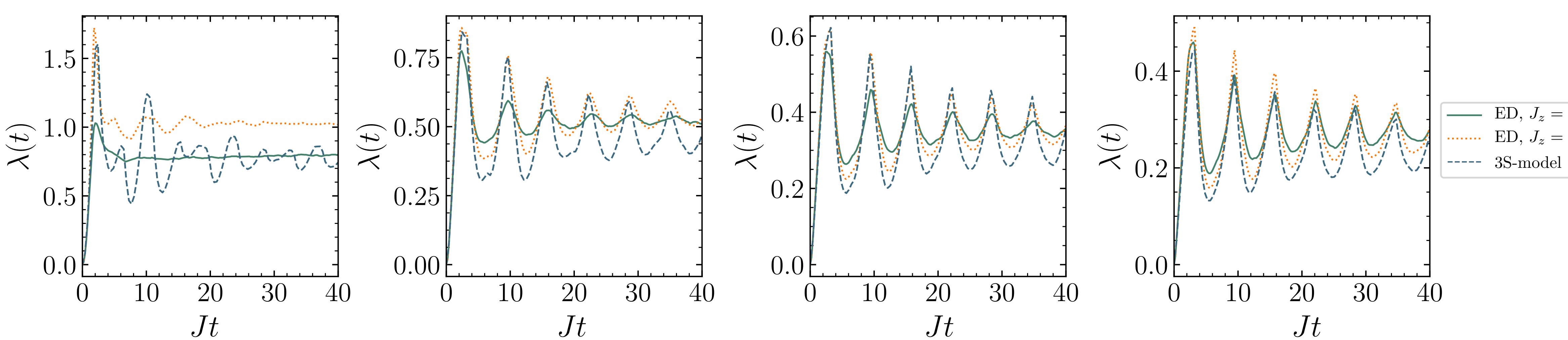
Extension: 3 sites cluster / 3-level systems

$$H_{101} = -\frac{J}{2}\left(c_1^\dagger c_2 + c_2^\dagger c_3 + h.c.\right) + \delta_{12} n_2 + (\delta_{23} - \delta_{12}) n_3$$
$$+ J_z\left[\left(n_1 - \frac{1}{2}\right)\left(n_2 - \frac{1}{2}\right) + \left(n_2 - \frac{1}{2}\right)\left(n_3 - \frac{1}{2}\right)\right]$$

$+$



$$H_{010} = -\frac{J}{2}\left(c_1^\dagger c_2 + c_2^\dagger c_3 + h.c.\right)$$
$$+ (V_1 + J_z) n_1 + V_2 n_2 + (V_3 + J_z) n_3$$

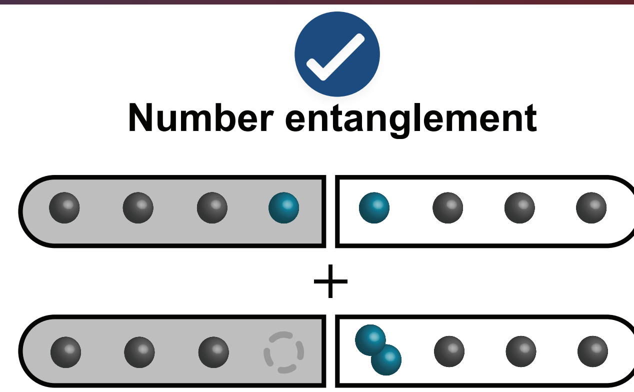


Entanglement entropy

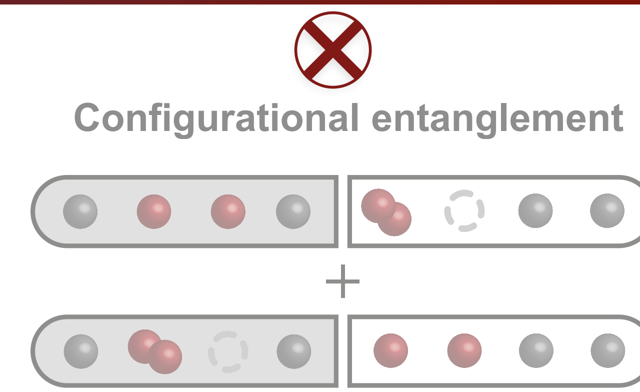
$$S_{N/2} = -\,\mathrm{Tr}_A \rho_A \log \rho_A$$

AL	MBL
No growth	$\sim \log t$

Number entanglement



Configurational entanglement



A. Lukin et al., Science 364, 6437, 256-260 (2019)

M. Kiefer-Emmanoulidis et al., PRL 124, 243601 (2020)

Entanglement entropy in the 2S-model

$$S_1(t;\delta) = L(t;\delta)\log L(t;\delta) + (1 - L(t;\delta))\log(1 - L(t;\delta)) \longrightarrow S^{2S}(t) = \int d\delta P(\delta) S_1(t;\delta)$$