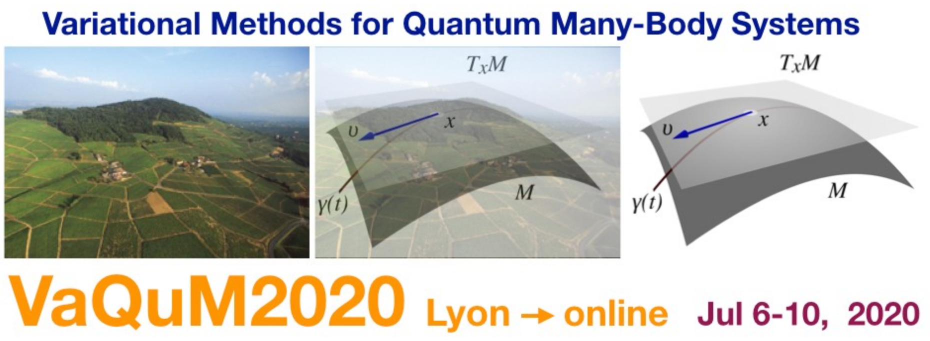
Basics of lattice gauge theories
with Tensor Networks



Luca Tagliacozzo

Based on:

L. Tagliacozzo G. Vidal
"Entanglement renormalization and gauge symmetry"
Phys. Rev. B 83, 115127 (2011)

L. Tagliacozzo, A. Celi, M. Lewenstein "Tensor Networks for Lattice Gauge Theories with continuous groups", Phys. Rev. X 4, 041024 (2014)

Outline

Gauge theories in HEP 5 min

Lattice gauge theory 5 min

Motivation for TN and LGT 1 min

Symmetries and superposition 15 min BB

Exotic phases of matter 5 min

Intro to Tensor Networks 5 min

Intro to LGT (Z2) 20 min BB

TN for Gauge theories (Z2) 20 min BB

Generalization 10 min

Example of results (2D MERA + PEPS) 5 min

Outline

Gauge theories in HEP

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Intro to LGT (Z2)

Tensor Networks for Gauge theories (Z2)

Generalization

Example of results (2D MERA + PEPS)

The "official review"

Simulating Lattice Gauge Theories within Quantum Technologies

M.C. Bañuls[1,2], R. Blatt[3,4], J. Catani[5,6,7], A. Celi[3,8], J.I. Cirac[1,2], M. Dalmonte[9,10], L. Fallani[5,6,7], K. Jansen[11], M. Lewenstein[8,12,13], S. Montangero[7,14 a], C.A. Muschik[3], B. Reznik[15], E. Rico[16,17 b], L. Tagliacozzo[18], K. Van Acoleyen[19], F. Verstraete[19,20], U.-J. Wiese[21], M. Wingate[22], J. Zakrzewski[23,24], and P. Zoller[3]

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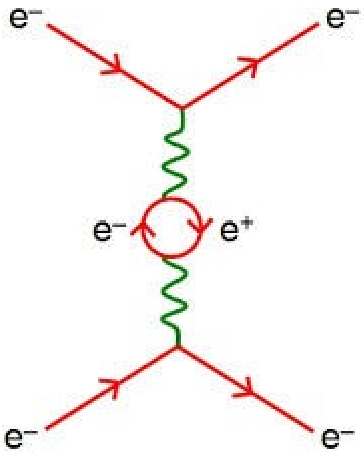
arXiv:1911.00003v1 [quant-ph] 31 Oct 2019

November 4, 2019

Abstract Lattice gauge theories, which originated from particle physics in the context of Quantum Chromodynamics (QCD), provide an important intellectual stimulus to further develop quantum information technologies. While one long-term goal is the reliable quantum simulation of currently intractable aspects of QCD itself, lattice gauge theories also play an important role in condensed matter physics and in quantum information science. In this way, lattice gauge theories provide both motivation and a framework for inter-

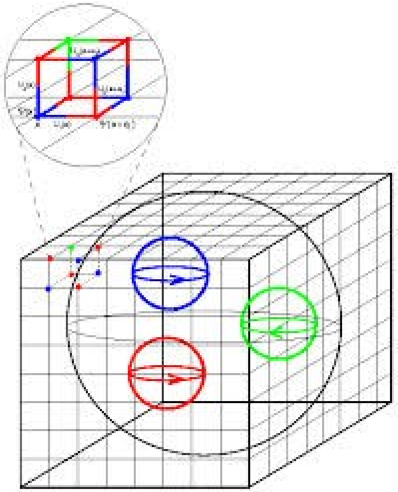
Gauge Theories



→ HEP, form QED, QCD, Standard Model, ele
gauge bosons

→ COND-MAT spin liquids, dimers (electrons in a material), emerging gauge bosons

→ Lattice allows for non-perturbative formulation of QCD

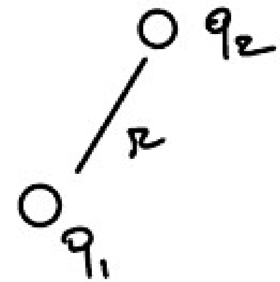


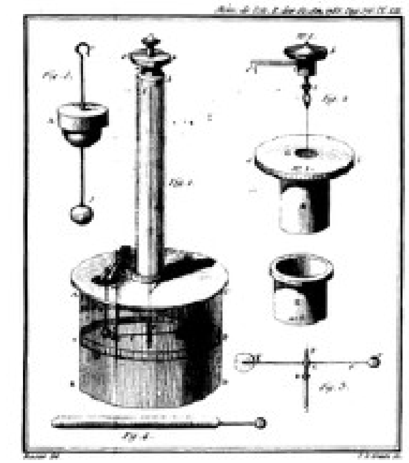
Wilson, K. G. Confinement of quarks.
Phys. Rev. D **10**, 2445–2459 (1974).

Electromagnetism and the Photon

1785 Coulomb

$$F \propto \frac{q_1 q_2}{r^2}$$

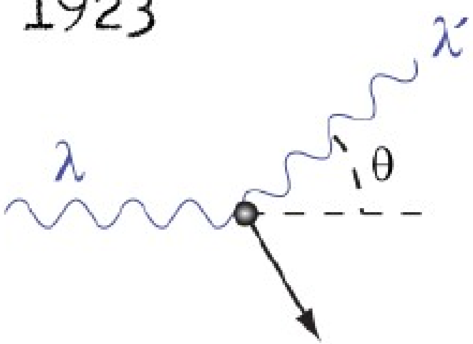




Light is made by particles

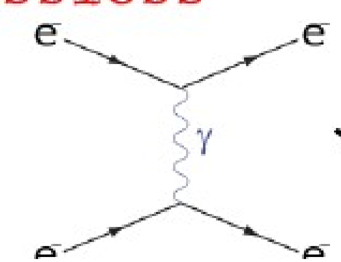
Einstein 1905

Compton 1923



Light is made by almost FREE particles

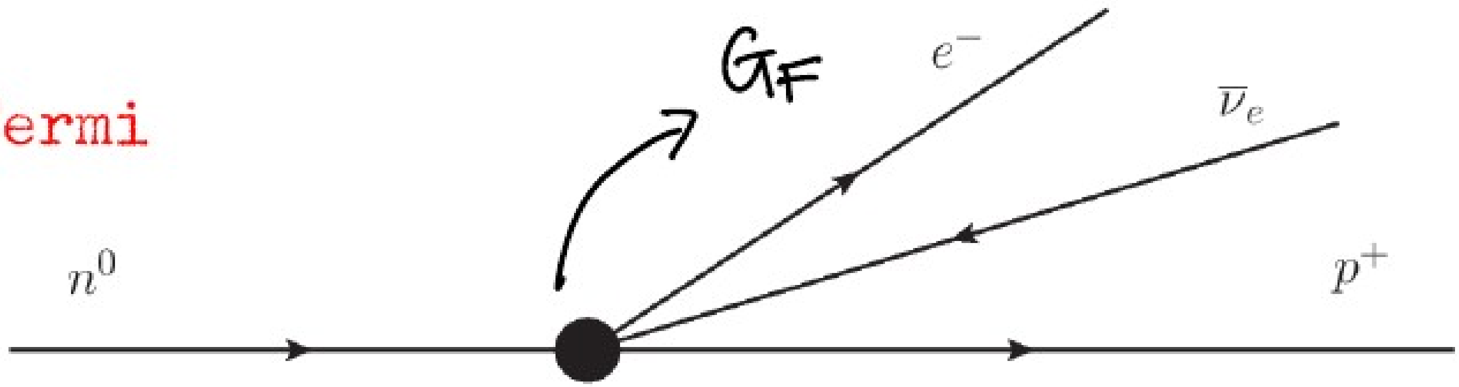
massless



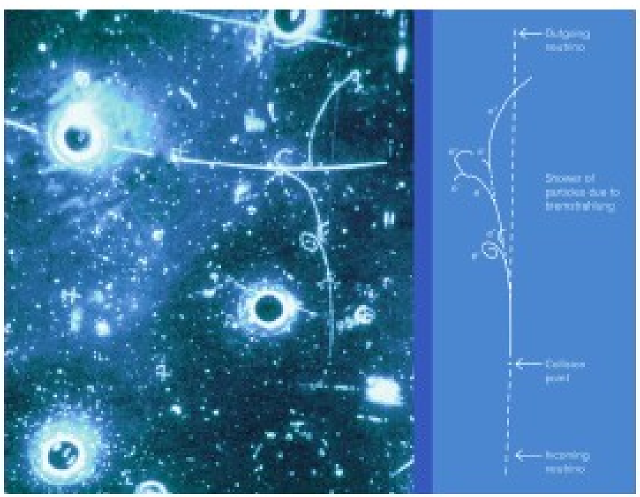
$$\frac{1}{\alpha} \sim 137$$

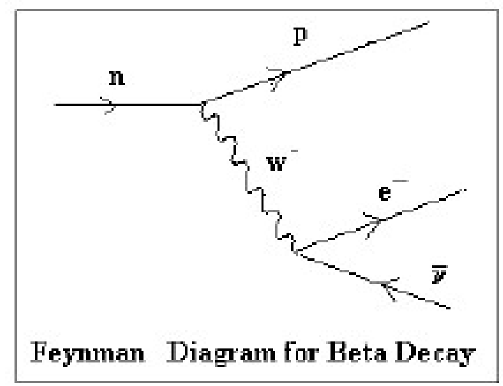
Beta Decay & The Fermi interaction

1933 Fermi



1973 Gargamelle

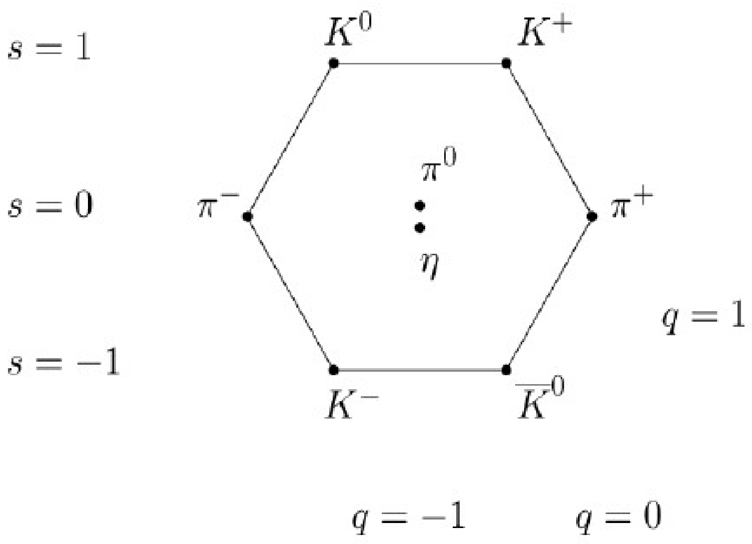


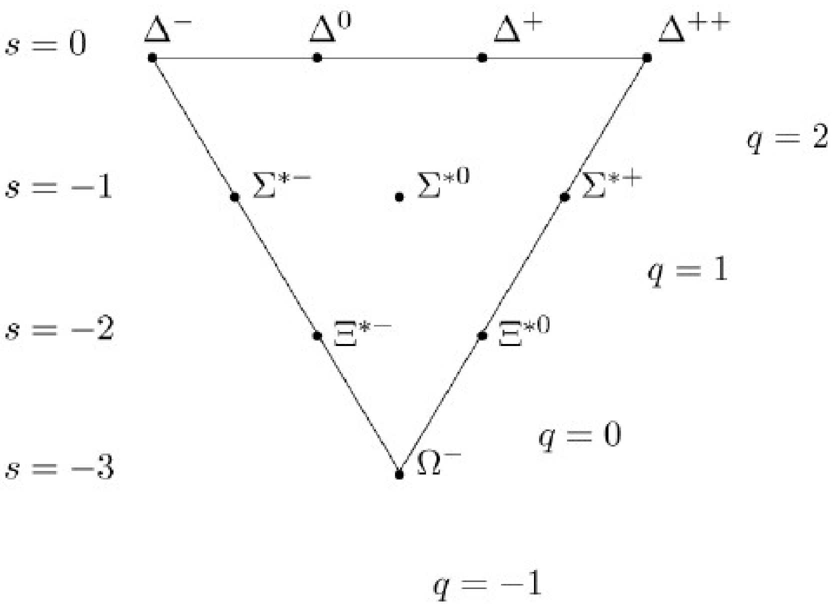


Salam, Glashow and Weinberg 1973

NON-ABELIAN GAUGE BOSONS

Quarks and Hadrons





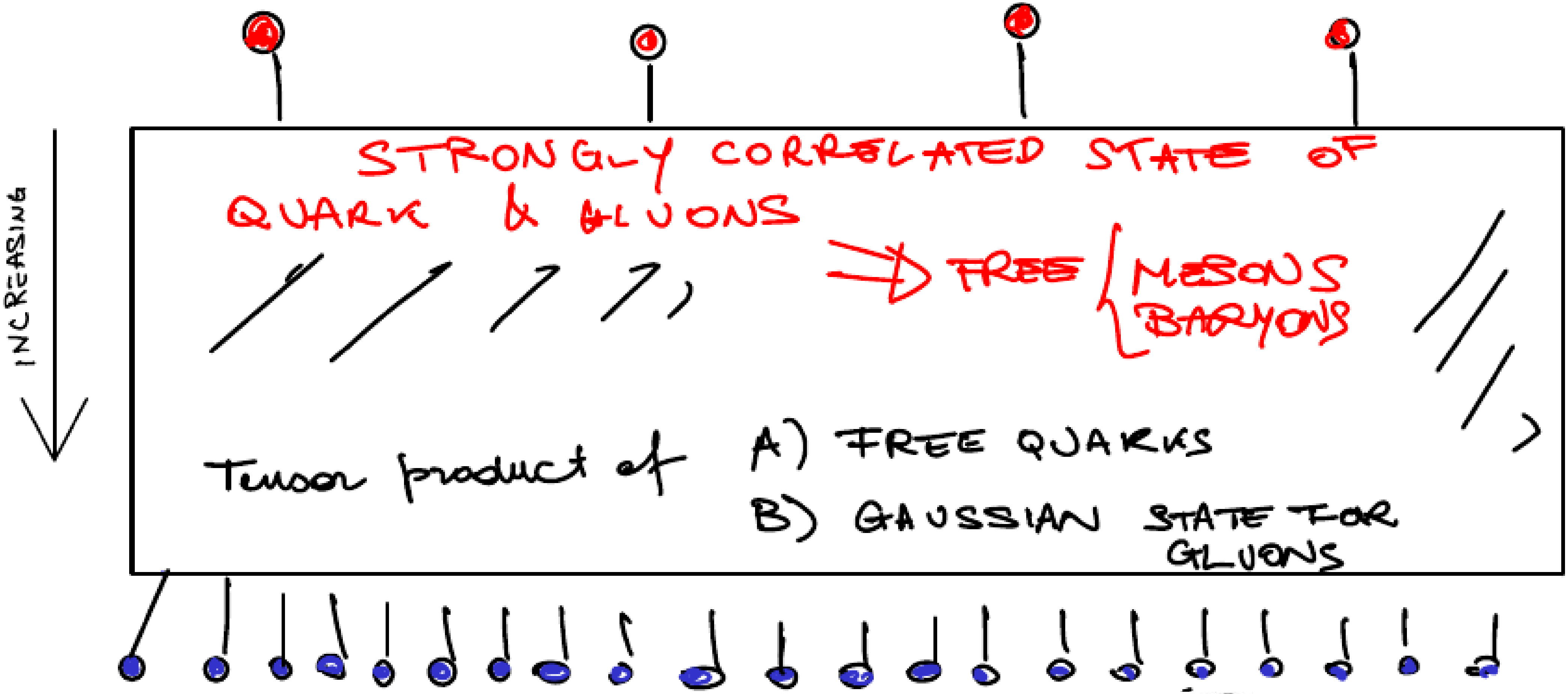
Mesons

Baryons

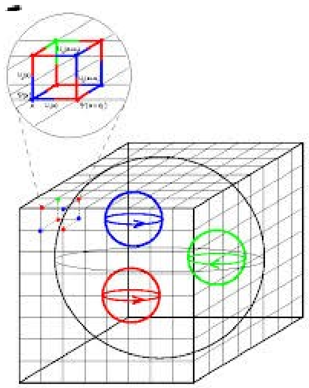
Despite partons in deep inelastic scattering

No experimental evidence of free quarks

Strongly coupled GAUGE THEORY



How do we compute?



Outline

Gauge theories in HEP

Lattice gauge theory

Motivation for TN and LGT

Symmetries and superposition

Exotic phases of matter

Intro to Tensor Networks

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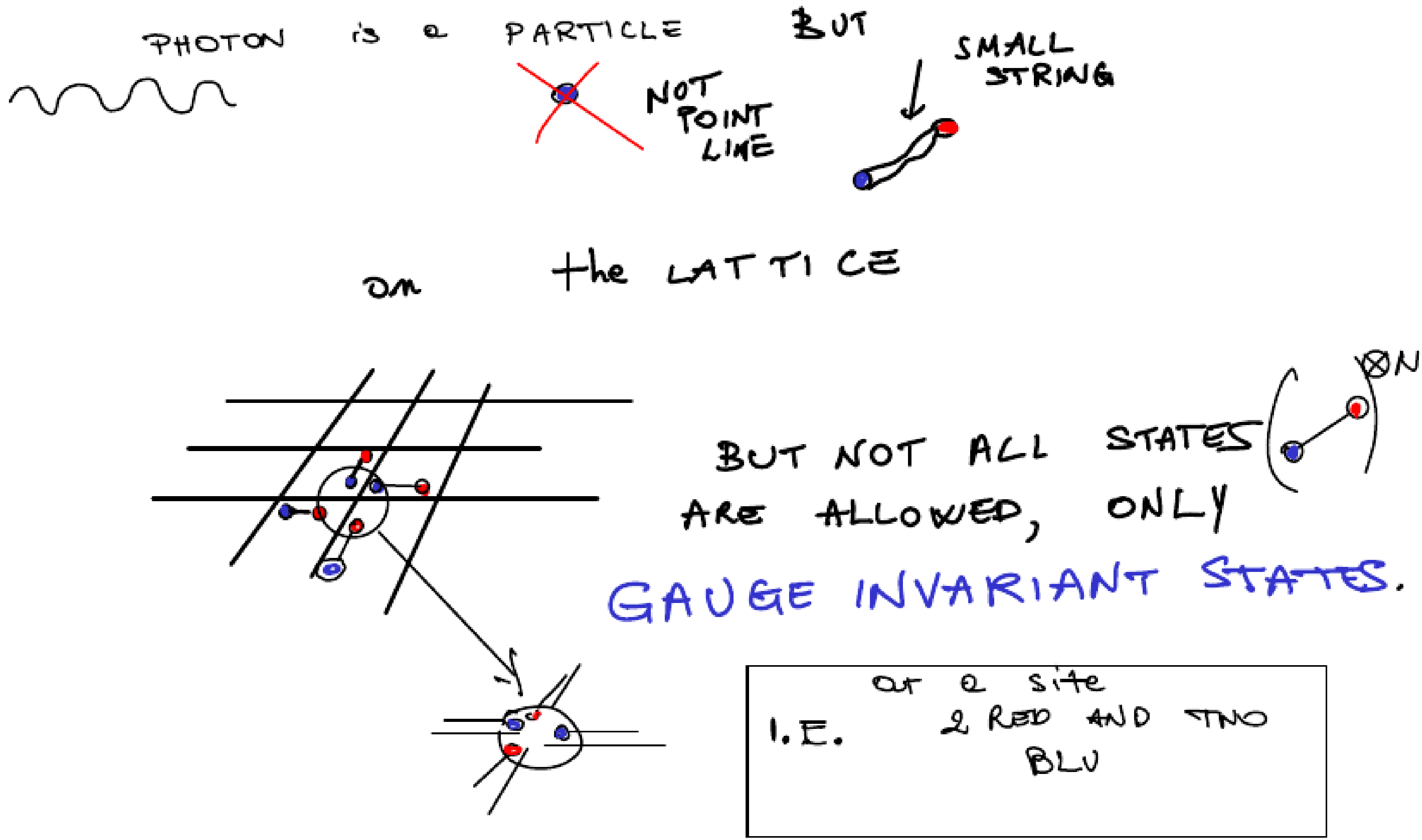
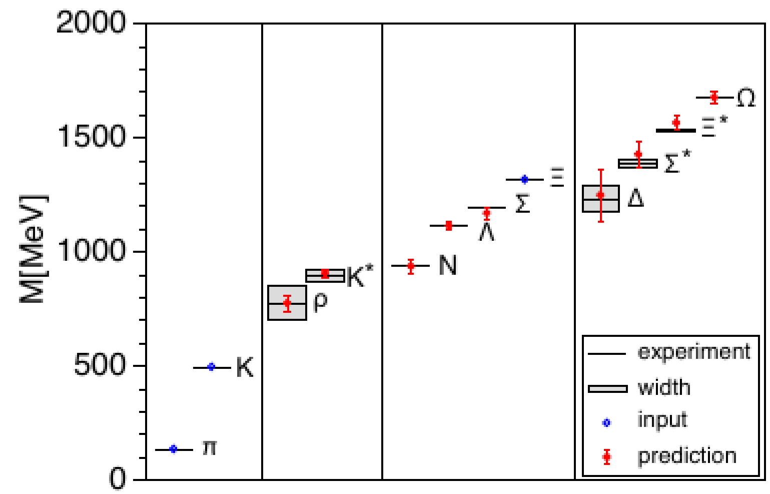
Lattice gauge theories



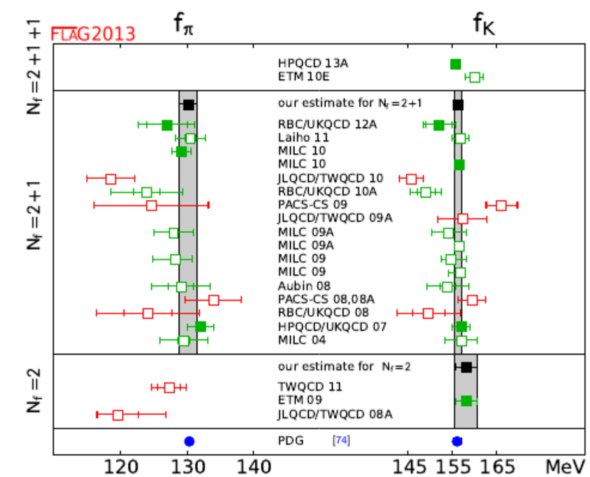
Achievements LGT

- Evidences of mass-gap in Yang Mills from first principles.



- Precise determination of the lowest excitations (agreement with experiments)

  Fodor, Z. & Hoelbling, C.
  Light Hadron Masses from Lattice QCD.
  Rev. Mod. Phys. 84, 449–495 (2012).

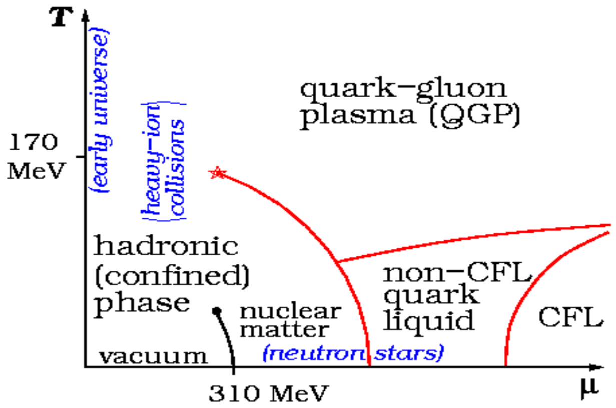


- Matrix elements input for phenomenology of Standard model

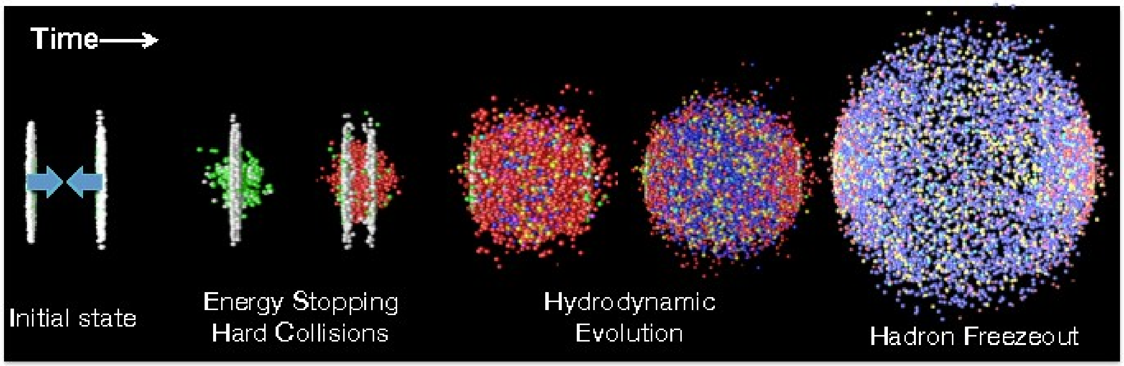
  Aoki, S. *et al.*
  Review of lattice results concerning
  low energy particle physics. *ArXiv:1310.8555*

Limitations LGT

- QCD at non-zero temperature and density (nuclear matter)?



- Real time dynamics (experiments at RICH and CERN)



- Classification of phases

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Achievements in TN/Quantum Many Body

- Study of frustrated and fermionic systems

Corboz, P., Evenbly, G., Verstraete, F. & Vidal, G.
Simulation of interacting fermions with entanglement renormalization.
Phys. Rev. A **81,** 010303 (2010).

- Out of equilibrium dynamics
- Vidal, G. Efficient Classical Simulation of Slightly Entangled Quantum Computations.

Phys. Rev. Lett. **91,** 147902 (2003).

- White, S. R. & Feiguin, A. E.

Real time evolution using the density matrix renormalization group.
Phys. Rev. Lett. **93,** (2004).

- Characterization of topological phases
  - Kitaev, A. & Preskill, J.

  Topological Entanglement Entropy. *Phys. Rev. Lett.* **96,** 110404 (2006).
  - Levin, M. & Wen, X.-G.

  Detecting Topological Order in a Ground State Wave Function.
  Phys. Rev. Lett. **96,** 110405 (2006).

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Symmetry and superpositon

- We can try to construct local H whose ground state has large superpositions
- One possibility is Hamiltonian with a symmetry

$$G = \prod_i \sigma_x^i$$

$$|\cdots \uparrow\uparrow \cdots\rangle \rightarrow |\cdots \downarrow\downarrow \cdots\rangle$$

$$H_x = \sum_i \sigma_x^i \qquad\qquad H_z = \sum_i \sigma_z^i \sigma_z^{i+1}$$

PRODUCT GROUND STATE

ENTANGLED GROUND STATE

Fate of large superpositions

- If there is a global discrete symmetry, it is spontaneously broken in the ground state (Absence of macroscopic cat states)

- If there is a local discrete symmetry the symmetry is not broken in the ground state (Presence of long range entanglement and short correlations)

- Phase transition without symmetry breaking....

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Simple LGT challenge UNIVERSALITY

Landau-Ginzburg 1950

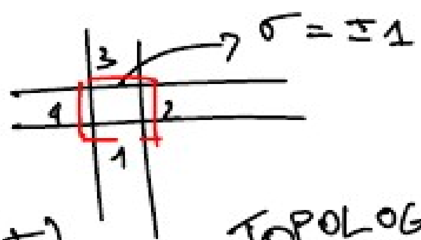
V.L. Ginzburg and L.D. Landau, Zh. Eksp. Teor. Fiz. 20, 1064 (1950).

Order parameter, based on universality

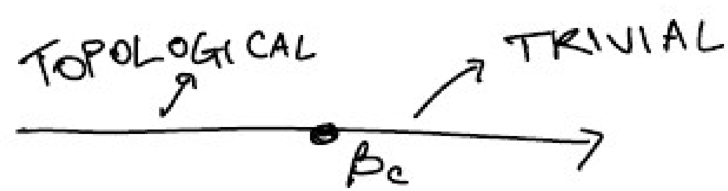
1) Symmetry (and its breaking pattern)
2) Dimensionality
3) Range of the interaction

F. Wegner,
Duality in Generalized Ising Models and Phase Transitions without Local Order Parameter,
J. Math. Phys. 12 (1971) 22592272

$$H = \sum_{\square} \sigma_1 \sigma_2 \sigma_3 \sigma_4$$



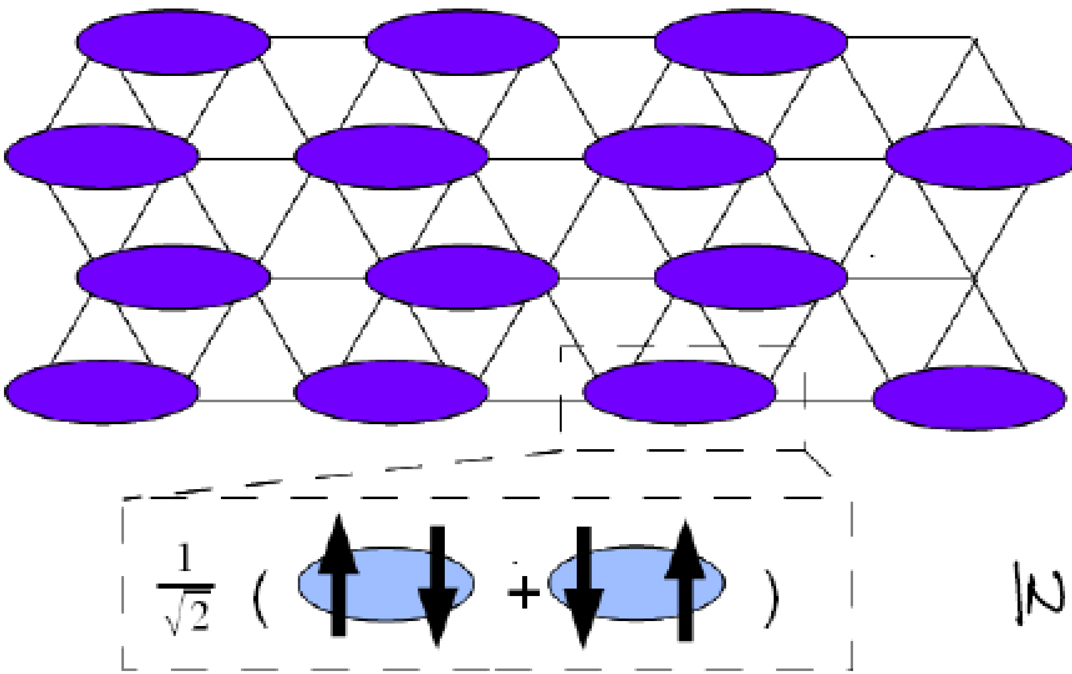
$$Z = \sum_{\{\sigma\}} e^{-\beta H(\sigma)}$$

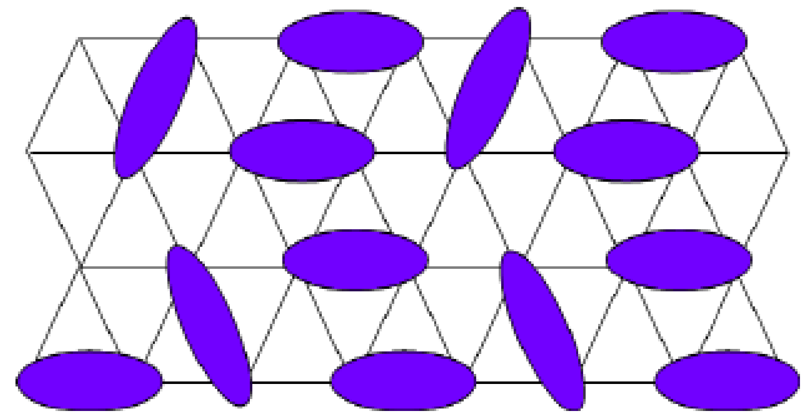


Resonating Valence Bond states

$$\mathcal{H} = J\, \vec{S}_i \cdot \vec{S}_j \qquad S = \frac{1}{2}$$

1973 Anderson





INSULATOR

↓

NO SYMMETRY BREAKING . - - -

TOPOLOGICAL STATE : SPIN LIQUID

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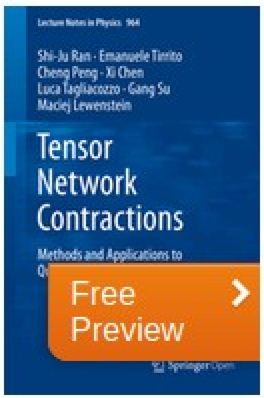
The book

» Physics » Theoretical, Mathematical & Computational Physics

Lecture Notes in Physics



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Tensor Network Contractions

Methods and Applications to Quantum Many-Body Systems

Authors: **Ran**, S.-J., **Tirrito**, E., **Peng**, C., **Chen**, X., **Tagliacozzo**, L., **Su**, G., **Lewenstein**, M.

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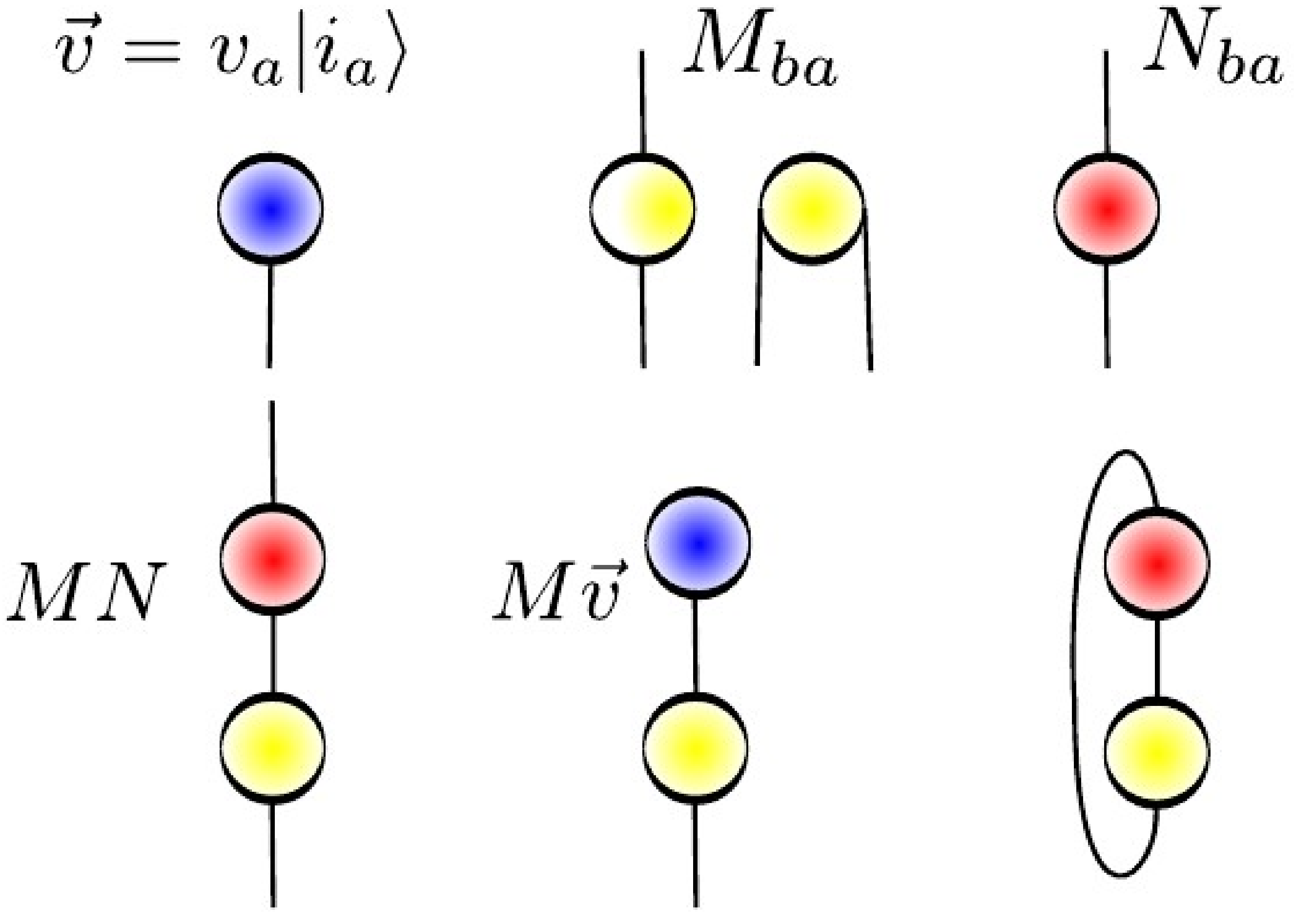
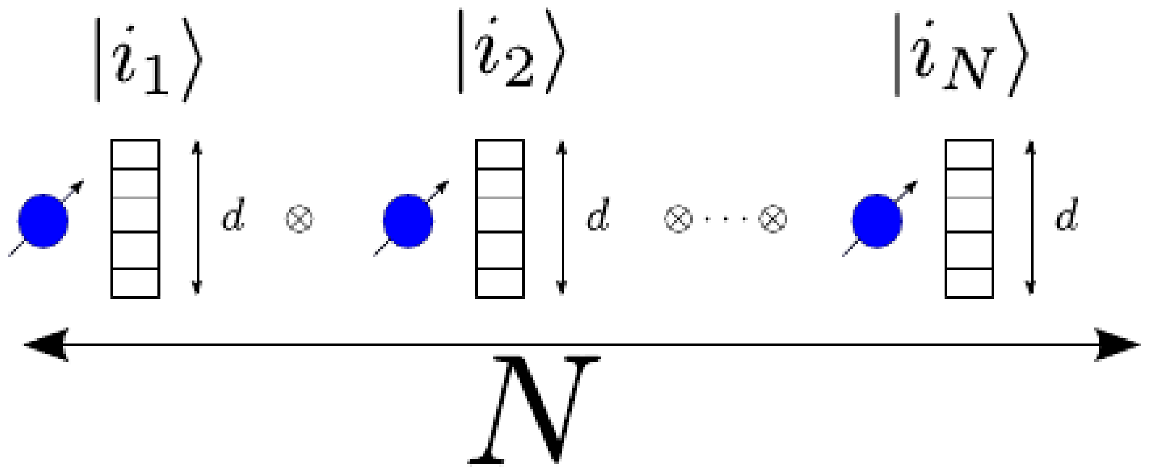
Notation



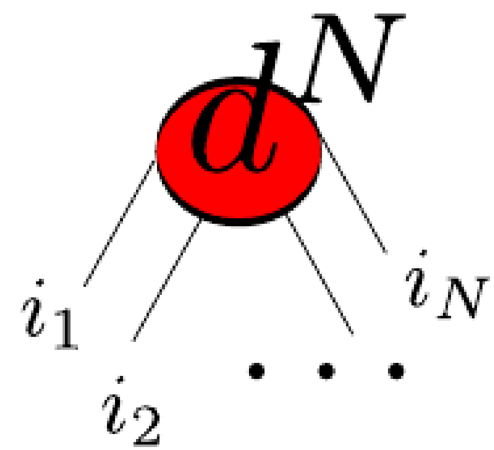
Quantum Many Body

Statement of the problem

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \cdots \otimes \mathcal{H}_N$$



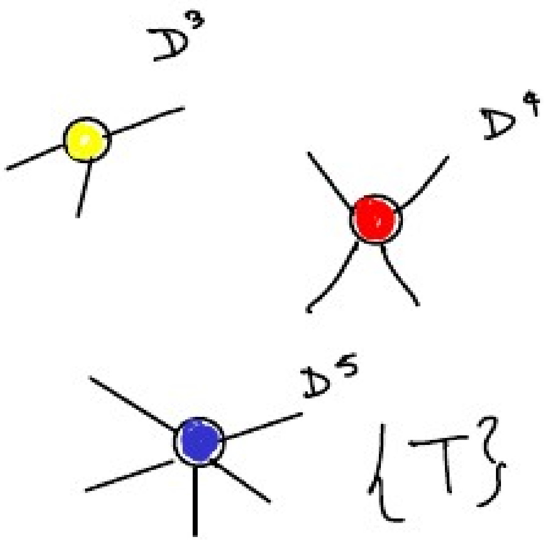
$$|\psi\rangle = c_{i_1 i_2 \cdots i_N} |i_1\rangle \otimes |i_2\rangle \cdots \otimes |i_N\rangle$$



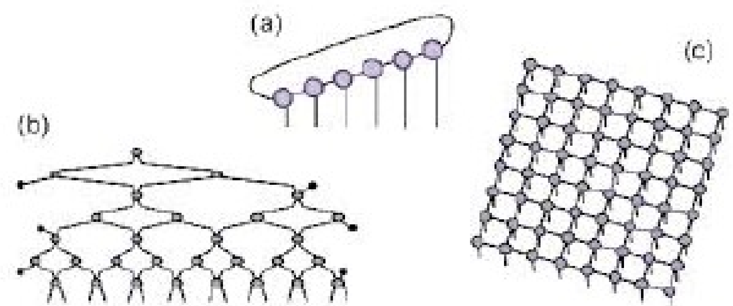
Tensor Networks = LEGO

POLYNOMIALLY FEW PARAMETERS $\Rightarrow$ EXPONENTIALLY MANY COEFFICIENTS

$\Rightarrow$



$\{T\} \Rightarrow$

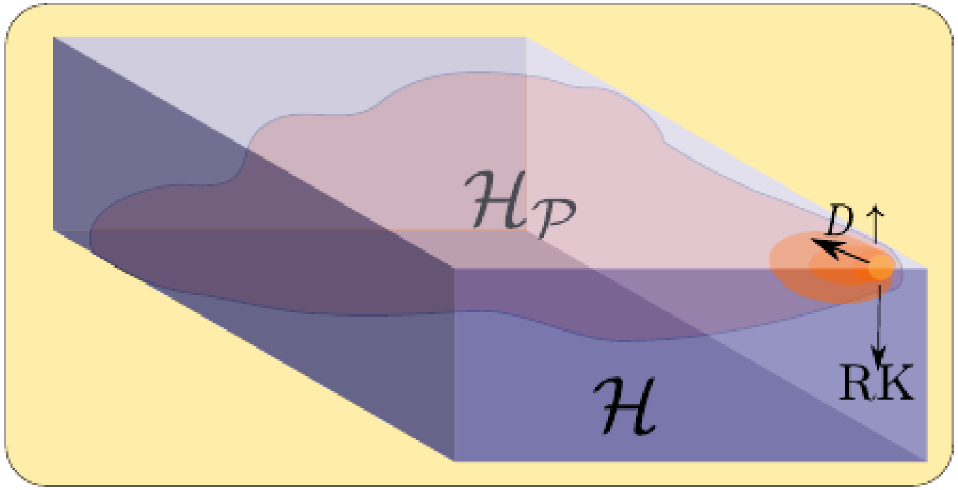


SMALL TENSORS

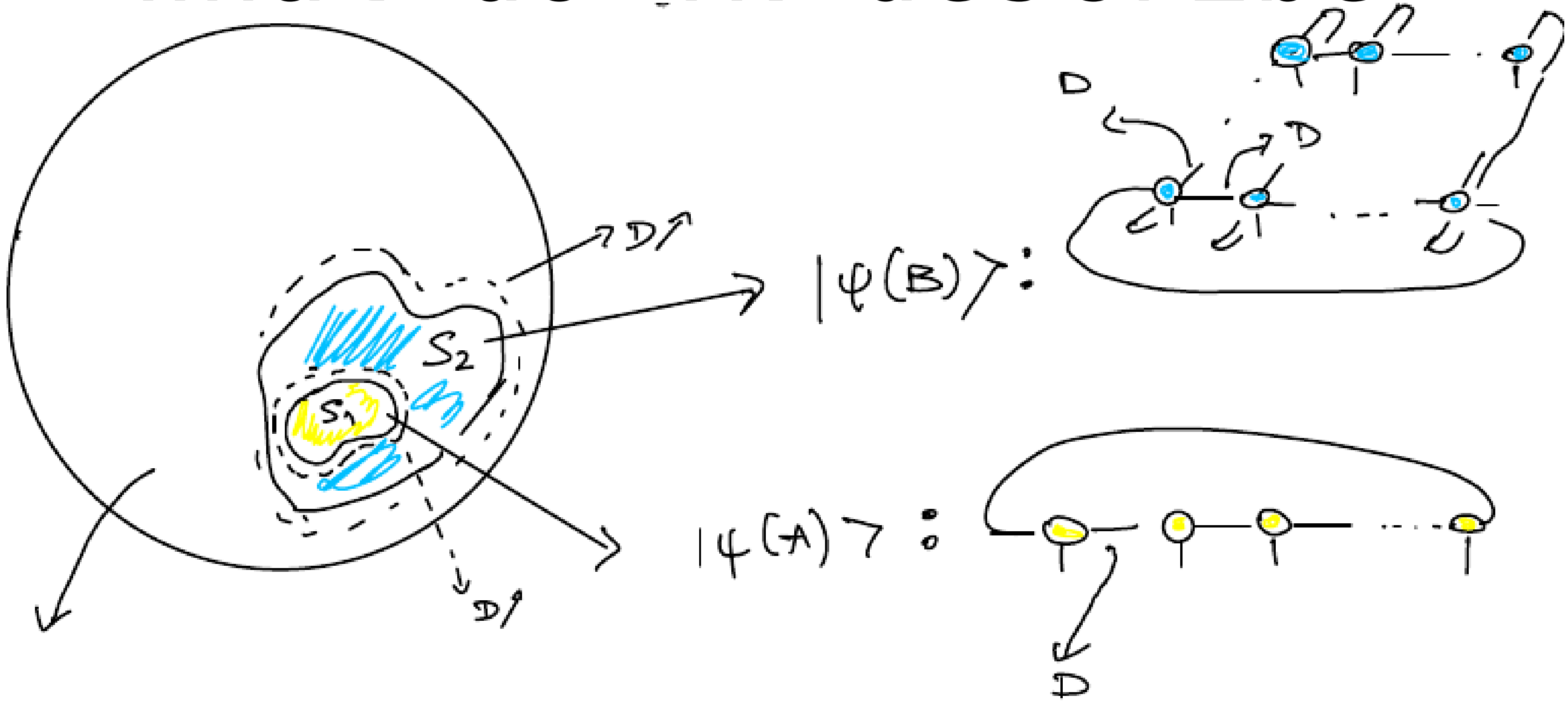
$|\psi\{T\}\rangle$

STATES OF LARGE SYSTEMS

Tensor Networks for LGT



What do TN describe



Full Hilbert space $\{|\psi\rangle\}$

S_1 : subspace of MPS

S_2 : " " PEPS

SPECIFIC STRUCTURE OF CORRELATIONS COMMONS TO LOCAL Hamilt.

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Constructing Z2 LGT

- Definition of a group
- Group algebra
- Building regular representation matrices
- Irreducible representations
- The local symmetry
- Interactions
- Hamiltonian
- Phases
- TN ansatz

Discussed by Kogut & Susskind, M. Creutz 70s

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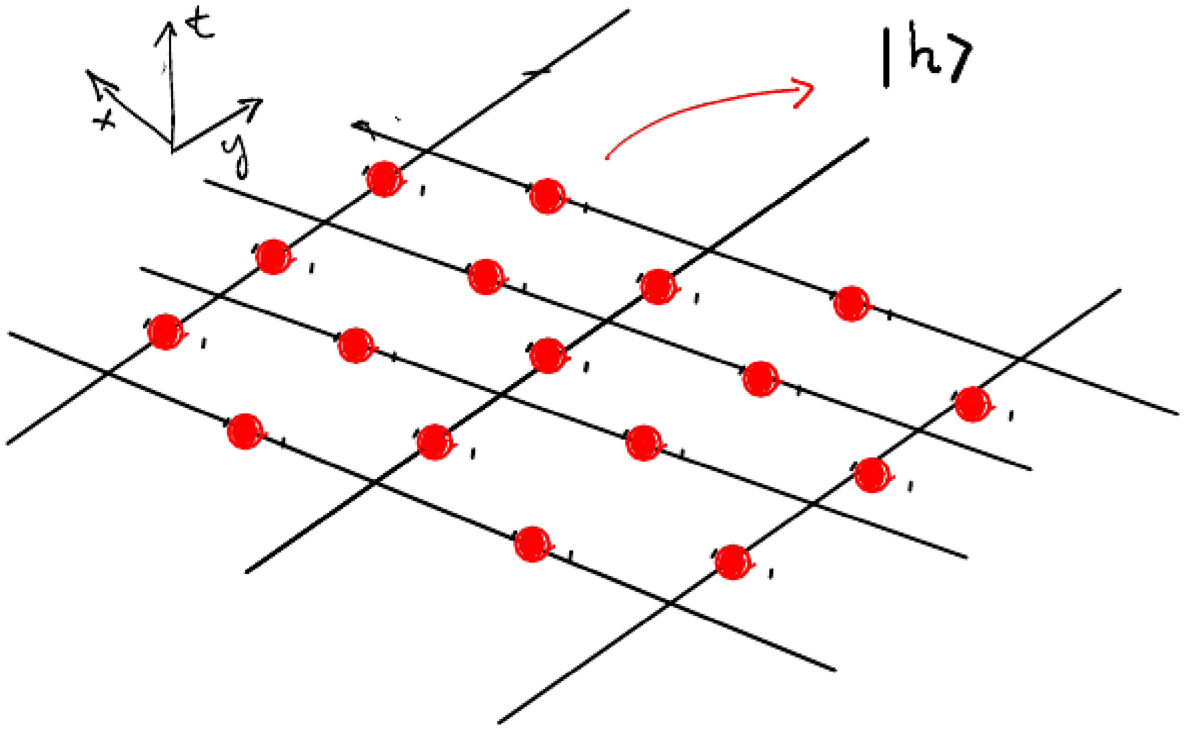
Hamiltonian LGT

$$h \in \mathcal{G} \to |h\rangle$$

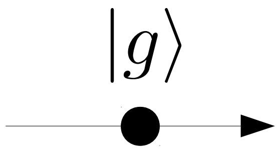


Discussed by Kogut & Susskind, M. Creutz 70s

Constructing a LGT

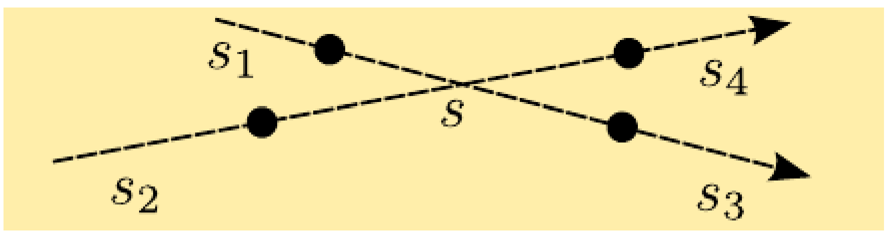
Notion of symmetry

$$h, g, k \in \mathcal{G}$$



- Constituents on links

- Local symmetry operators $A_s(h)|\psi\rangle = |\psi\rangle$



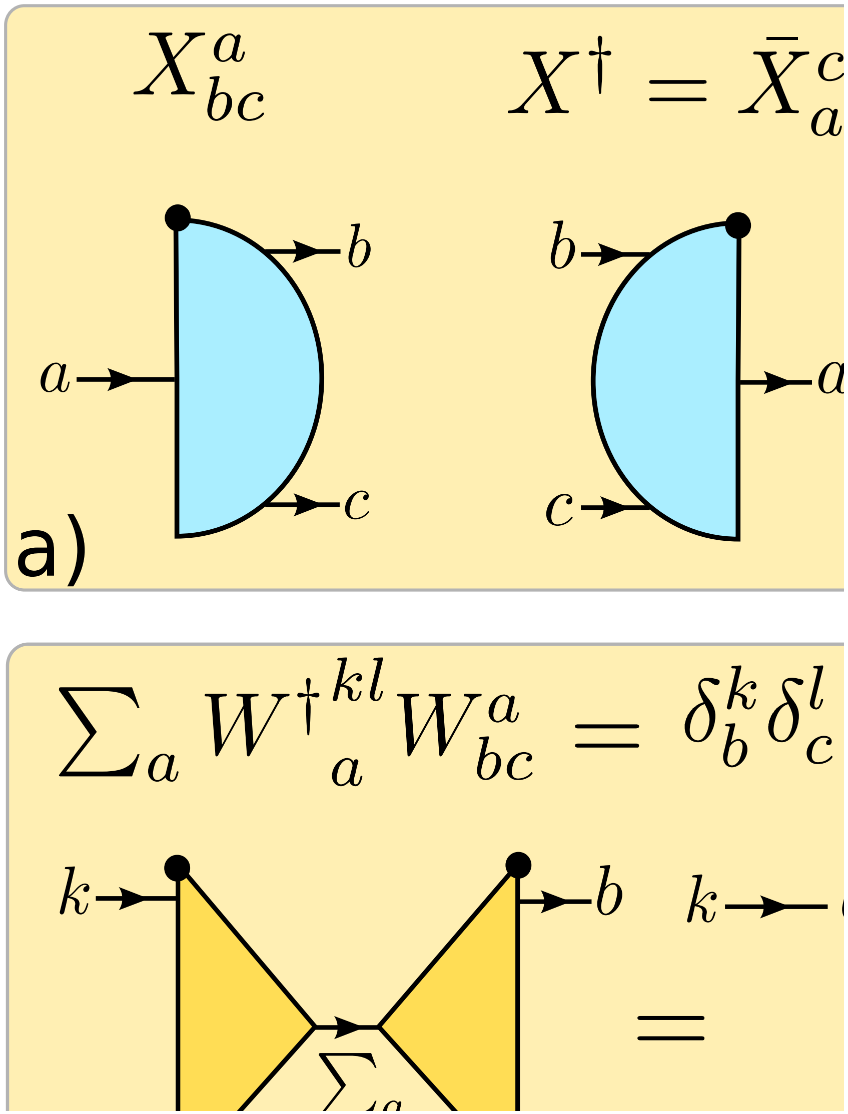
$$A_s(h) = R(h)_{s_1} \otimes R(h)_{s_2} \otimes L(h^{-1})_{s_3} \otimes L(h^{-1})_{s_4}$$

- Left right rotations of the state

$$L(h^{-1})R(k)\,|g\rangle \equiv |h^{-1}gk\rangle$$

Tagliacozzo, L., Celi, A. & Lewenstein, M.
TN for LGT with continuous groups.
ArXiv:1405.4811

Tensors



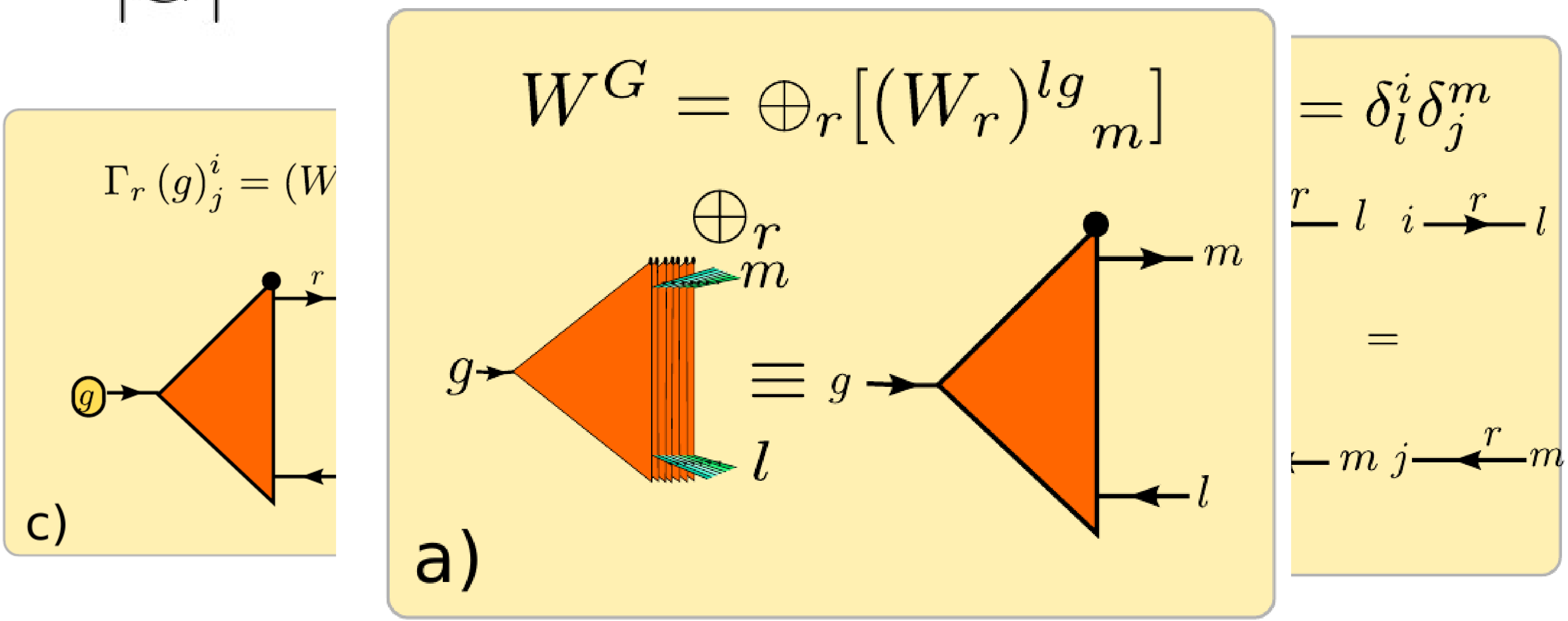
X^a_{bc}
$X^\dagger = \bar{X}^c_a$
a
b
c
a)
$\sum_a W^{\dagger}{}^{kl}_{a} W^a_{bc} = \delta^k_b \delta^l_c$
k
b
k
=

Orthogonality theorem

Serre, J.-P. *Linear representations of finite groups*. (Springer-Verlag, 1977).

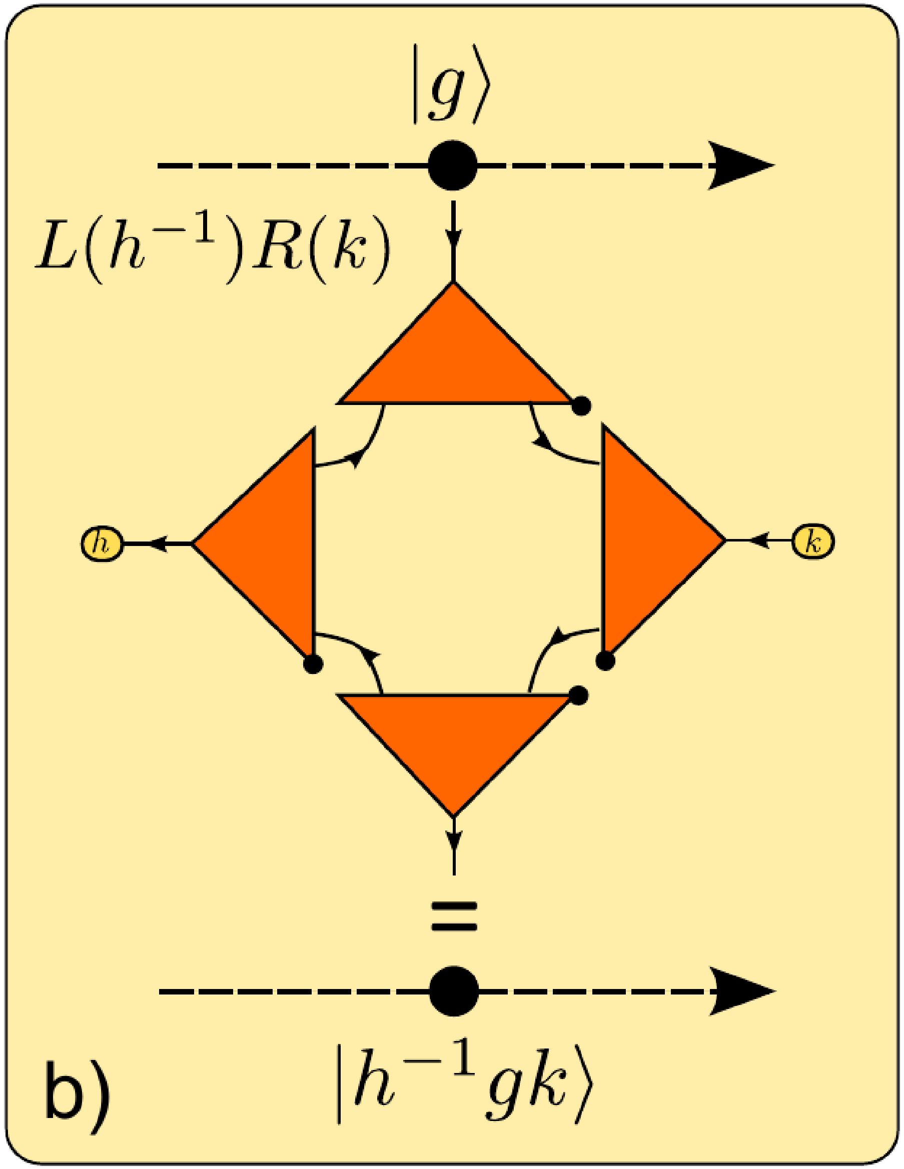
Matrix representation of g in irrep r: $\Gamma_r(g)$

$$\frac{\sqrt{n_r n_{r'}}}{|G|}\sum \Gamma_r(g^{-1})^i_j \Gamma_{r'}(g)^l_k = \delta^i_k \delta^j_l \delta(r,r')$$



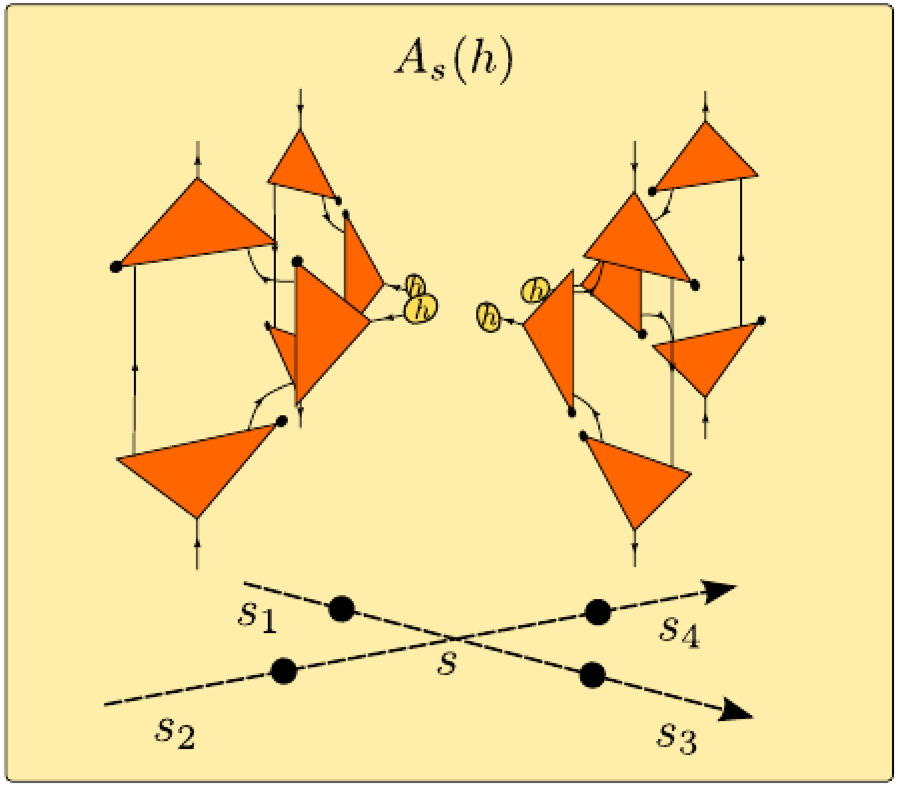
LR multiplication

$$L(h^{-1})R(k)\,|g\rangle \equiv |h^{-1}gk\rangle$$



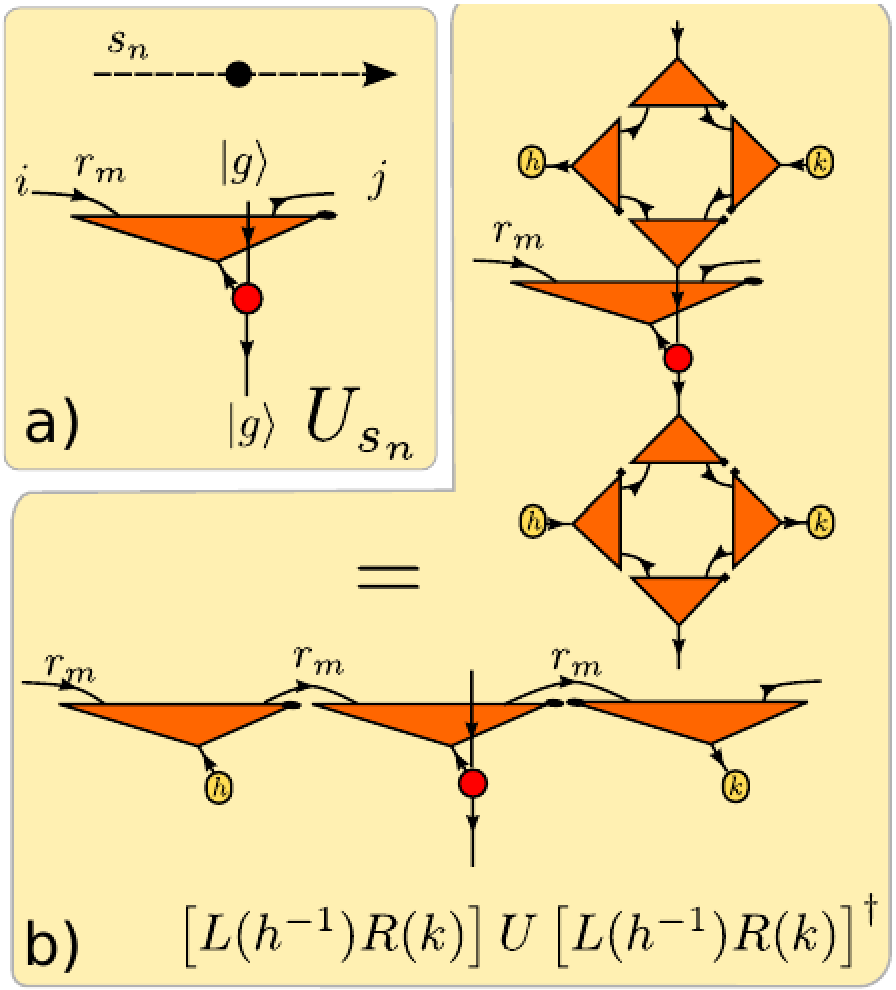
Generalized cross operators

$$A_s(h) = R(h)_{s_1} \otimes R(h)_{s_2} \otimes L(h^{-1})_{s_3} \otimes L(h^{-1})_{s_4}$$



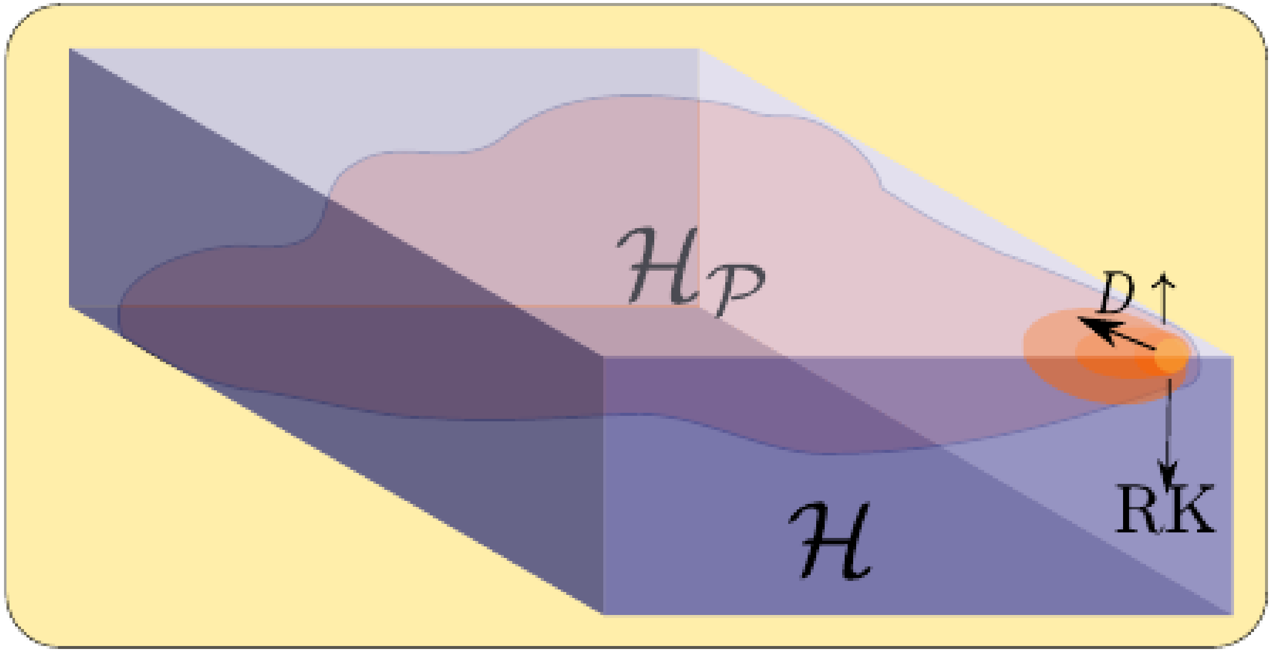
Generalized disentanglers

- U operators



Gauge invariant Hilbert space

$$\mathcal{H}_P = \{|\psi\rangle : A_s(h)|\psi\rangle = |\psi\rangle\, \forall s, h\}$$



Dynamic on Hp

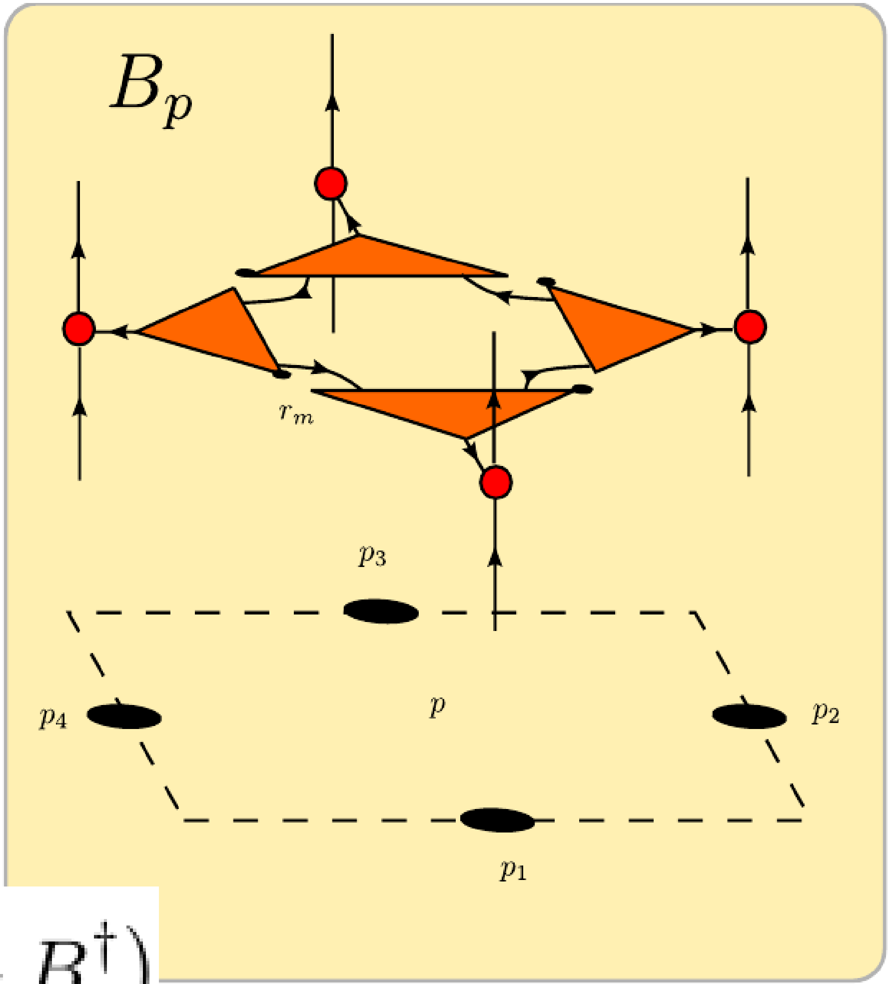
Kogut, J. & Susskind, L. *Phys. Rev. D* **11,** 395–408 (1975).
Creutz, M. *Phys. Rev. D* **15,** 1128 (1977).

$$H = E^2 + B^2$$

$$\mathcal{E}^2_{s_n} = \left[W_G^\dagger \oplus_r \left[c^r \, \mathbb{I}_r \otimes \mathbb{I}_{\bar{r}} \right] W_G^\dagger \right]_{s_n}$$

$$U_{s_n} = \sum_g |g\rangle \langle g|_{s_n} \otimes \Gamma_{r_m}(g)^i_j$$

$$B_p = \mathrm{tr}_{r_m} \left(U_{p_1} U_{p_2} U^\dagger_{p_3} U^\dagger_{p_4} \right)$$



$$H_{LGT} = \sum_l \mathcal{E}^2_l + \frac{1}{\alpha^2} \sum_p \left(B_p + B_p^\dagger \right)$$

Outline

Gauge theories in HEP

Lattice gauge theory

Motivation for TN and LGT

Symmetries and superposition

Exotic phases of matter

Intro to Tensor Networks

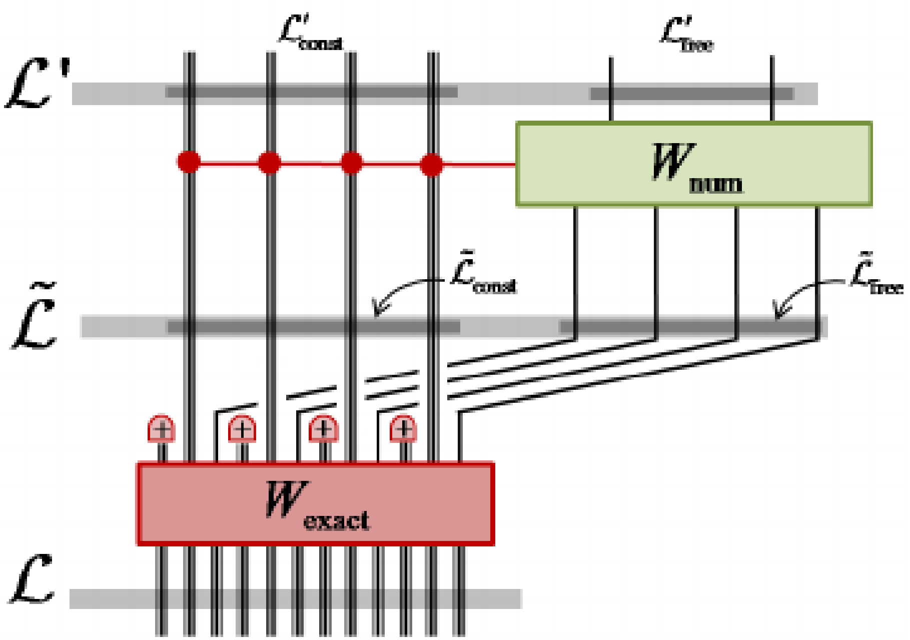
Intro to LGT (Z2) (Blackboard)

Tensor Networks for Gauge theories (Z2)

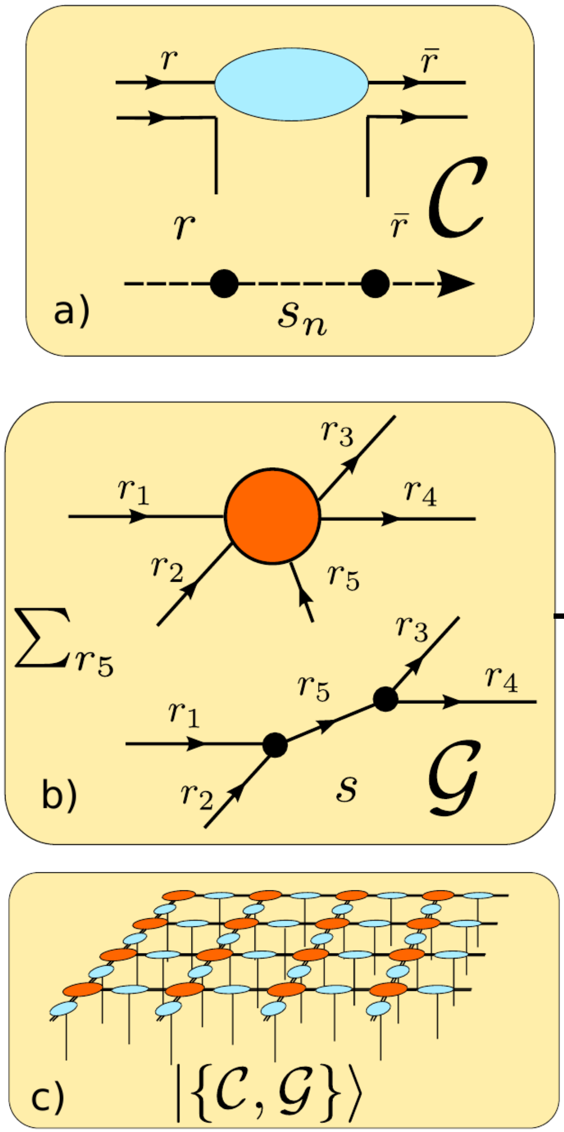
Generalization

Example of results (2D MERA + PEPS)

The two ways



MERA,Hierarchical TN

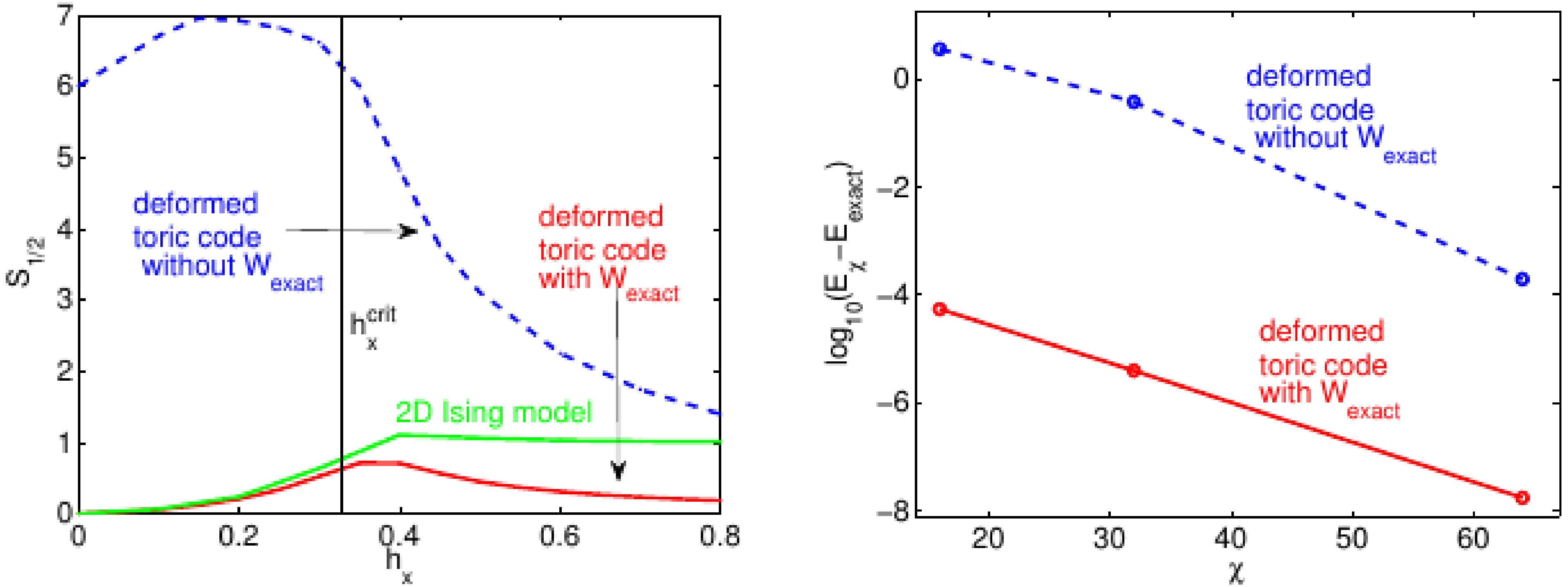


TPS/PEPS

Tagliacozzo, L. & Vidal, G.
Phys. Rev. B **83,** 115127 (2011)

Tagliacozzo, L., Celi, A. & Lewenstein, M.
ArXiv:1405.4811

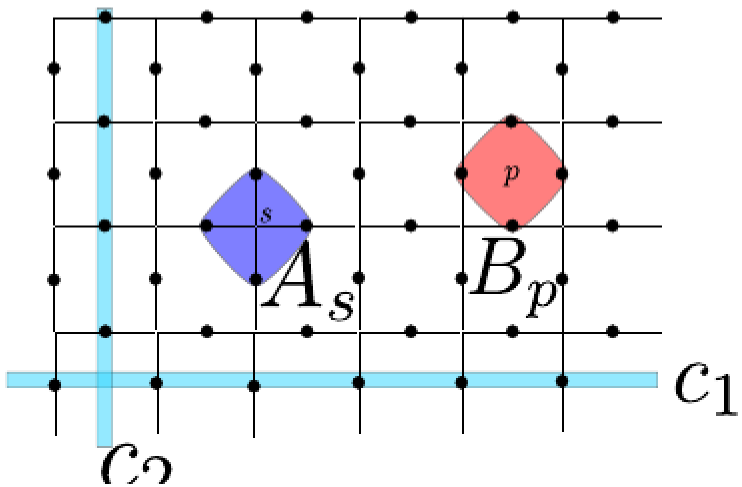
Variational Ansatz for gauge invariant states

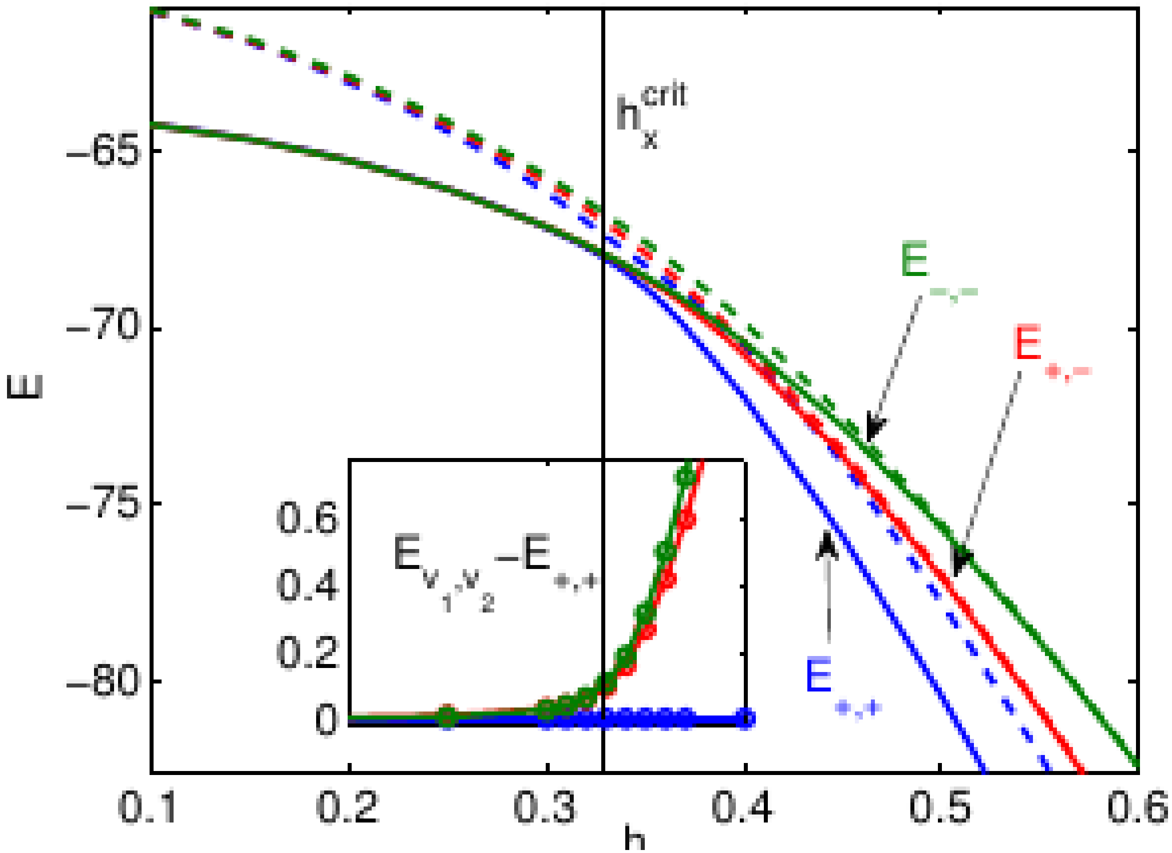


Phys. Rev. B **83,** 115127 (2011)

Low energy spectrum MERA

Z2 LGT 8x8 torus



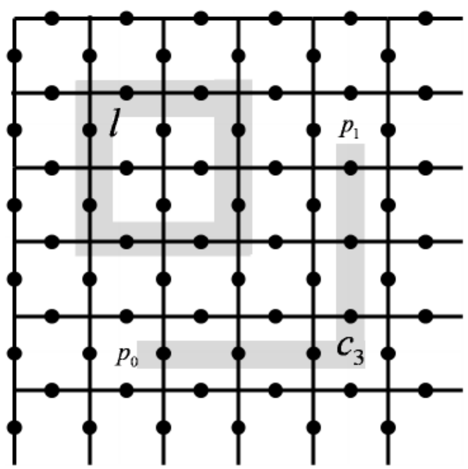


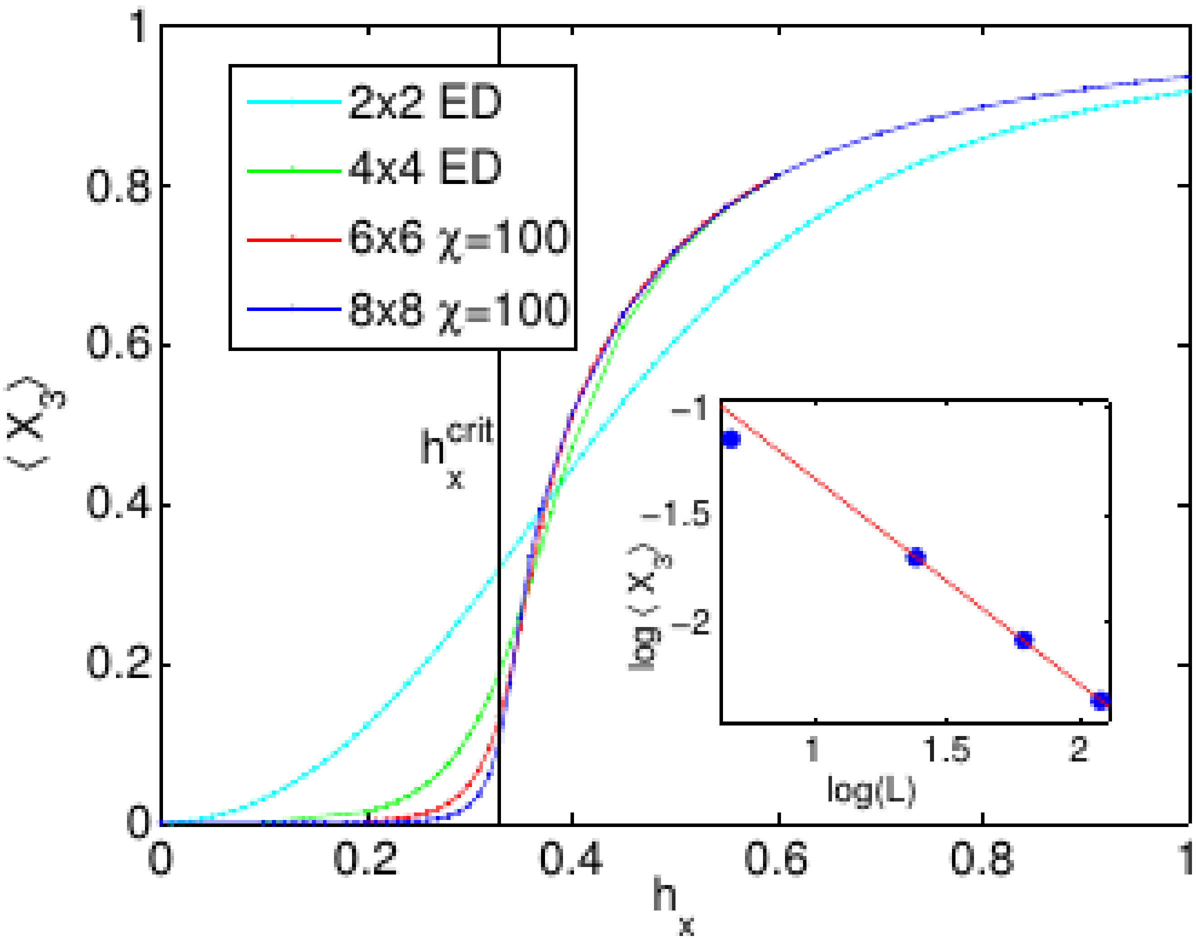
Phys. Rev. B **83,** 115127 (2011)

Disorder parameter MERA

Z2 LGT 8x8 torus

$$X_3 \equiv \prod_{j \in c_3} \sigma_j^x ,$$

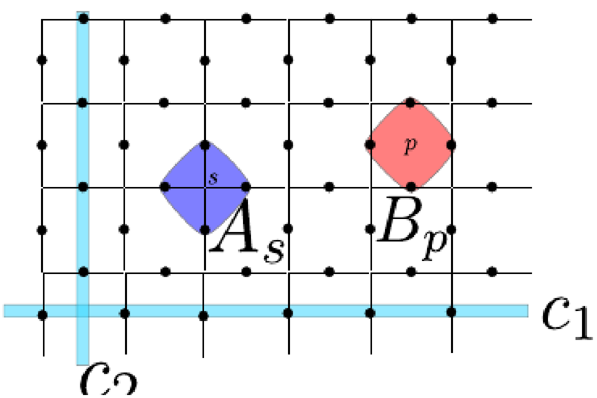


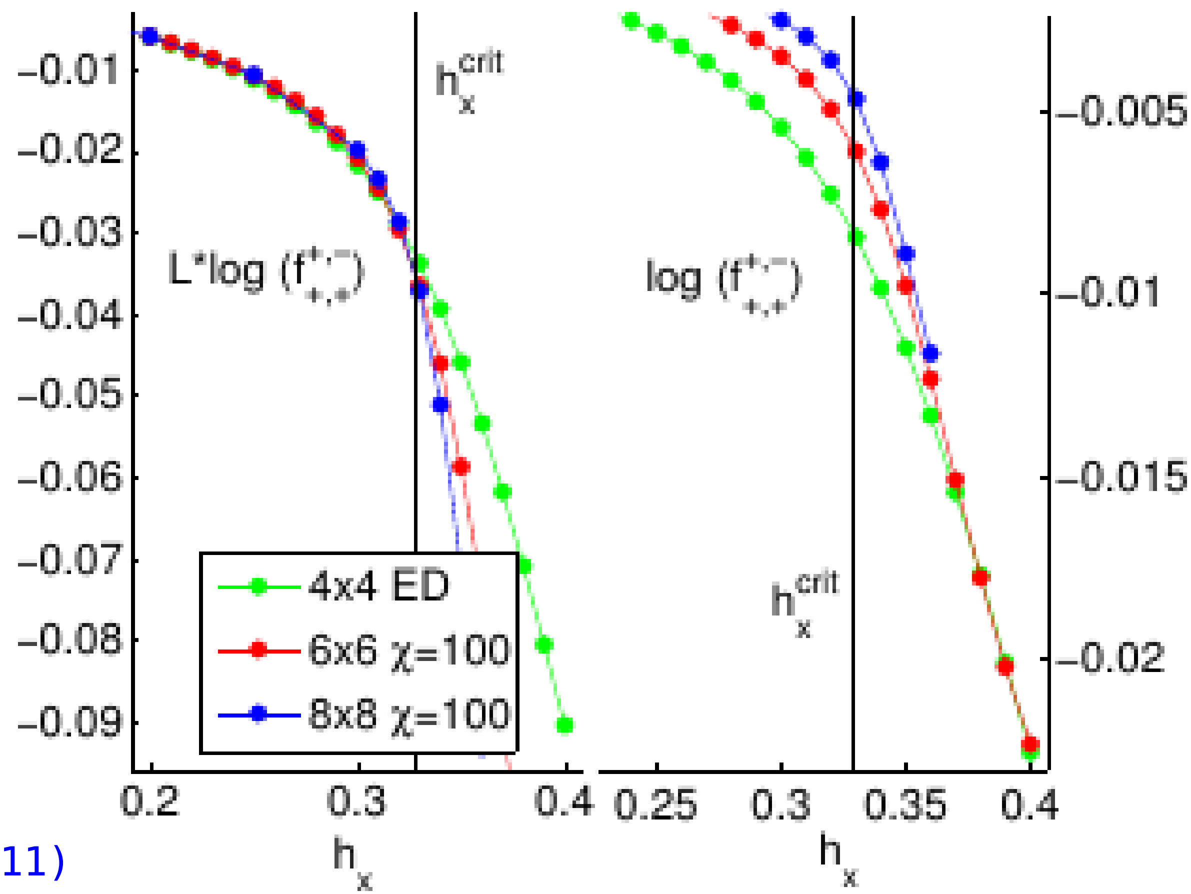


Phys. Rev. B **83,** 115127 (2011)

Topological fidelities MERA

$$\log(f_{+,+}^{+,-}) \equiv \frac{1}{L^2}\log\left(\langle\Phi_{+,+}|Z_2|\Phi_{+,-}\rangle\right)$$





Phys. Rev. B **83,** 115127 (2011)

Topological QPT with TPS

From the ground state of

$$H_{TC} = \sum_s A_s + \sum_p B_p$$

to the ground state of

$$H_{GM} = \sum_p \Big[\big(a_{p_1} a_{p_2} a^\dagger_{p_3} a^\dagger_{p_4} + H.c. \big) - \big(a_{p_1} a_{p_2} a^\dagger_{p_3} a^\dagger_{p_4} + H.c. \big)^2 \Big]$$

Through a wave function modification

$$|\psi(\theta)\rangle$$

$$|\psi(0)\rangle = |\Omega_{TC}\rangle \longrightarrow |\psi(\pi/2)\rangle = |\Omega_{GM}\rangle$$

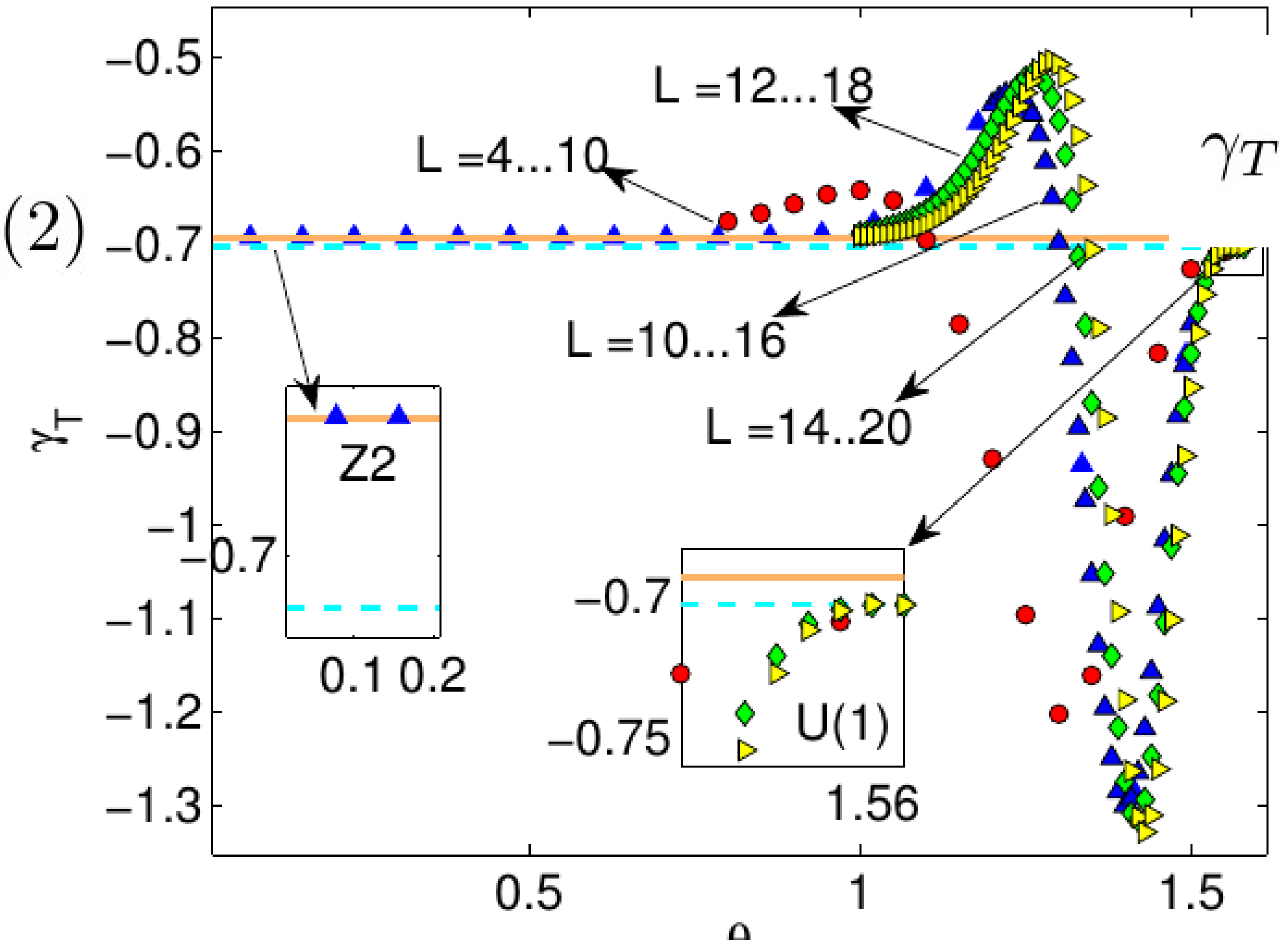
$$H(\theta) = H_{TC} + \mathcal{V}(\theta)$$

$$S = c_1 L + \gamma_T + c_2/L + \ldots$$

ArXiv:1405.4811

Topological Entropy

$\gamma_T = -\log(2)$



$\gamma_T = \log(R) - \frac{1}{2}$

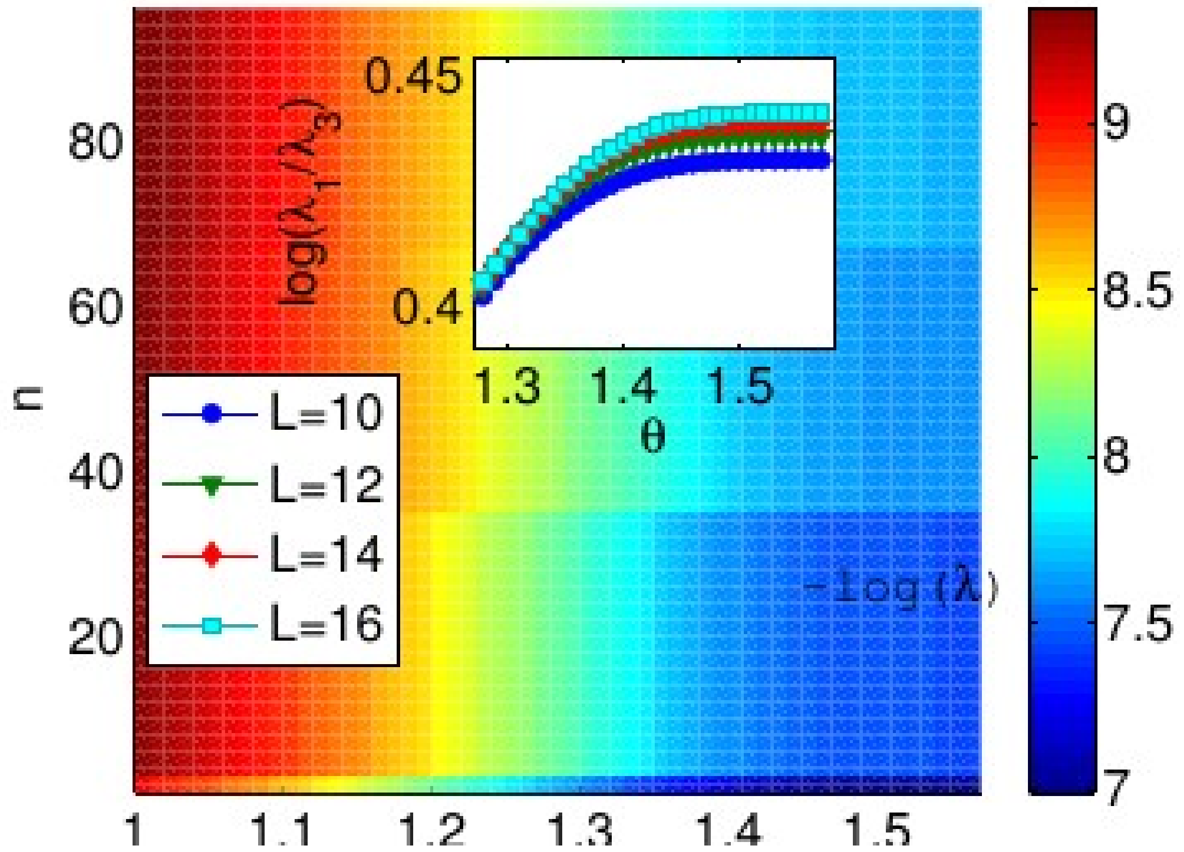
Stéphan et. al. *Phys. Rev. B* **80,** 184421 (2009).
Stéphan et. al. *J. Stat* **2012,** P02003 (2012).

ArXiv:1405.4811

Schmidt-gap

Li, H. & Haldane, F. D. M. *Phys. Rev. Lett.* **101,** 010504 (2008).
De Chiara et. al *Phys. Rev. Lett.* **109,** (2012).
A. Läuchli, arXiv:1303.0741

$$\rho_A = \sum_\alpha \lambda_\alpha |\alpha\rangle\langle\alpha|$$



Does not detect the topological phase transition

ArXiv:1405.4811

Luitz, D. et al. *J. Stat.* **2014,** P08007 (2014).

Conclusions

- I have justified the need of TN framework to analyze LGT
- It is suited both for theoretical analysis and to design numerical ansatz
- Discrete, continuous Abelian and Non-Abelian model can be considered
- Both hierarchical TN and TPS/PEPS
- Already have benchmark numerical results in 2D
- Easily extended to include matter
- Interesting time to come...

THANKS FOR THE ATTENTION !!!