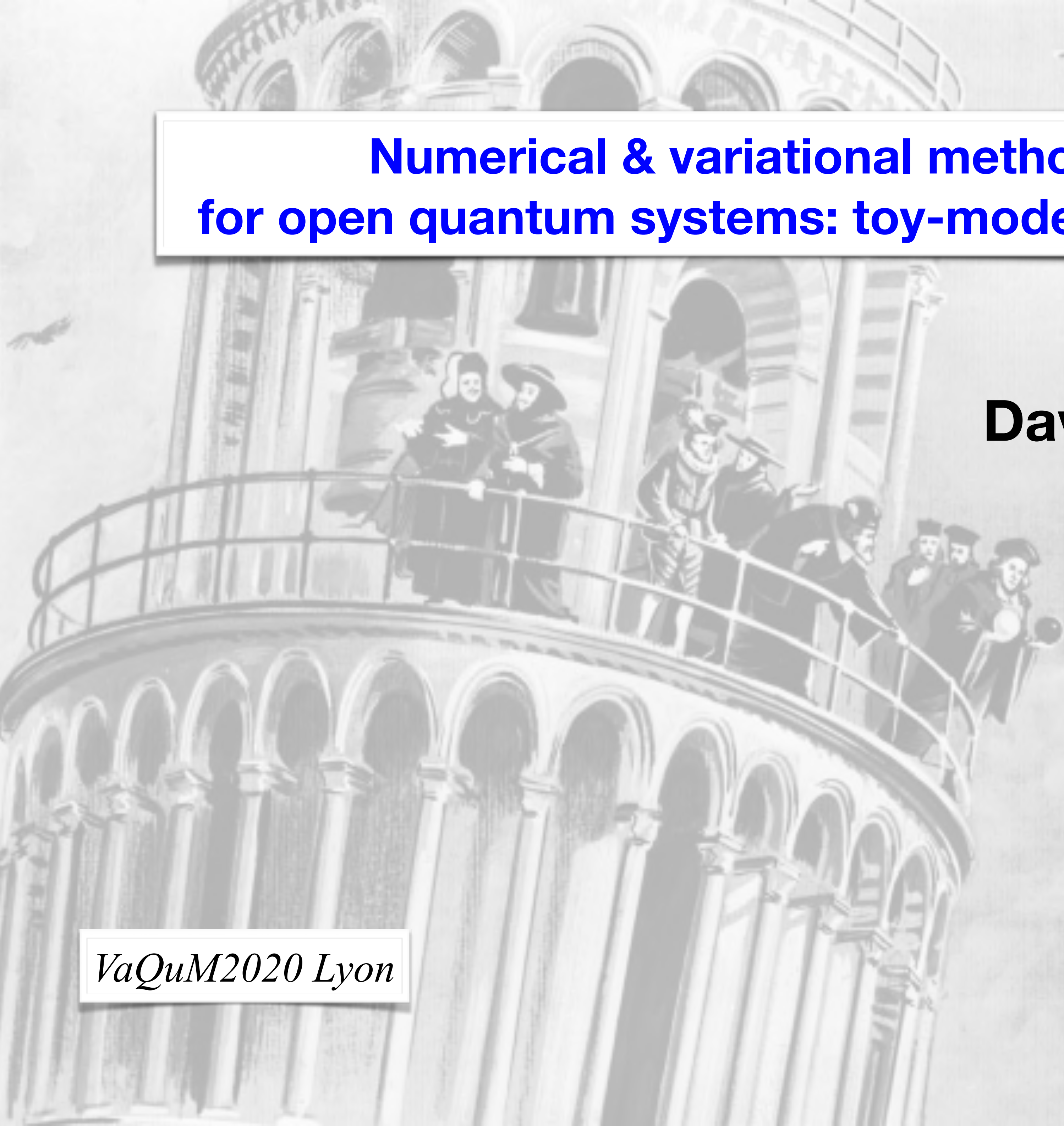


Numerical & variational methods for open quantum systems: toy-model example

Davide Rossini

IN SUPREMÆ DIGNITATIS 1343

UNIVERSITÀ DI PISA

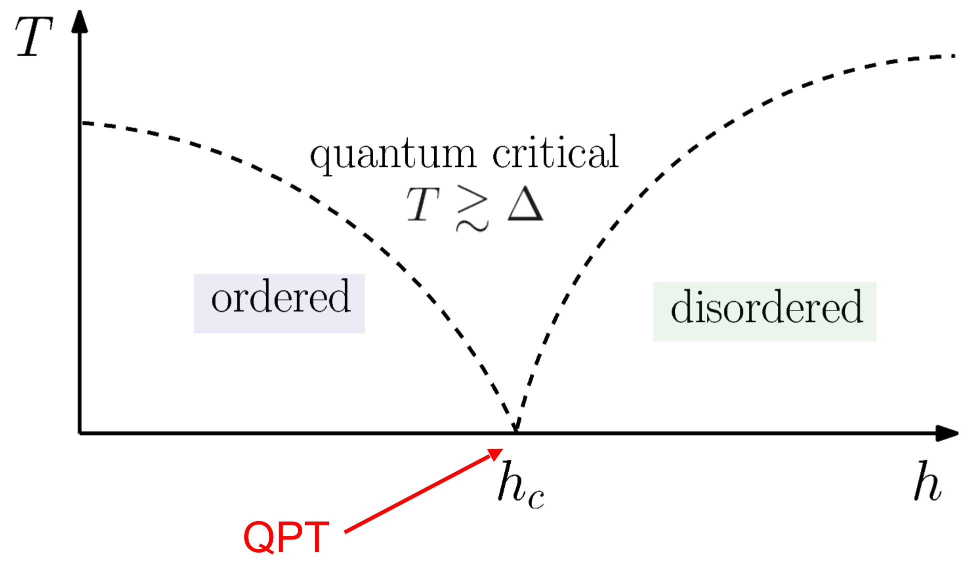
INFN
Istituto Nazionale
Fisica Nucleare
Sezione di Pisa

VaQuM2020 Lyon

Quantum phase transitions

- Ruled by **quantum fluctuations** at $T=0$
- Discontinuous thermodynamic properties
$\longrightarrow$ different symmetries
$\longrightarrow$ order/disorder transition
- Energy-gap closure $\Delta \sim |\lambda|^{z\nu}$
- Diverging length scale
$\longrightarrow$ ***long-range correlations*** $\xi \sim |\lambda|^{-\nu}$ where $\lambda = h - h_c$

Ginzburg-Landau:
minimize ground-state energy
$\longrightarrow$ free-energy analysis



S. Sachdev (1998)
"*Quantum phase transitions*"
Cambridge Univ. Press

Quantum phase transitions in a dissipative framework?

Criticality: a different paradigm?

→ the system may want to reach a steady state ($t \to \infty$)

→ ordering, if exists, has a *dynamical origin*

→ **short-range correlations** could be important

A spin-system toy model

We consider a spin-1/2 XYZ anisotropic Heisenberg model

$$H = \sum_{\langle i,j \rangle} (J_x \sigma_i^x \sigma_j^y + J_y \sigma_i^y \sigma_j^y + J_z \sigma_i^z \sigma_j^z)$$

in the presence of incoherent dissipative spin-flips (*z axis*)

$$\partial_t \rho = -i[H, \rho] + \gamma \sum_i \left(\sigma_i^- \rho \sigma_i^+ - \tfrac{1}{2} \{ \sigma_i^+ \sigma_i^-, \rho \} \right)$$

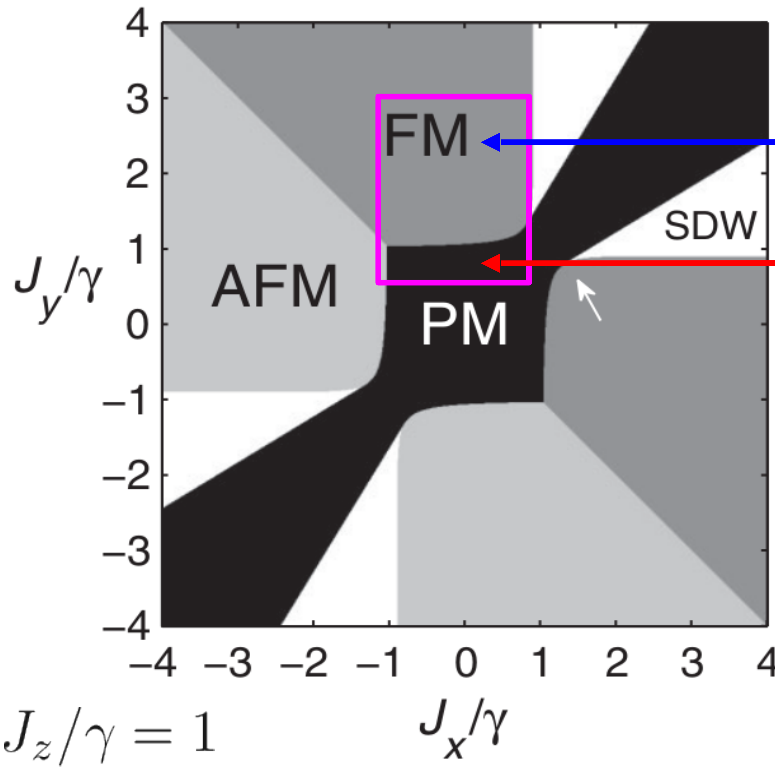
- *x-y* Hamiltonian anisotropy generates a **competition** between coherent dynamics & dissipative effects
- experimentally implementable with trapped ions
- The master equation has a $\mathbb{Z}_2$ symmetry $\sigma_j^x, \sigma_j^y \to -\sigma_j^x, -\sigma_j^y$ which may **spontaneously break** in *ordered phases*

(*e.g.*: the paramagnetic / ferromagnetic transition in the *Ising model*)

The master equation has a $\mathbb{Z}_2$ symmetry $\sigma_j^x, \sigma_j^y \rightarrow -\sigma_j^x, -\sigma_j^y$

which may **spontaneously break** in *ordered phases*

(*e.g.*: paramagnetic / ferromagnetic transition in the *Ising model*)



$J_z/\gamma = 1$

We study the paramagnet (PM) / ferromagnet (FM)

steady-state phase transition

Single-site mean-field phase diagram

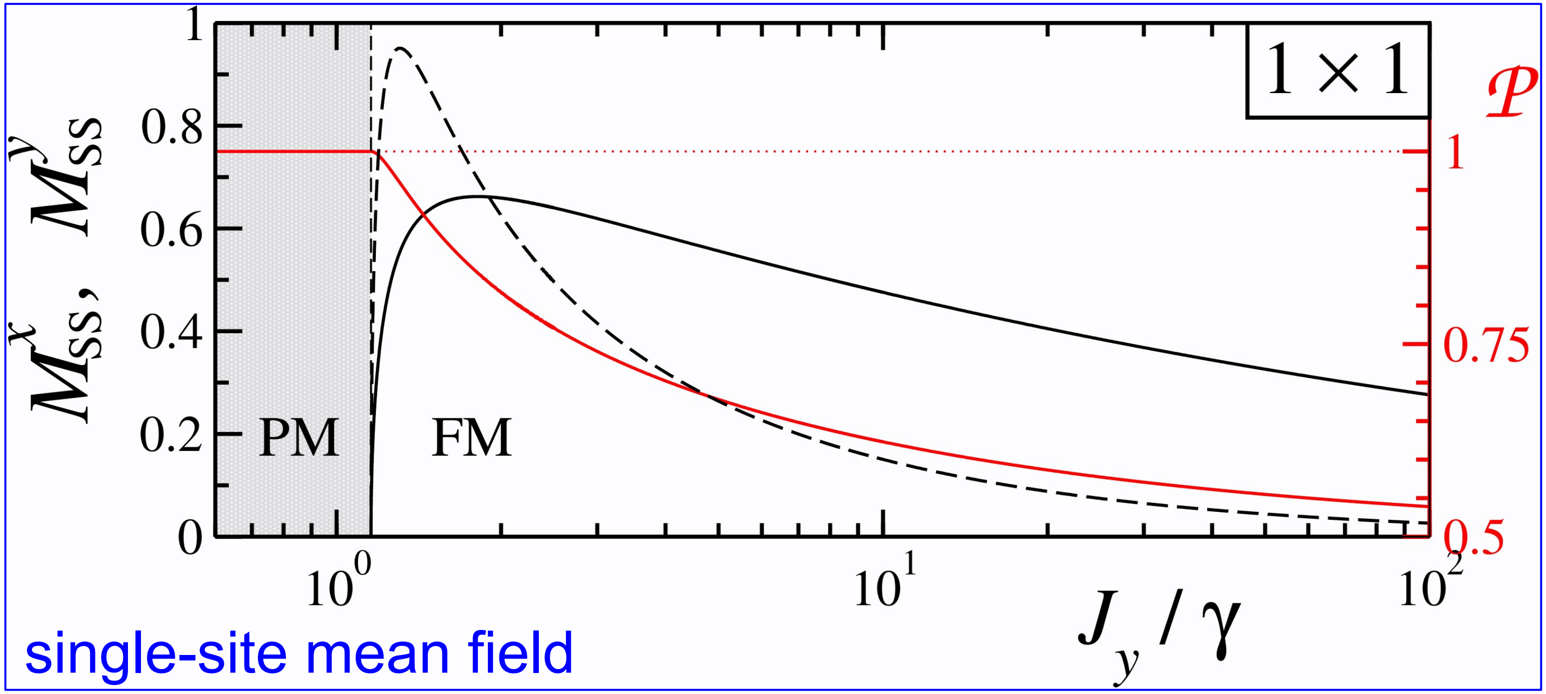
T.E. Lee, S. Gopalakrishnan, M.D. Lukin, *PRL* (2013)

Physical mechanism

importance of short-range correlations

J. Jin, A. Biella, O. Viyuela, L. Mazza, J. Keeling, R. Fazio, DR, *PRX* **6**, 031011 (2016)

The 1 x 1 mean-field

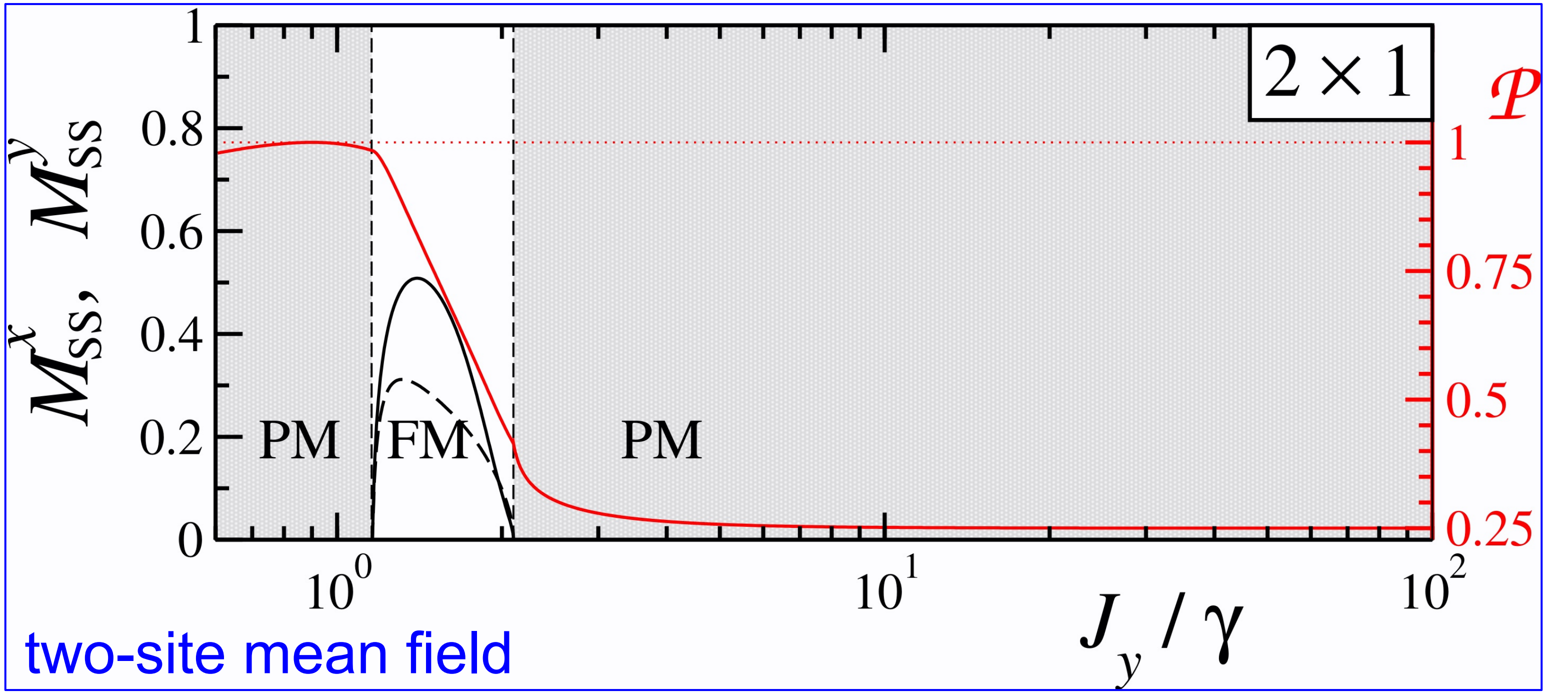


- the <u>ferromagnet</u> extends over a ***semi-infinite range of J_y***
- for $J_y \to +\infty$ the magnetization vanishes (PM)
- progressive deterioration of the purity $\mathcal{P} = \mathrm{Tr}[\rho^2_{\mathrm{SS}}]$:

PM @ $J_y < J_y^c$
pure state
(fully polarized along *z*)

PM @ $J_y \to +\infty$
fully mixed state
(unpolarized)

The 2 x 1 cluster mean-field



- the <u>ferromagnet</u> extends over a ***finite range of J_y***
- the PM at $J_y \to +\infty$ stabilizes over an <u>**extended region**</u>
- different nature of the two PM regions (purity):

PM @ $J_y < J_y^{c_1}$
nearly pure state
(fully polarized along *z*)

PM @ $J_y > J_y^{c_2}$
nearly fully mixed state
(unpolarized)

The 2 x 1 cluster mean-field @ large J_y

$$\partial_t \langle \sigma_j^\beta \rangle = -2 \sum_{\alpha=x,y,z} J_\alpha \epsilon_{\alpha\beta\gamma} \left[\langle \sigma_j^\gamma \rangle \langle \sigma_{j+1}^\alpha \rangle + \boxed{\langle \sigma_j^\gamma \sigma_{j+1}^\alpha \rangle} \right] - \frac{\gamma}{2} \left[\langle \sigma_j^\beta \rangle + \delta_{\beta z} (\langle \sigma_j^\beta \rangle + 2) \right]$$

coherent part

dissipative part

The steady state for $J_y > J_y^{c_2}$ is almost fully mixed: $\rho_{\text{SS}}^{[2\times1]} \approx \rho^{[1]} \otimes \rho^{[2]}$

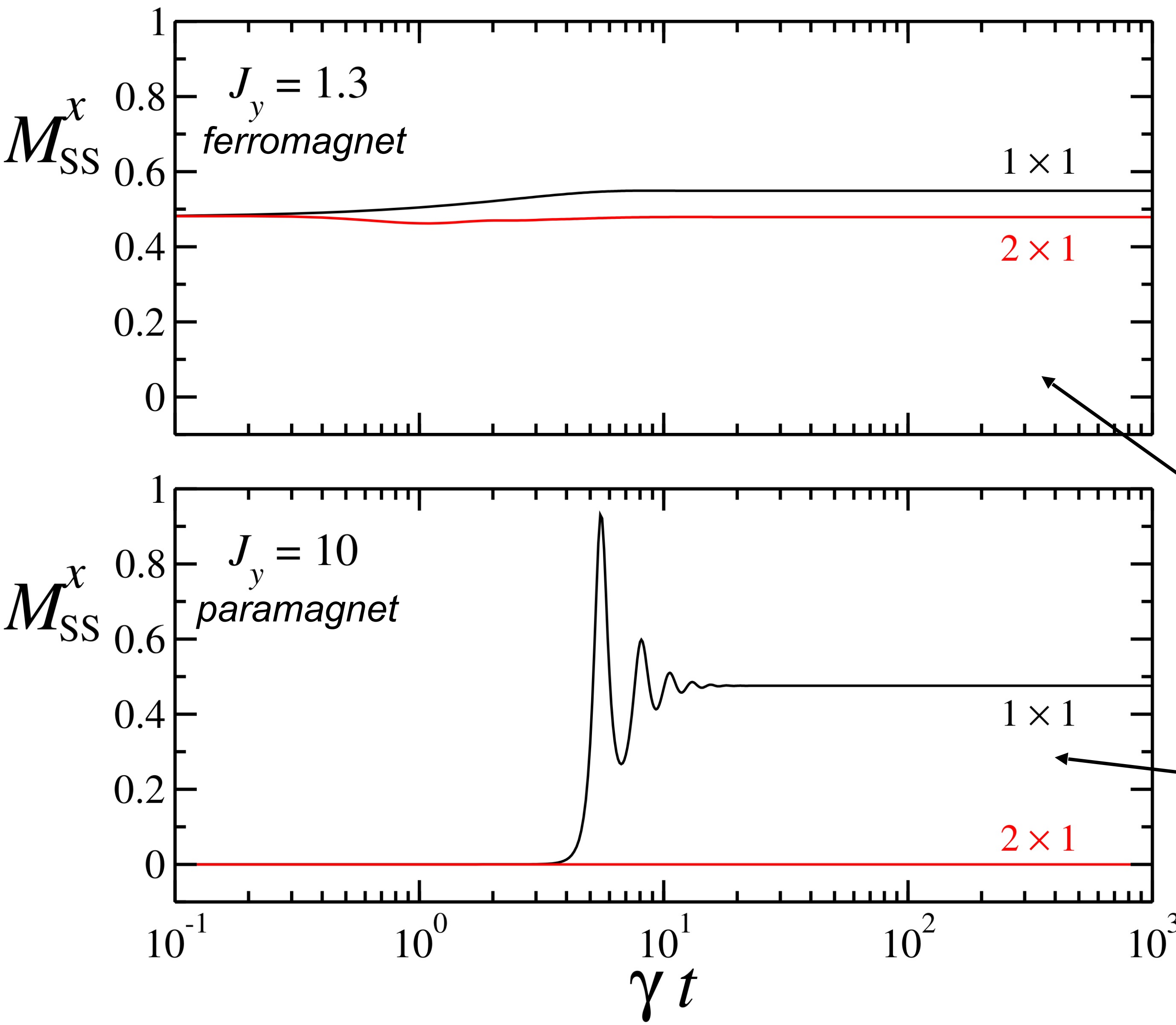
$$\Longrightarrow \langle \sigma_j^\gamma \sigma_{j+1}^\alpha \rangle = \langle \sigma_j^\gamma \rangle \langle \sigma_{j+1}^\alpha \rangle + \langle \Sigma_{j,j+1}^{\gamma,\alpha} \rangle \quad \text{where} \quad \underline{|\langle \Sigma_{j,j+1}^{\gamma,\alpha} \rangle| \ll 1}$$

therefore $\partial_t \langle \sigma_j^\beta \rangle = \mathcal{L}_{[1\times1]}^\beta \boxed{- 2 \sum_\alpha J_\alpha \epsilon_{\alpha\beta\gamma} \langle \Sigma_{j,j+1}^{\gamma,\alpha} \rangle}$

- correlations $\Sigma_{j,j+1}^{\gamma,\alpha}$ **drastically modify** the steady-state structure for $t \to +\infty$
- dynamically-induced **purity reduction**

→ ***dynamical suppression of ferromagnetic ordering***

The 2 x 1 cluster mean-field @ large J_y



$$\rho_{SS}^{[2\times 1]} \approx \rho^{[1]} \otimes \rho^{[2]}$$

Initial condition:

$$\rho_{\rm in}^{[1]} = {\rm Tr}_2[\rho_{SS}^{[2\times 1]}]$$

$$\rho_{\rm in}^{[2\times 1]} = \rho_{\rm in}^{[1]} \otimes \rho_{\rm in}^{[1]}$$

dynamics **does not** strongly affect the magnetization

dynamics **strongly** affects magnetization @ long time

instability in the initial condition

Results for different dimensionalities

One-dimensional geometry

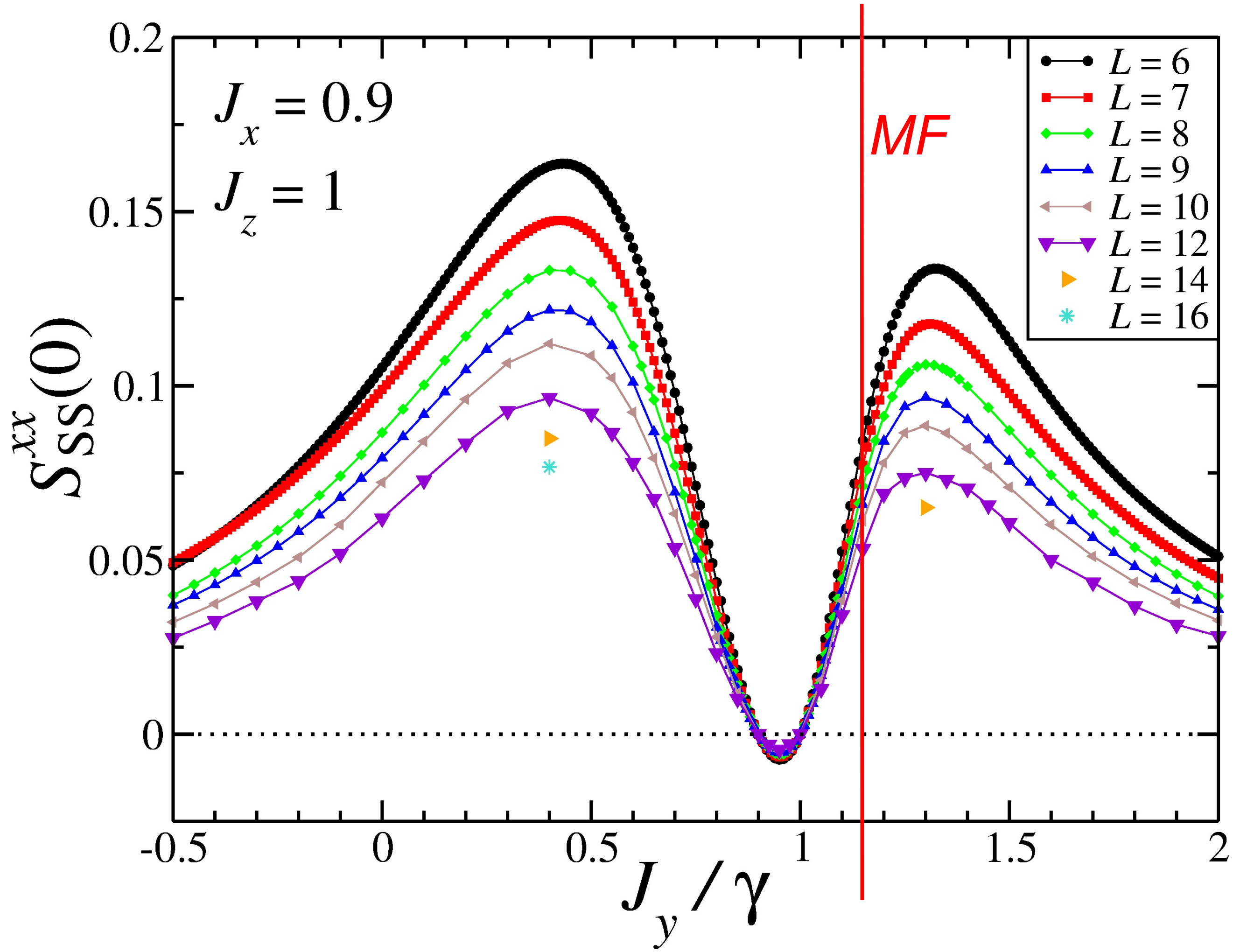
- Due to extremely reduced dimensionality, mean field should **fail**
- *Reminiscence* of features predicted by mean field
- *Presumably* **no phase transition**

One-dimensional geometry

Spin ordering signaled by the **structure factor**

$$S^{xx}_{\rm SS}(k) = \frac{1}{L^2}\sum_{j,l} e^{-ik(j-l)}\,\langle \sigma^x_j \sigma^x_l \rangle_{\rm SS}$$

($k = 0 \leftrightarrow$ FM $\qquad k = \pi \leftrightarrow$ AFM)

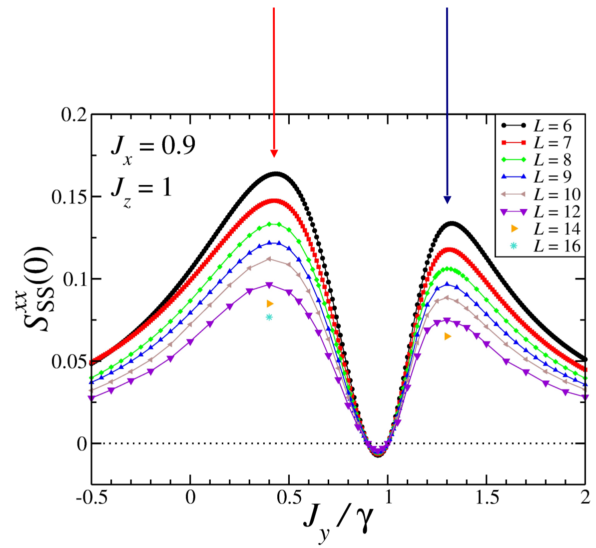


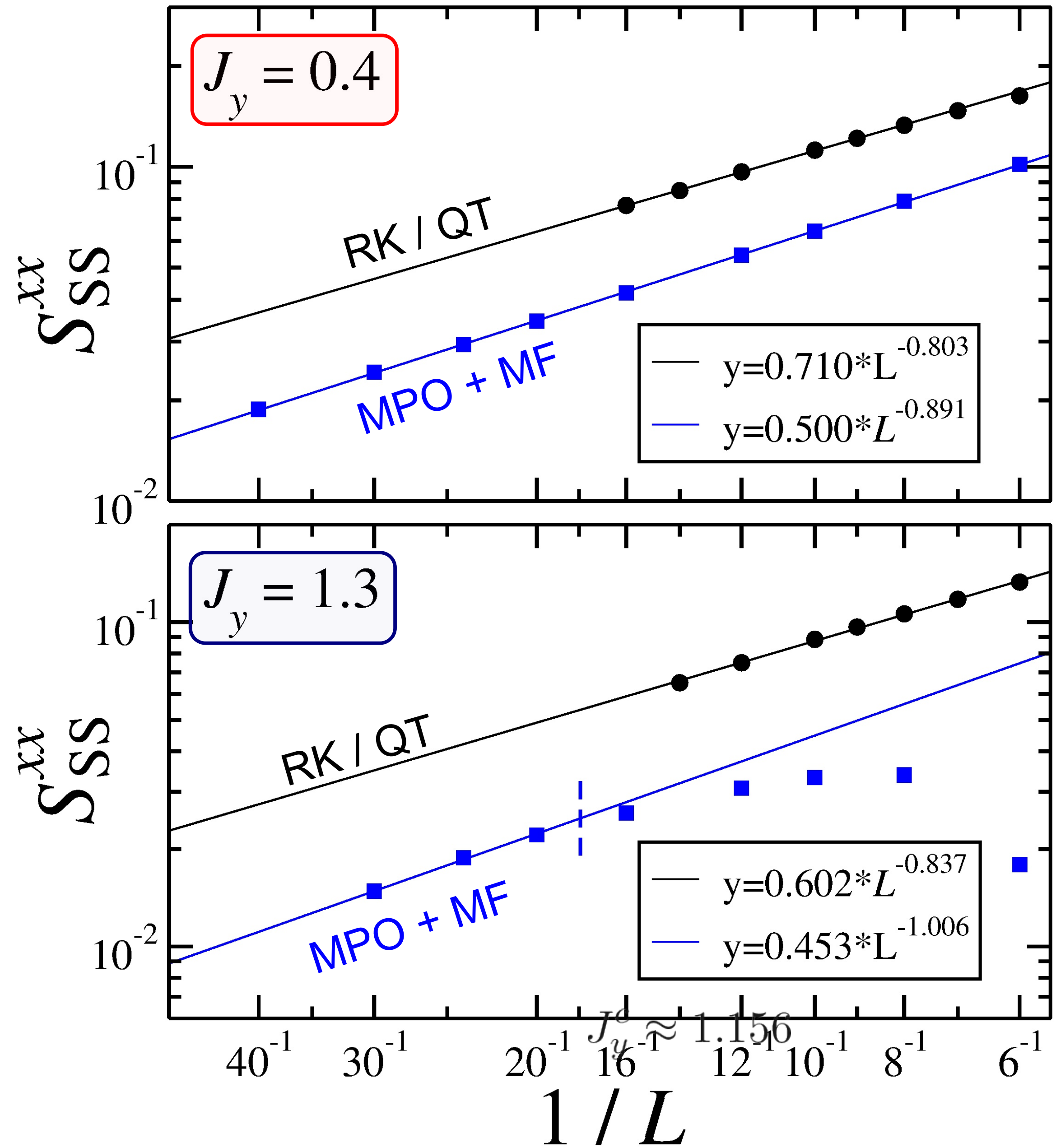
Mean field prediction:

PM–to–FM transition

$$\frac{J^c_y}{J_z} = \frac{37}{12} \approx 1.156$$

One-dimensional geometry



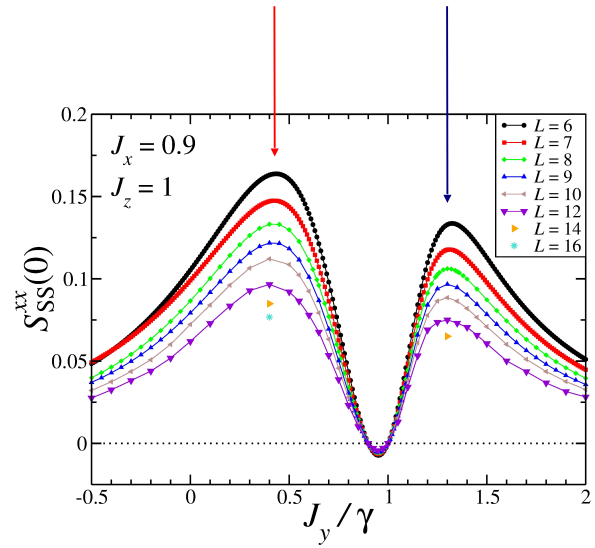


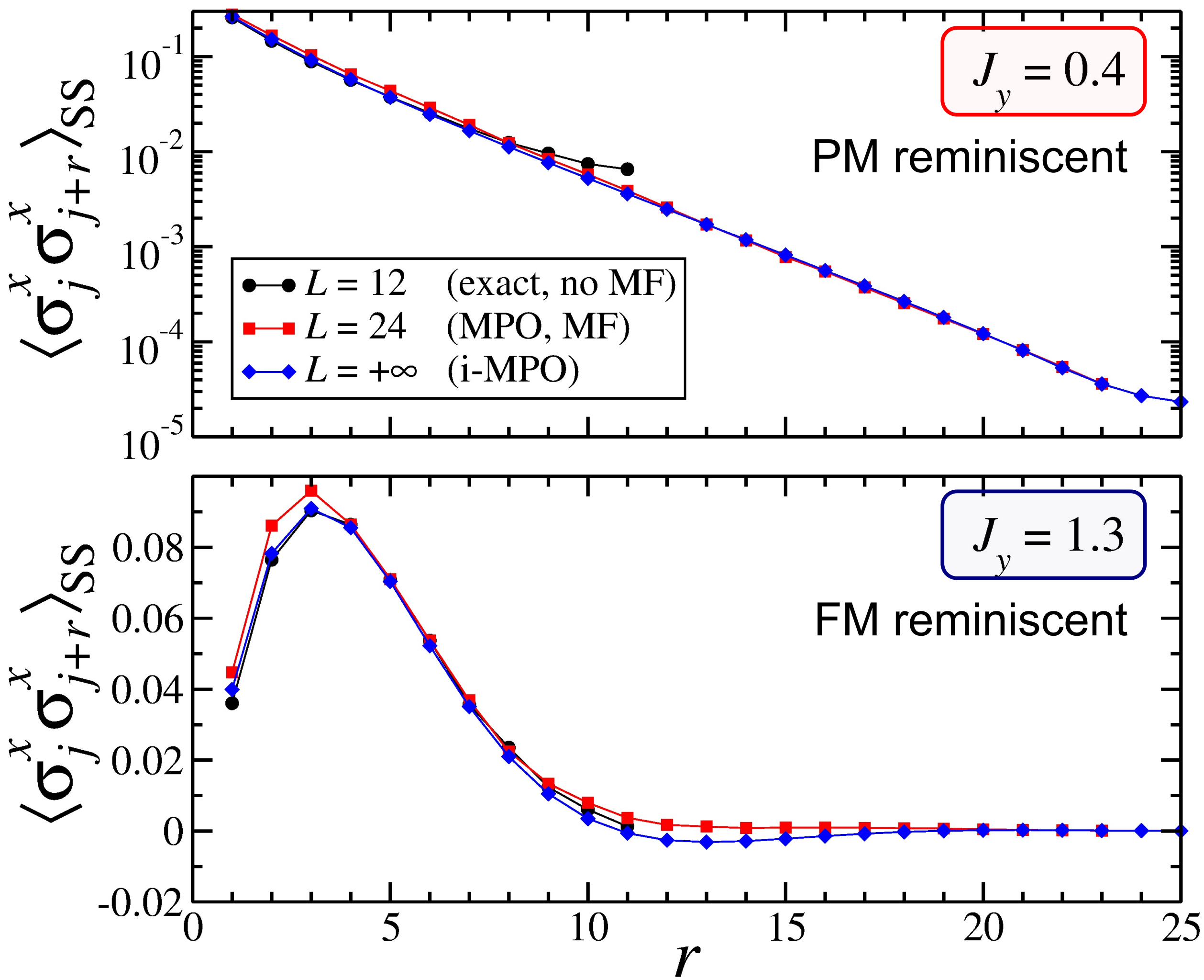
Finite-size scaling of numerical data

→ power-law decay

$$S^{xx}_{\rm SS}(0) \sim \kappa L^{-\gamma}$$

One-dimensional geometry





***xx* correlators**

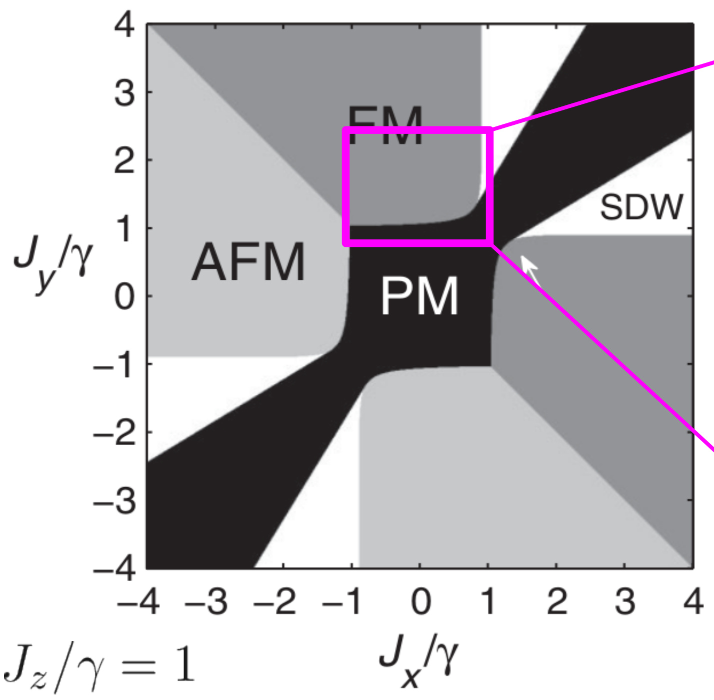
$J_y < J_y^c$
clearly exponential

$J_y > J_y^c$
exponential @ large r

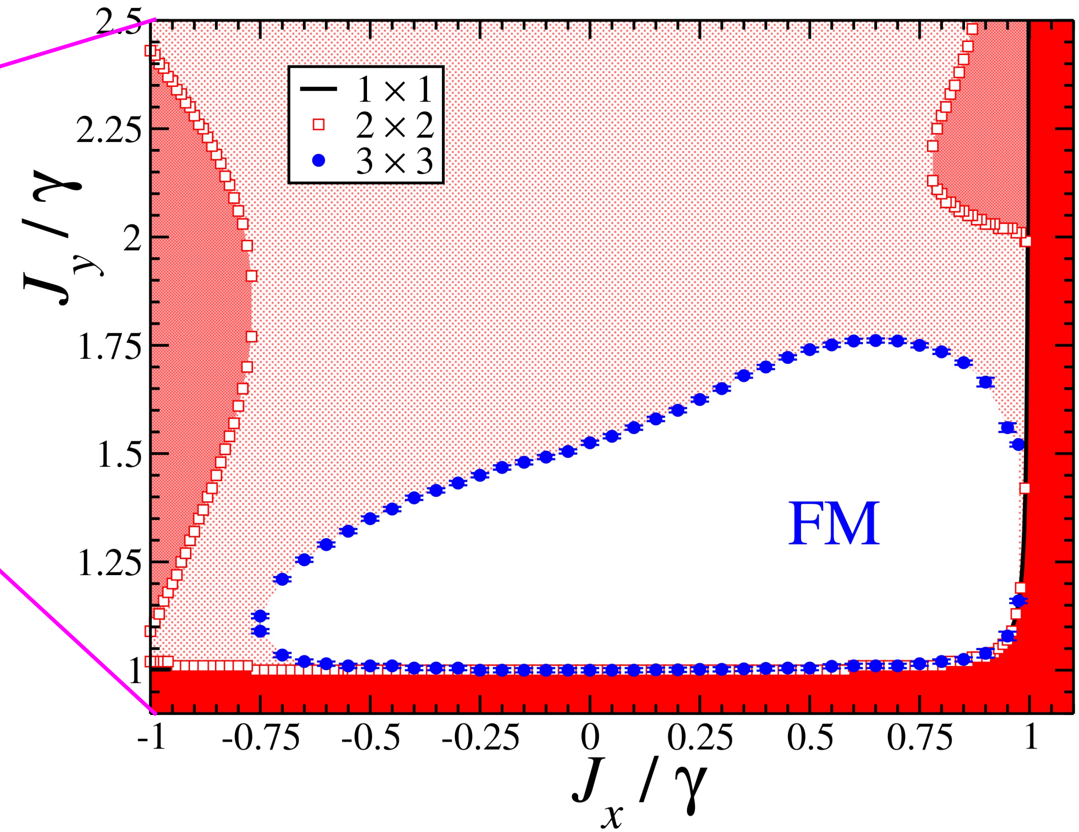
Two-dimensional geometry

- What is the **fate** of mean field?
- *Presumably* **there is a phase transition**
  (existence of a symmetry-broken phase)

Two-dimensional geometry



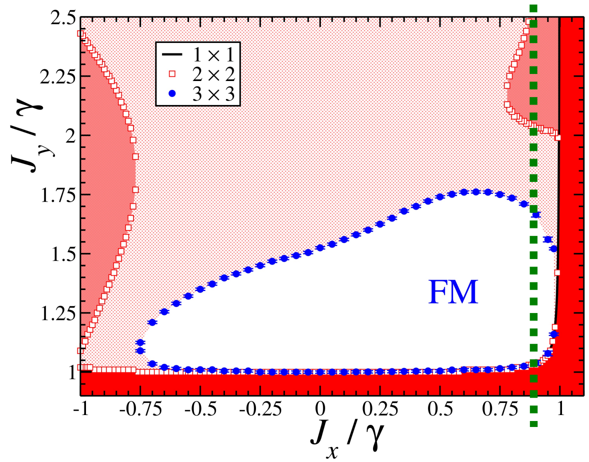
Mean-field phase diagram



Cluster mean-field on a 2D square lattice

- Extension of the **symmetry-broken phase** ***drastically reduced***
- **Boundaries & topology** of the phase diagram ***change a lot***

Two-dimensional geometry

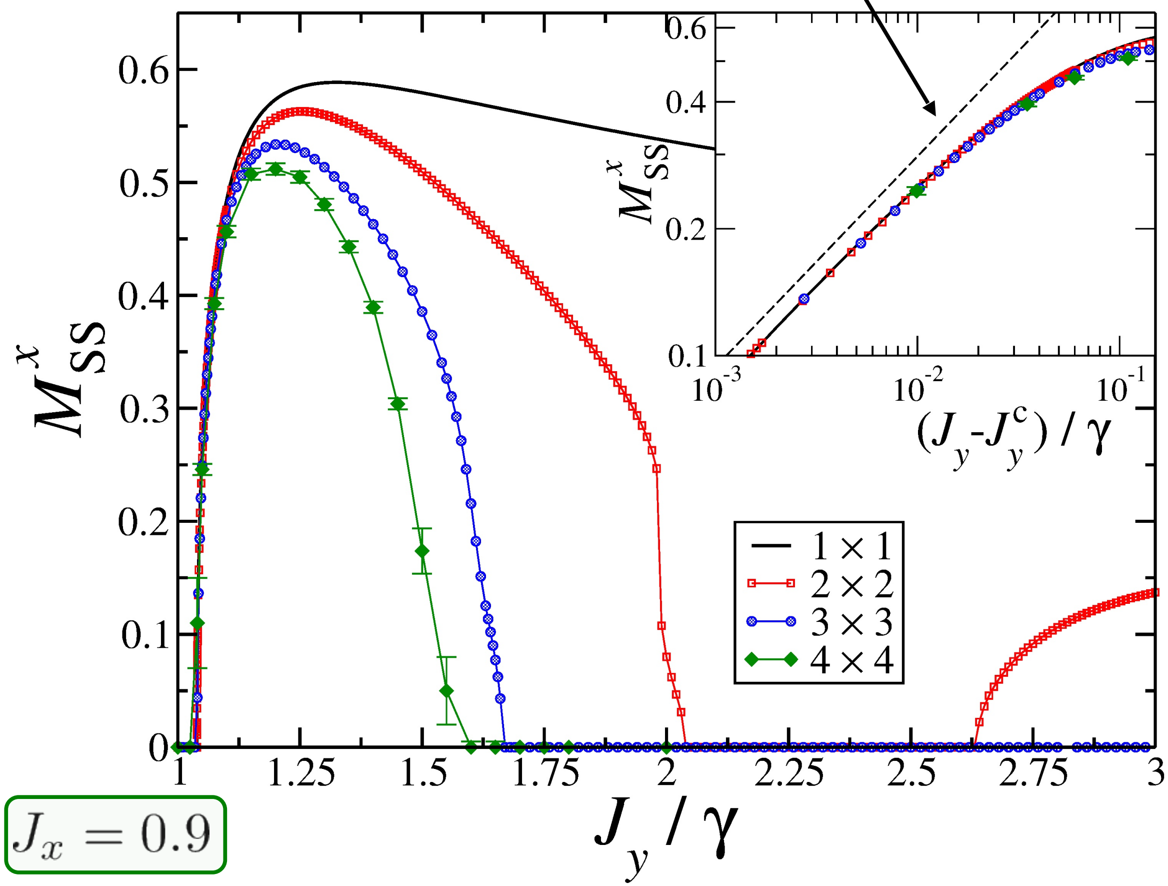


$$M^x_{\rm SS} = \frac{1}{\ell^2} \sum_{j=1}^{\ell^2} \langle \sigma^x_j \rangle_{\rm SS}$$

$$J_y^{(c)} \approx 1.03$$

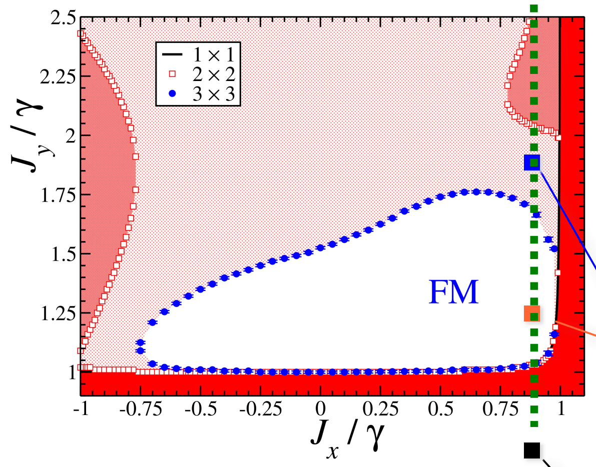
$J_x = 0.9$

$$M^x_{\rm SS} \sim m \sqrt{\frac{J_y - J_y^c}{J_y^c}}$$



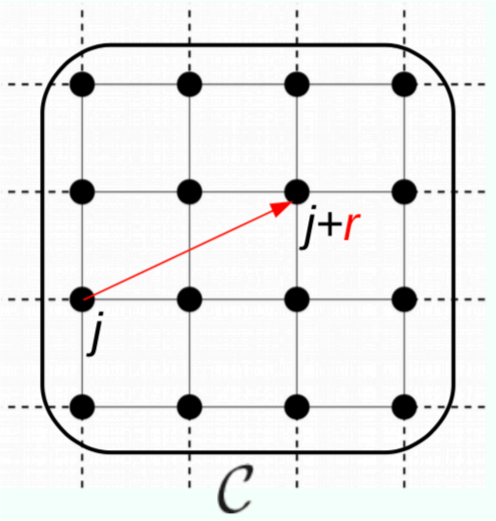
- FM phase seems to survive in the thermodynamic limit
- no revival @ large *J* observed for larger clusters

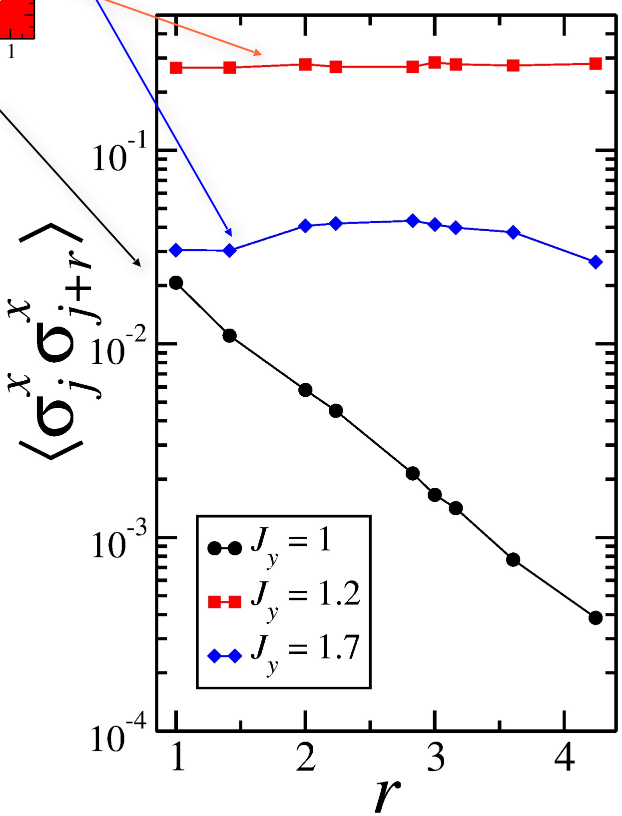
Two-dimensional geometry

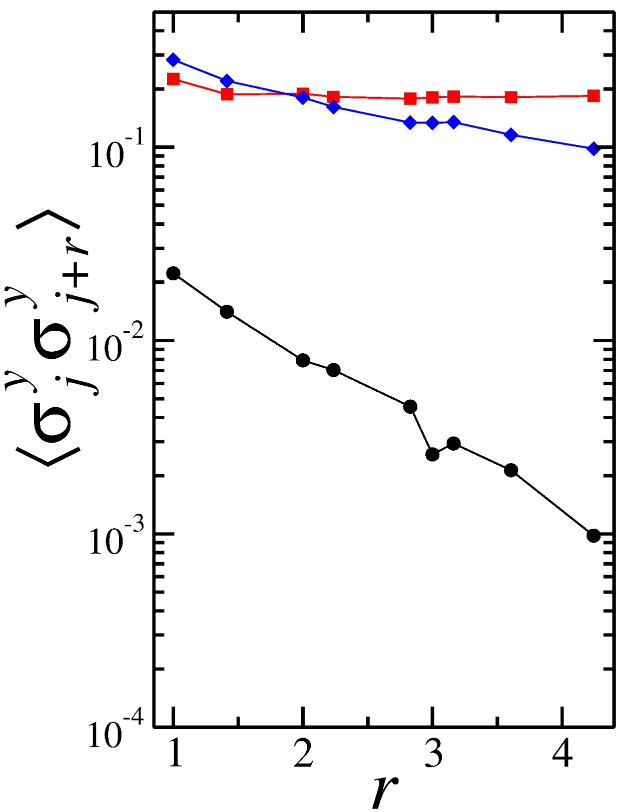


FM phase corroborated by correlation functions:

→ PM phase: exponential decay
→ FM phase: saturation
→ crossover region: unclear at small size







Corner-space renormalization

Susceptibility tensor:

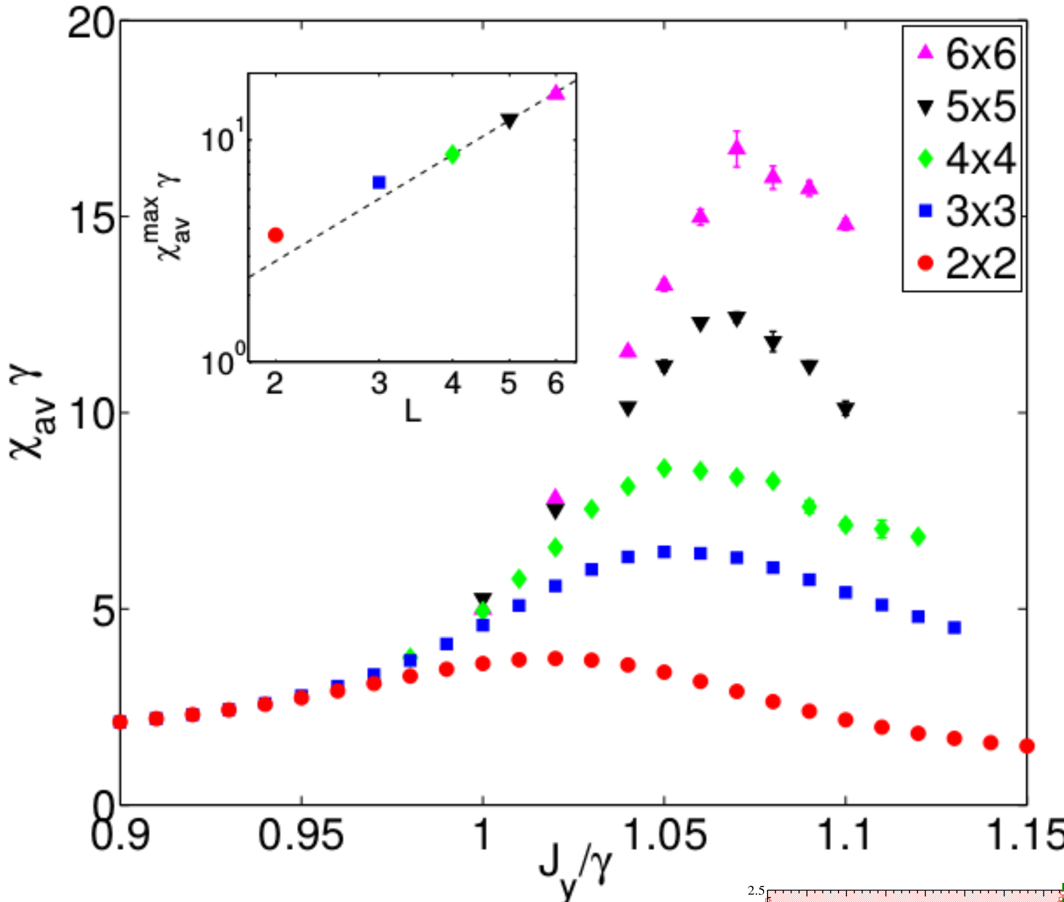
$$\chi_{\alpha\beta} = \left.\frac{\partial M_\alpha}{\partial h_\beta}\right|_{h\to 0}$$

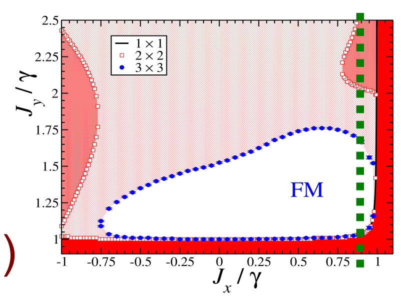
Angularly averaged susceptibility:

$$\chi_{\rm av} = \int d\theta \; \partial_h \underbrace{\left|\vec{M}(h,\theta)\right|_{h\to 0}}_{\left|\begin{pmatrix} \chi_{xx}\cos\theta + \chi_{xy}\sin\theta \\ \chi_{yx}\cos\theta + \chi_{yy}\sin\theta \end{pmatrix}\right|}$$

$$\chi_{\rm av}^{\rm max}(L) \propto L^{\kappa} \quad (\kappa \simeq 1.59)$$

$$J_y^{(c)} \approx 1.07$$





R. Rota, F. Storme, N. Bartolo, R. Fazio, C. Ciuti, PRB **95**, 134431 (2017)

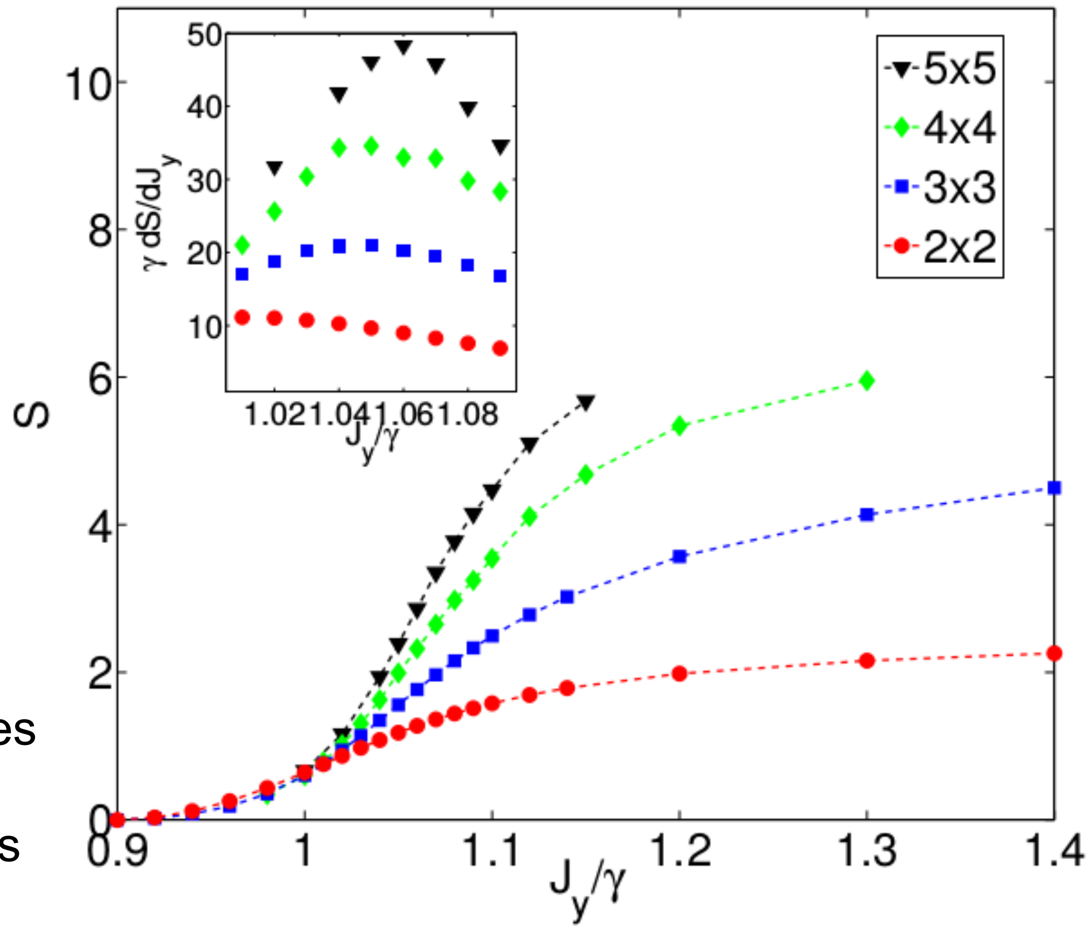
Corner-space renormalization

Von Neumann entropy:

$$S = -\mathrm{Tr}(\rho \log \rho)$$

$$\max\left(\frac{\partial S}{\partial J_y}\right) \propto L^{\lambda}$$

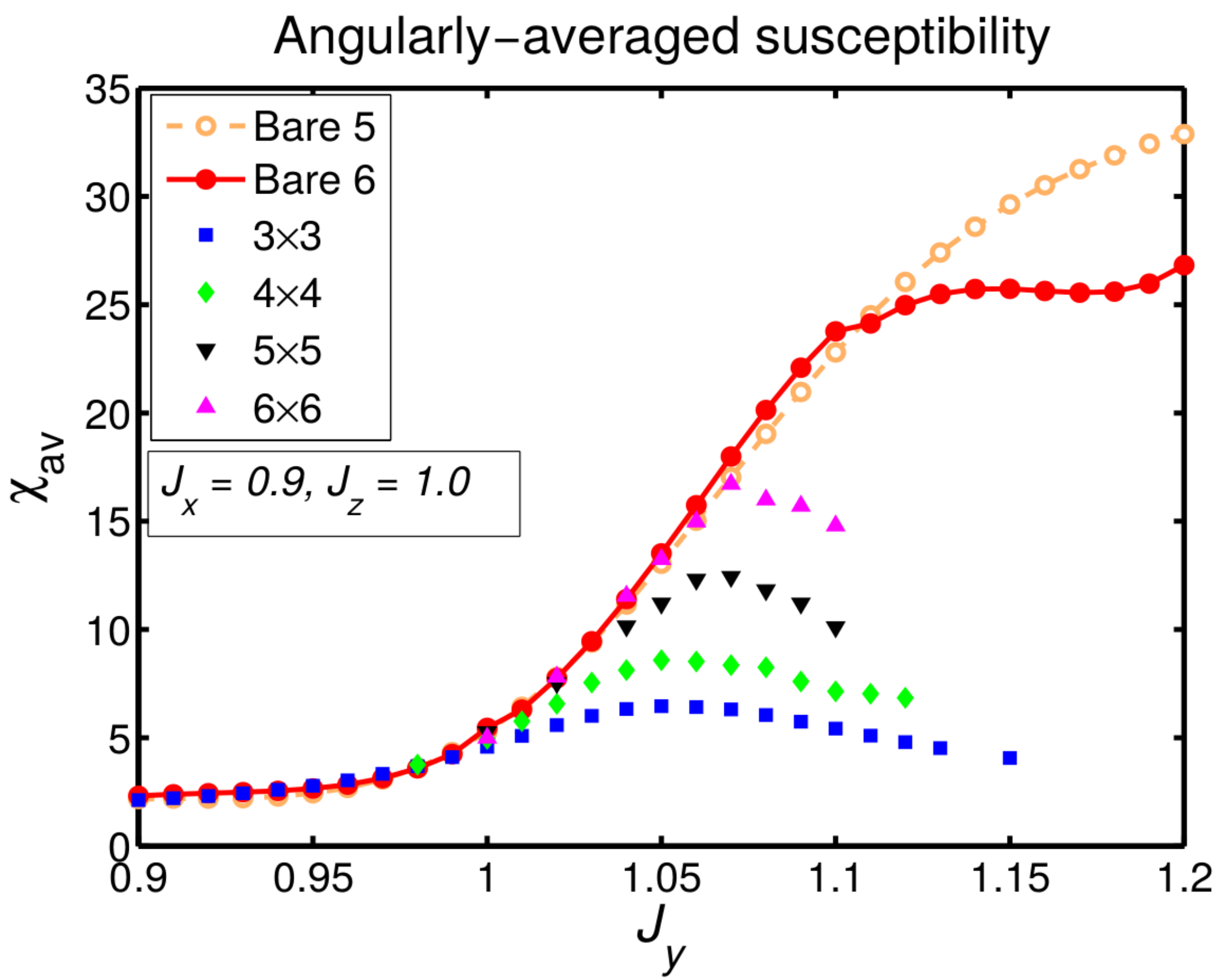
$$\lambda = 1.6 \pm 0.2$$



The behavior of the S vs. J_y resembles that of the entropy vs. temperature in 2nd order thermal phase transitions

Contrary to conventional transition (where the FM phase has a lower entropy), here the ferromagnetic phase has larger entropy than the paramagnetic one.

Numerical linked cluster expansions

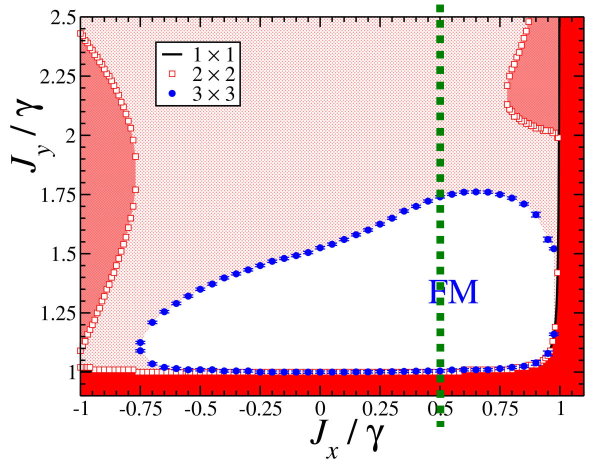


A. Biella, J. Jin, O. Viyuela, C. Ciuti, R. Fazio, DR, PRB **97** 035103 (2018)

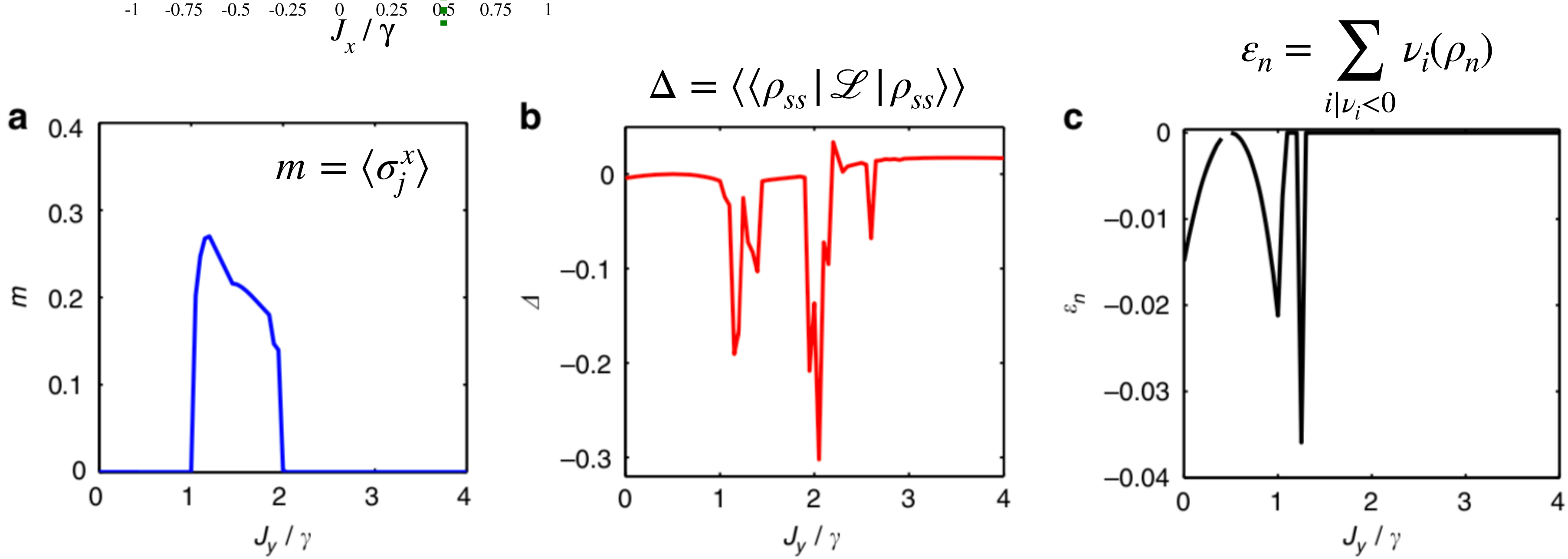
It is also possible to extrapolate the *critical exponent* associated to the phase transition → Padé analysis

$$\chi_{\rm av} \sim |J_y - J_y^{(\rm c)}|^{-\gamma}$$

Projected pair entangled operators (PEPOs)

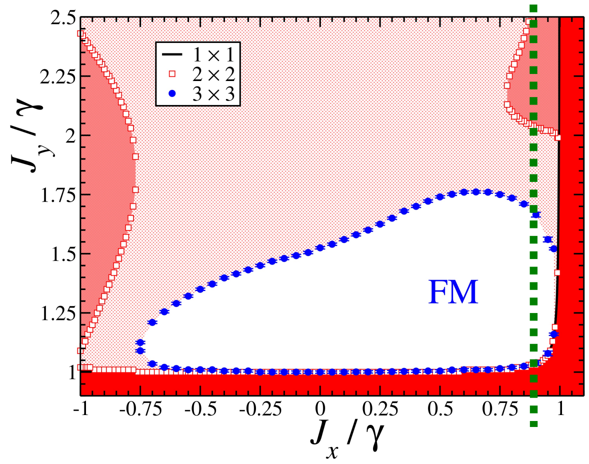


PEPO bond dimension: D = 4
(results consistent also with larger values of D)



A. Kshetrimayum, H. Weimer, R. Orus, Nat. Commun. **8**, 1291 (2017)

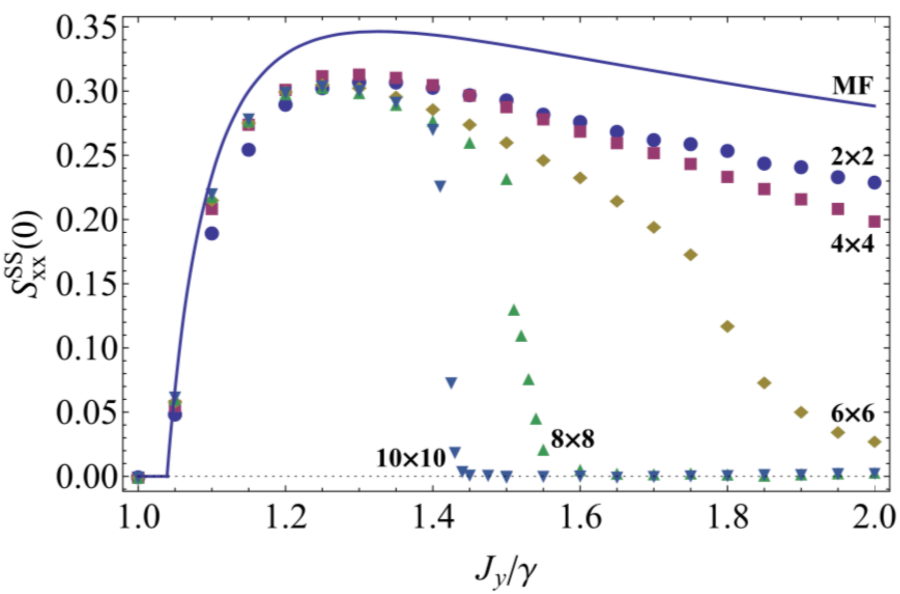
Gutzwiller MC approach

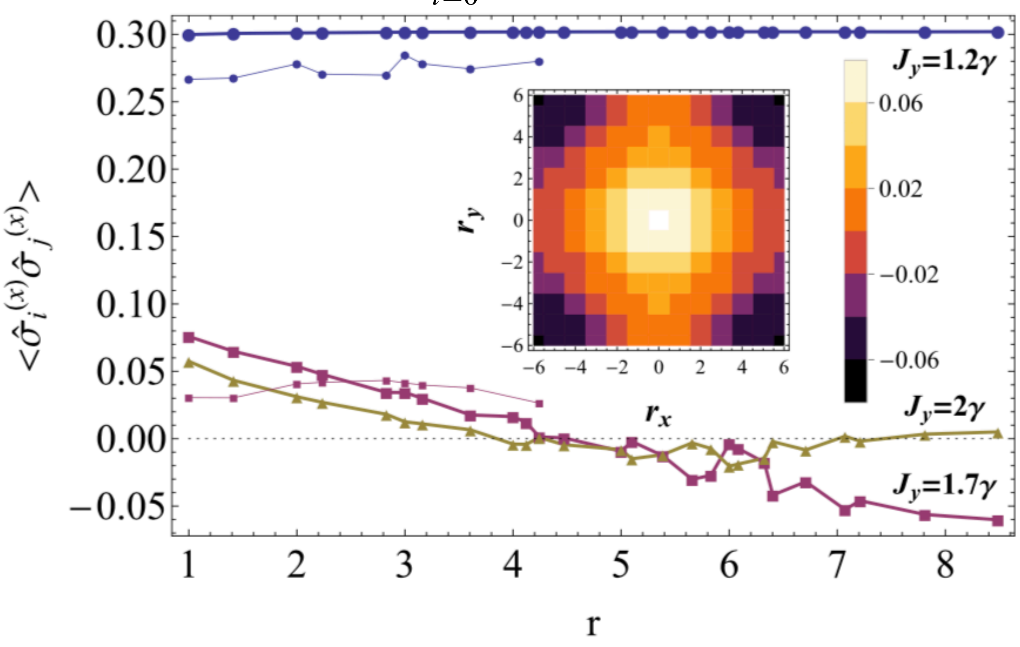


extensive mean-field Gutzwiller wave-function MC substantially agree with the previous picture

$$|\Psi_{GW}(\{\mathbf{i}\})\rangle = \prod_{\mathbf{i}} |\psi_{\mathbf{i}}\rangle$$

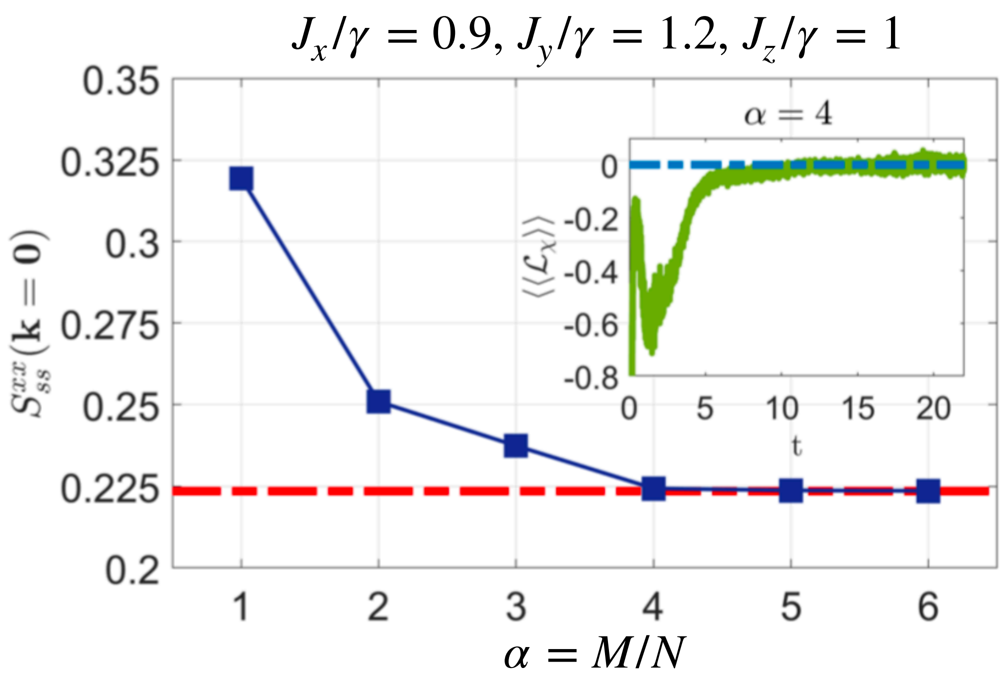
$$\langle \sigma_{\mathbf{i}}^{\alpha} \sigma_{\mathbf{j}}^{\beta} \rangle \sim \frac{1}{\mathscr{T}} \sum_{t=0}^{\mathscr{T}} \langle \psi_{\mathbf{i}}^{(k)} | \sigma_{\mathbf{i}}^{\alpha} | \psi_{\mathbf{i}}^{(k)} \rangle \, \langle \psi_{\mathbf{j}}^{(k)} | \sigma_{\mathbf{j}}^{\beta} | \psi_{\mathbf{j}}^{(k)} \rangle$$

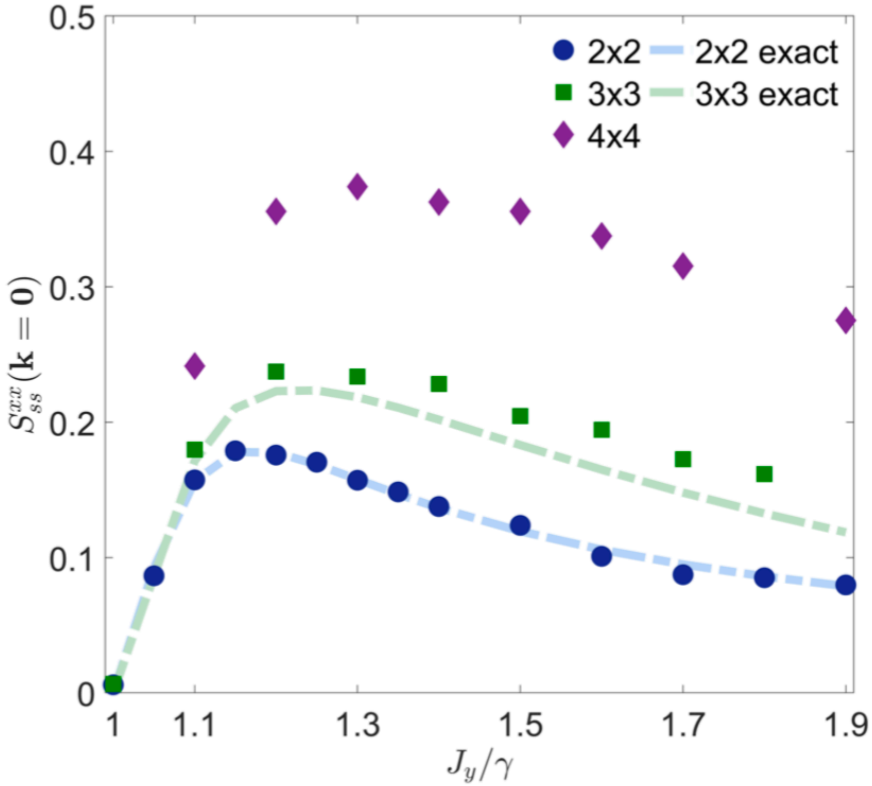




W. Casteels, R. M. Wilson, M. Wouters, PRA **97**, 062107 (2018)

Variational MC with neural networks





The large$-J_y$ behavior seems to be not compatible with a paramagnetic phase (at least for the small considered sizes)…

A. Nagy, V. Savona, PRL **122**, 250501 (2019)