# Gaussian variational Ansätze for many-body quantum dynamics

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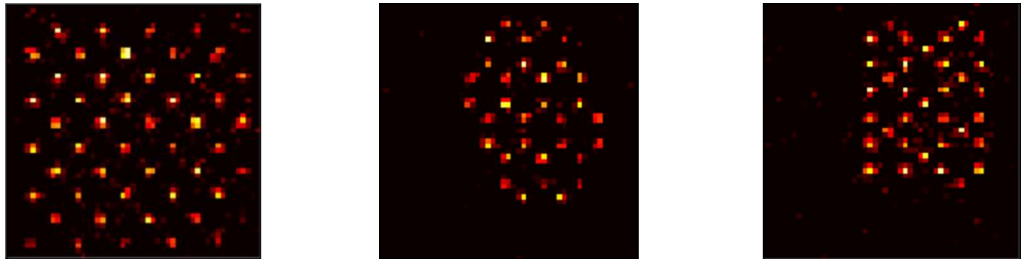
# Quantum spin systems : linear spin-wave theory



*D. Barredo & al. Science, **354** 6315 (2016)*

Rydberg simulators : tunable geometries, two levels systems ( $|g\rangle$, $|r\rangle$), Rydberg blockade

→ Mappable on spins 1/2

Spin-boson transformation

$$\hat{S}_i^z = S - \hat{b}_i^\dagger \hat{b}_i$$
$$\hat{S}_i^- = \hat{b}_i^\dagger \sqrt{2S - \hat{b}_i^\dagger \hat{b}_i}$$
$$\hat{S}_i^+ = \sqrt{2S - \hat{b}_i^\dagger \hat{b}_i}\ \hat{b}_i$$

$$\hat{\mathcal{H}} = -\frac{1}{2}\sum_{i,j} J_{ij} \hat{S}_i^z \hat{S}_j^z - \Omega \sum_i \hat{S}_i^x$$

Dilution hypothesis

$$\hat{b}_i^\dagger \hat{b}_i \ll 2S$$

$$\hat{\mathcal{H}} = E_{\mathrm{MF}} + \frac{1}{2}\sum_{i,j} \left(\hat{b}_i^\dagger \ \ \hat{b}_i\right) \mathcal{M}_{ij} \begin{pmatrix} \hat{b}_j \\ \hat{b}_j^\dagger \end{pmatrix} + O\left(b^3\right)$$

Linear spin-wave theory : **integrable** but restricted by dilution (small quenches)

# Bosonic Gaussian Ansatz

LSW theory provides a description in terms of **Gaussian state**

$$\hat{\rho} = \frac{1}{\mathcal{Z}} \exp\left[ -\frac{1}{2} \sum_{i,j} \left( \hat{b}_i^\dagger \mathcal{A}_{ij} \hat{b}_j + \hat{b}_i \mathcal{B}_{ij} \hat{b}_j + \mathrm{h.c.} \right) - \sum_i \left( \mathcal{C}_i \hat{b}_i + \mathrm{h.c.} \right) \right]$$

which enables the use of **Wick's theorem** : all boils down to two-points correlators and mean-field

$$c_i = \langle \hat{b}_i \rangle \qquad G_{ij} = \langle \hat{\beta}_i^\dagger \hat{\beta}_j \rangle \qquad F_{ij} = \langle \hat{\beta}_i \hat{\beta}_j \rangle \qquad \text{where} \quad \hat{\beta}_i = \hat{b}_i - c_i$$

**We assume that the state remains Gaussian beyond the linear description of spin-waves**

**Wick**

$$\frac{\mathrm{d}}{\mathrm{d}t} c_i = i \langle [\hat{\mathcal{H}}, \hat{b}_i] \rangle = \mathcal{J}_i(\{c_i\}, \{G_{ij}\}, \{F_{ij}\})$$

$$\frac{\mathrm{d}}{\mathrm{d}t} G_{ij} = i \langle [\hat{\mathcal{H}}, \hat{\beta}_i^\dagger \hat{\beta}_j] \rangle = \mathcal{G}_{ij}(\{c_i\}, \{G_{ij}\}, \{F_{ij}\})$$

$$\frac{\mathrm{d}}{\mathrm{d}t} F_{ij} = i \langle [\hat{\mathcal{H}}, \hat{\beta}_i \hat{\beta}_j] \rangle = \mathcal{F}_{ij}(\{c_i\}, \{G_{ij}\}, \{F_{ij}\})$$

The evolution of two-points correlators is governed by non-linear ordinary differential equations

# Transverse-field Ising chain

Benchmarking with the transverse-field ising chain

$$\hat{\mathcal{H}} = -J\sum_i \hat{S}_i^x \hat{S}_{i+1}^x - \Omega \sum_i \hat{S}_i^z$$

Initiated in the state

$$|\Psi_0\rangle = \bigotimes_i |\rightarrow\rangle_i$$

Indeed the transverse-field Ising chain :

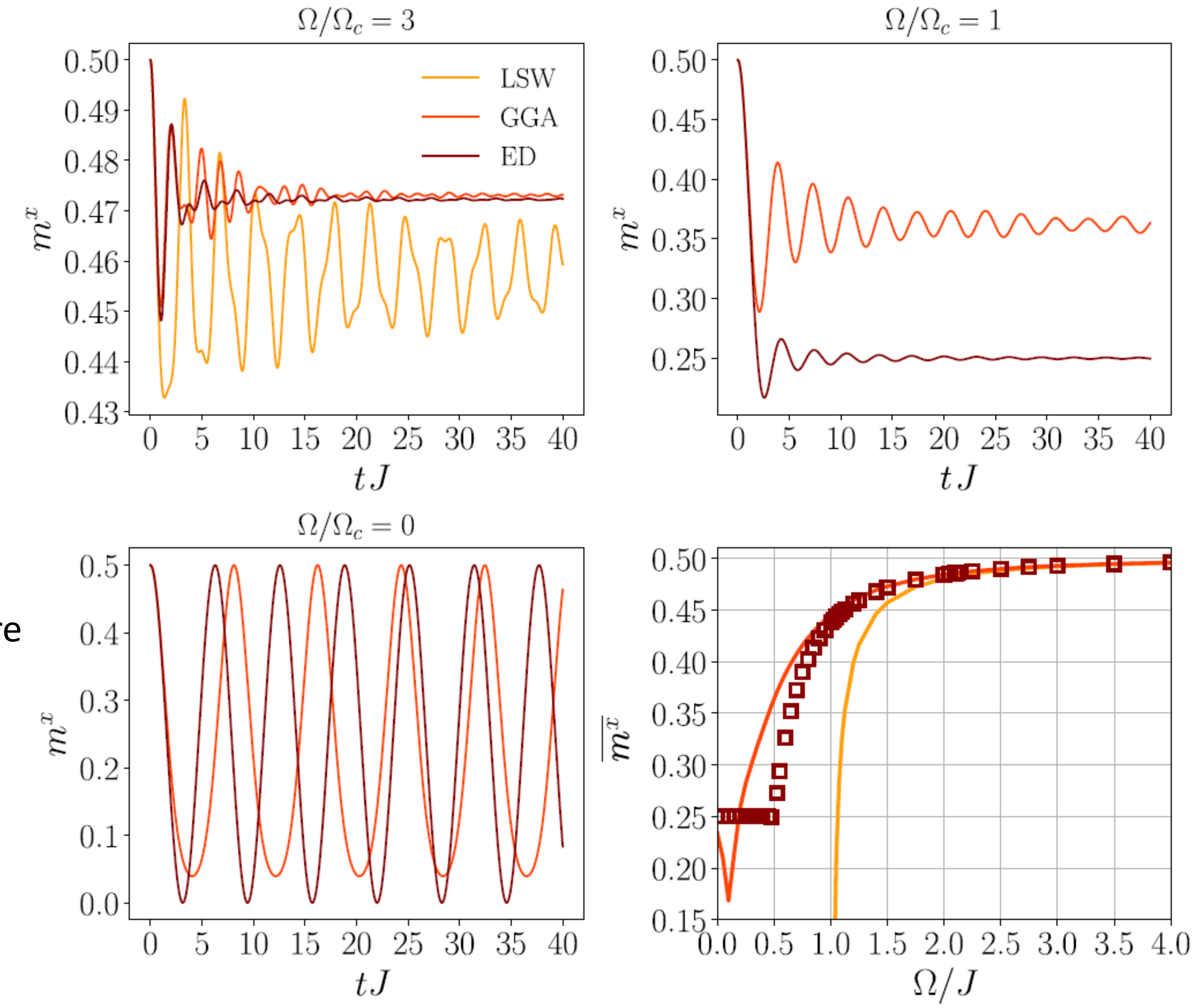
- is integrable (free fermions image)
- Is strongly governed by interactions (non-linearities)

Transverse-magnetization and its time-average are

$$m^x = \frac{1}{N}\sum_i \langle \hat{S}_i^x \rangle$$

$$\overline{m^x} = \frac{1}{T}\int_0^T m^x(t)\mathrm{d}t$$

Numerical instabilities at low fields



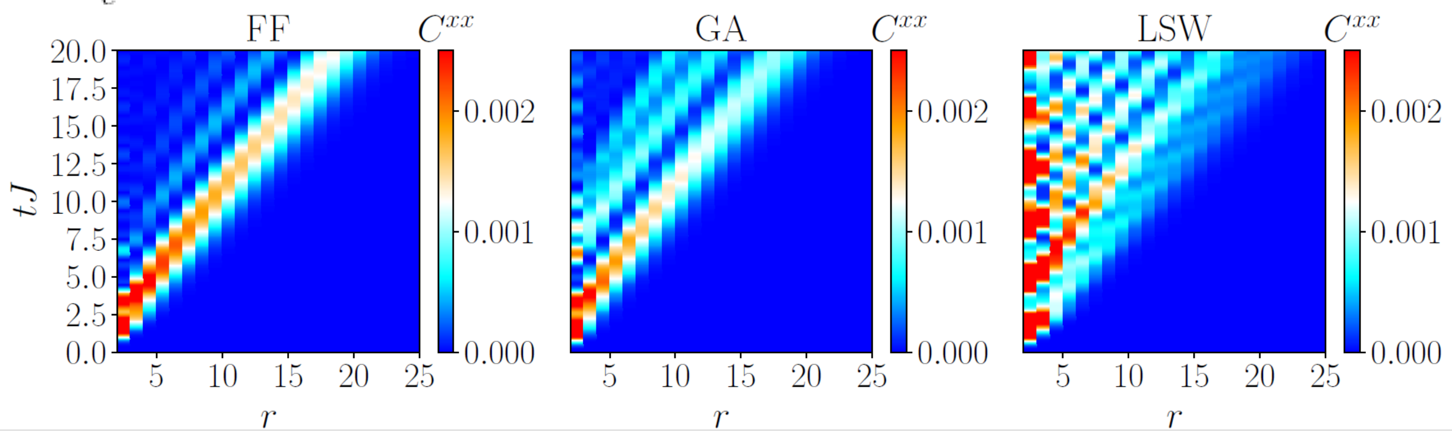
# Correlations and entanglement

The off-diagonal elements of the covariance are also reproduced

$$C^{xx}(|r_i - r_j|, t) = \langle \hat{S}_i^x \hat{S}_j^x \rangle - \langle \hat{S}_i^x \rangle \langle \hat{S}_j^x \rangle$$

$$S_{eq}^{xx}(k) = \frac{1}{T} \int_0^T \frac{\mathrm{d}t}{N} \sum_{i,j} e^{ik(r_i - r_j)} C^{xx}(r_i - r_j, t)$$

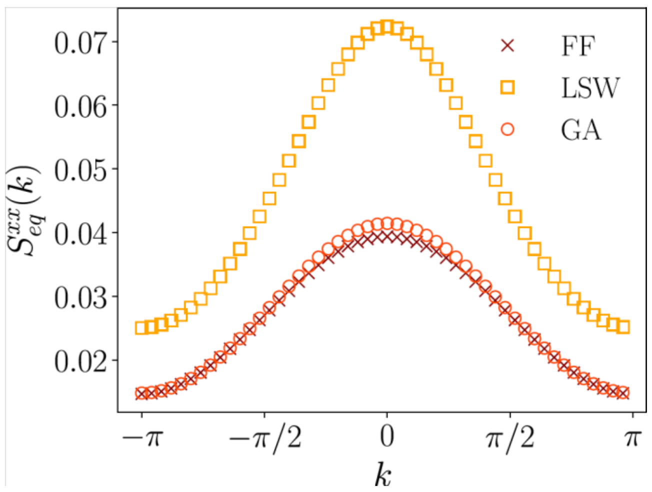
$\Omega = 3\Omega_c$

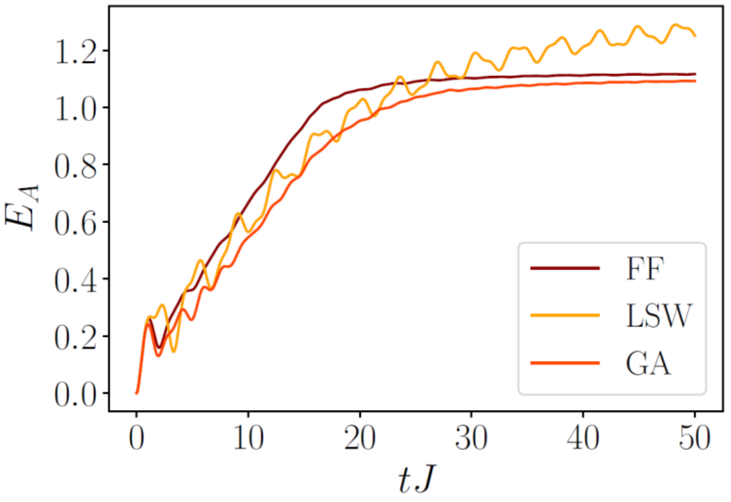



The Gaussian state also gives a straighforward access to **entanglement entropy**

$$\begin{pmatrix} -1 - G^* & F \\ -F^* & G \end{pmatrix} = U \begin{pmatrix} \mathrm{diag}(-1 - n_\alpha) & 0 \\ 0 & \mathrm{diag}(n_\alpha) \end{pmatrix} U^{-1}$$

$$E_A = \sum_\alpha \left[ (1 + n_\alpha) \ln(1 + n_\alpha) - n_\alpha \ln n_\alpha \right]$$



# Conclusions and outlook

Conclusions:

- Gaussian Ansatz reproduces accurately the dynamics of an integrable model
- It also provides a description of its equilibration (prethermalization)
- Presence of numerical instabilities for extreme quenches

Perspectives:

- Extension of the Gaussian Ansatz to fermionic systems
- Example: study of the ETH-MBL dynamical phase transition

$$\hat{\mathcal{H}} = -t\sum_i \left(\hat{f}_i^\dagger \hat{f}_{i+1} + \mathrm{h.c.}\right) + U\sum_i \hat{n}_i\hat{n}_{i+1} + \sum_i W_i \hat{n}_i$$